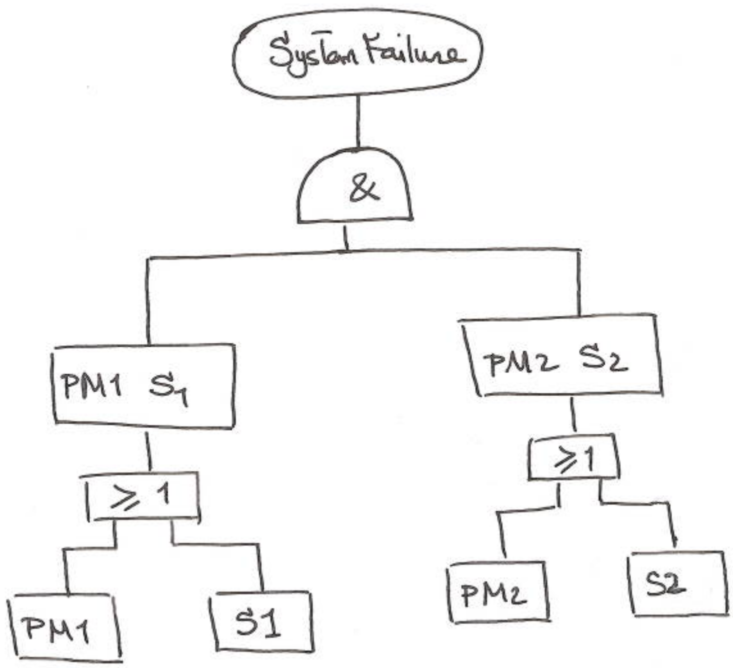
1.a) There are <u>four</u> error containment regions

1. PM1 + S1
2. PM2 + S2
3. Communication Bus 1
4. Communication Bus 2

PM1 and S1 are in series with each other so as are PM2 and S2. Failure of PM1/S_1 results in Sensor 1/PM1 to be unusable. The same is valid for PM2 and S2.

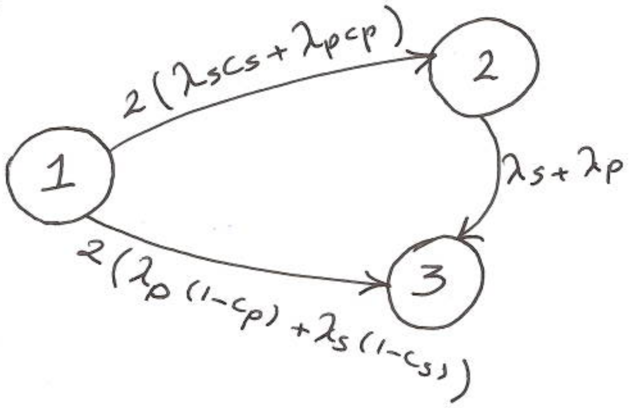
Communication Buses are working in active redundancy and are in parallel with each other. So they should be in separate error containment regions.

1.b)



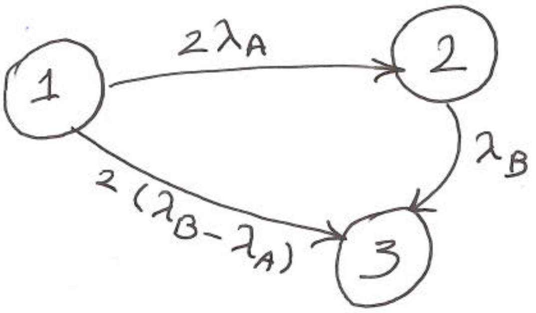
Fault tree for the FTU

1.c)



$$\lambda_A = \lambda_S \cdot c_S + \lambda_P c_P$$

$$\lambda_B = \lambda_S + \lambda_P$$



Markov Model for FTU

$$Q = \begin{bmatrix} -2\lambda_B & 2\lambda_A & 2(\lambda_B - \lambda_A) \\ 0 & -\lambda_B & \lambda_B \\ 0 & 0 & 0 \end{bmatrix}$$

Transition rate Matrix

$$P'(t) = P(t) \cdot Q$$

$$P(0) = [1 \quad 0 \quad 0]$$

$$\mathcal{L}\{f'(t)\} = s F(s) - f(0)$$

$$s P(s) - P(0) = P(s) \cdot Q$$

$$\begin{cases} sP_1(s) - 1 = -2\lambda_B P_1(s) & (1) \\ sP_2(s) - 0 = 2\lambda_A P_1(s) - \lambda_B P_2(s) & (2) \\ sP_3(s) - 0 = 2(\lambda_B - \lambda_A) P_1(s) + \lambda_B P_2(s) & (3) \end{cases}$$

$$(1): \quad P_1(s) = \frac{1}{s + 2\lambda_B} \xrightarrow{\mathcal{L}^{-1}} P_1(t) = e^{-2\lambda_B t}$$

$$(2),(1): \quad (s + \lambda_B) P_2(s) = 2\lambda_A \cdot \frac{1}{s + 2\lambda_B}$$

$$P_2(s) = 2\lambda_A \cdot \frac{1}{s + 2\lambda_B} \cdot \frac{1}{s + \lambda_B}$$

1.c)
cont

$$P_2(s) = \frac{2\lambda_A}{\lambda_B}\left(\frac{1}{s+\lambda_B} - \frac{1}{s+2\lambda_B}\right)$$

$$\mathcal{L}^{-1}\left[P_2(s)\right] = \frac{2\lambda_A}{\lambda_B}\left(e^{-\lambda_B t} - e^{-2\lambda_B t}\right)$$

$$R_{FTU}(t) = P_1(t) + P_2(t)$$

$$= e^{-2\lambda_B t} + \frac{2\lambda_A}{\lambda_B}\left(e^{-\lambda_B t} - e^{-2\lambda_B t}\right)$$

$$= \frac{2\lambda_A}{\lambda_B} e^{-\lambda_B t} + \frac{\lambda_B - 2\lambda_A}{\lambda_B} e^{-2\lambda_B t}$$

$$\lambda_A = \lambda_s c_s + \lambda_p \cdot c_p$$

$$\lambda_B = \lambda_s + \lambda_p$$

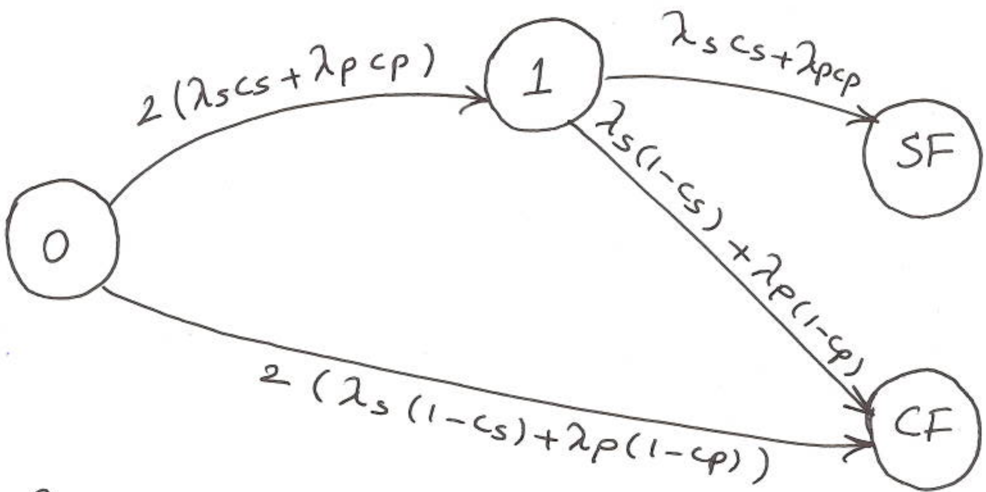
$$R_{FTU}(t) =$$

$$\frac{2(\lambda_s c_s + \lambda_p c_p)}{\lambda_s + \lambda_p} e^{-(\lambda_s+\lambda_p)t} +$$

$$\frac{\lambda_s + \lambda_p - 2(\lambda_s c_s + \lambda_p c_p)}{\lambda_s + \lambda_p} e^{-2(\lambda_s+\lambda_p)t}$$

$$R_{FTU}(t) = \frac{2\lambda_s c_s + 2\lambda_p c_p}{\lambda_s + \lambda_p} e^{-(\lambda_s+\lambda_p)t} + \frac{\lambda_s(1-2c_s) + \lambda_p(1-2c_p)}{\lambda_s + \lambda_p} e^{-2(\lambda_s+\lambda_p)t}$$

1.d)



$$\lambda_A = \lambda_s c_s + \lambda_p c_p$$

$$\lambda_B = \lambda_s + \lambda_p$$

$$S(\infty) = \text{Prob}\,(S_0 \rightarrow S_1) * \text{Prob}\,(S_1 \rightarrow S_{SF})$$

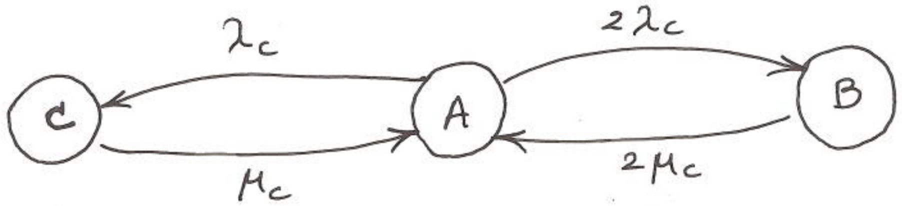
$$= \frac{2\lambda_A}{2\lambda_A + 2(\lambda_B - \lambda_A)} * \frac{\lambda_A}{\lambda_B - \lambda_A + \lambda_A}$$

$$= \frac{\lambda_A}{\lambda_B} * \frac{\lambda_A}{\lambda_B} = \frac{\lambda_A^2}{\lambda_B^2}$$

$$= \frac{(\lambda_s c_s + \lambda_p c_p)^2}{(\lambda_s + \lambda_p)^2}$$

$$= \frac{\lambda_s^2 c_s^2 + \lambda_p^2 c_p^2 + 2\lambda_s c_s \lambda_p c_p}{\lambda_s^2 + \lambda_p^2 + 2\lambda_s \lambda_p}$$

2.a)



Markov Model of the control unit

$$\pi_A = \frac{\mu_c}{\lambda_c}\pi_c \qquad (1)$$

$$\pi_B = \frac{2\lambda_c}{2\mu_c}\pi_A \Rightarrow \pi_B = \frac{\lambda_c}{\mu_c}\cdot\frac{\mu_c}{\lambda_c}\pi_c \qquad (2)$$

$$\sum_i \pi_i = 1 \quad : \pi_A + \pi_B + \pi_c = 1 \qquad (3)$$

(2) $\pi_B = \pi_c$

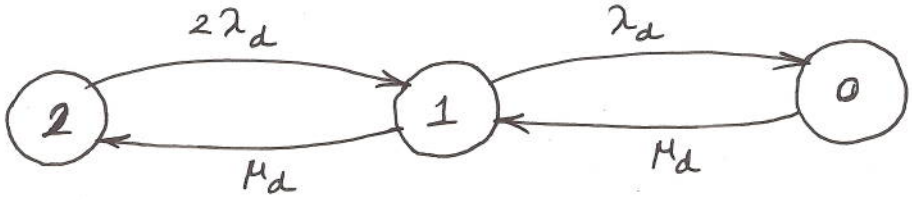
(3)(2) $\pi_A + \pi_B + \pi_c = \left(\frac{\mu_c}{\lambda_c} + 2\right)\pi_B = 1$

$$\pi_B \left(\frac{2\lambda_c + \mu_c}{\lambda_c}\right) = 1 \Rightarrow \pi_B = \frac{\lambda_c}{2\lambda_c + \mu_c} = \pi_c$$

(1) $\pi_A = \frac{\mu_c}{\lambda_c}\cdot\frac{\lambda_c}{2\lambda_c + \mu_c} = \frac{\mu_c}{2\lambda_c + \mu_c}$

$$A_{\infty\,(\text{control unit})} = \pi_A = \frac{\mu_c}{2\lambda_c + \mu_c}$$

2.b)



$$\Pi_1 = \frac{2\lambda_d}{\mu_d}\Pi_2 \qquad (1)$$

$$\Pi_0 = \frac{\lambda_d}{\mu_d}\Pi_1 \qquad (2)$$

(1)(2) $$\Pi_0 = \frac{\lambda_d}{\mu_d}\cdot\frac{2\lambda_d}{\mu_d}\Pi_2 = \frac{2\lambda_d^2}{\mu_d^2}\Pi_2 \qquad (3)$$

$$\sum_{i=0}^{2}\Pi_i = 1 \qquad (4)$$

(1)(3)(4) $$\Pi_0 + \Pi_1 + \Pi_2 = 1$$

$$\Pi_2\left(\frac{2\lambda_d^2}{\mu_d^2} + \frac{2\lambda_d}{\mu_d} + 1\right) = 1$$

$$\Pi_2\left(\frac{2\lambda_d^2 + 2\lambda_d\mu_d + \mu_d^2}{\mu_d^2}\right) = 1$$

$$\Pi_2 = \frac{\mu_d^2}{2\lambda_d^2 + 2\lambda_d\mu_d + \mu_d^2} \qquad (5)$$

(1)(5) $$\Pi_1 = \frac{2\lambda_d}{\mu_d}\cdot\frac{\mu_d^2}{2\lambda_d^2 + 2\lambda_d\mu_d + \mu_d^2}$$

$$= \frac{2\lambda_d\mu_d}{2\lambda_d^2 + 2\lambda_d\mu_d + \mu_d^2}$$

$$A_\infty(\text{disks}) = \Pi_1 + \Pi_2 = \frac{2\lambda_d\mu_d + \mu_d^2}{2\lambda_d^2 + 2\lambda_d\mu_d + \mu_d^2}$$

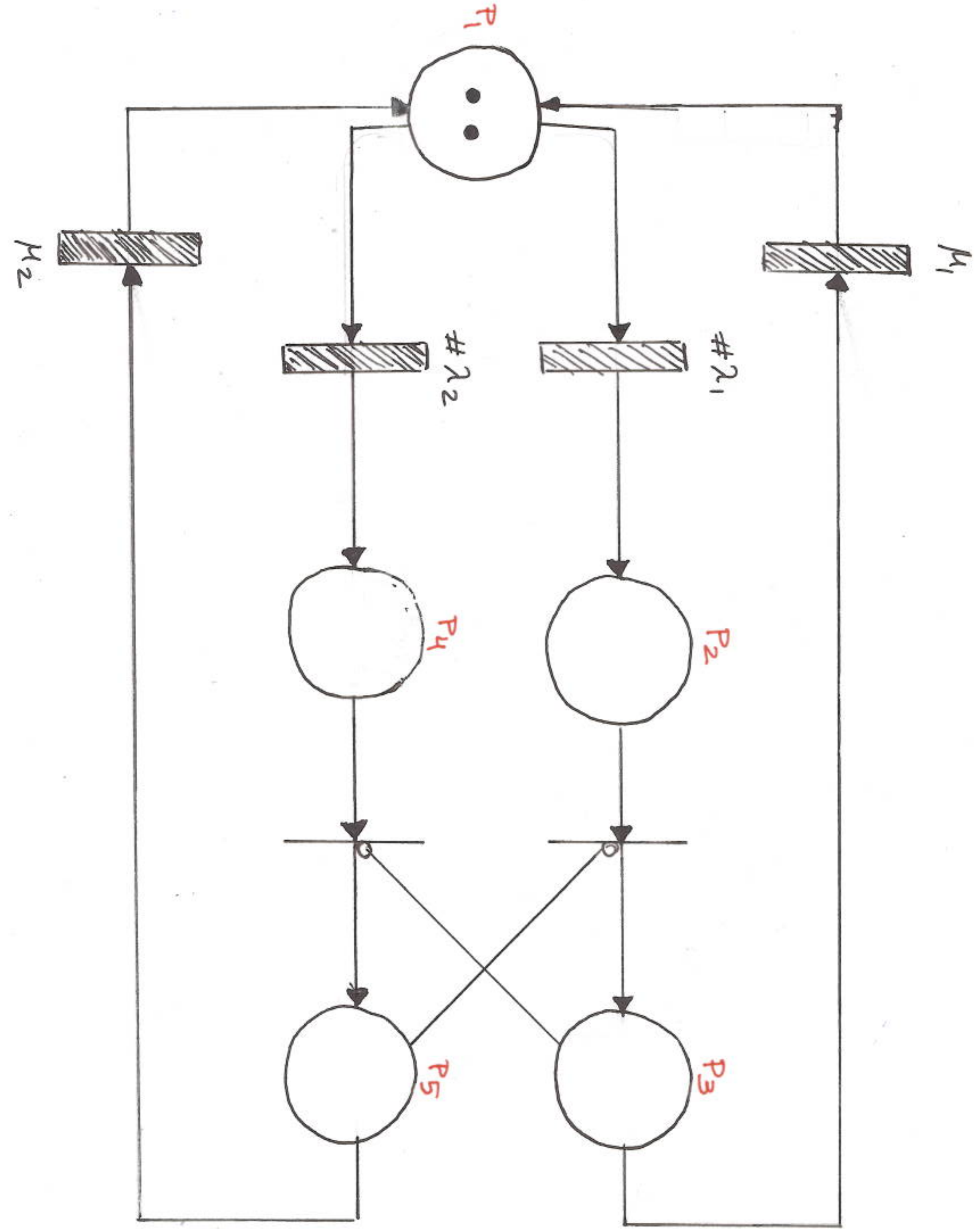
2.c)

$$A_\infty(\text{File server}) = A_\infty(\text{disks}) * A_\infty(\text{control unit})$$

$$A_\infty(\text{File server}) = \frac{2\lambda_d\mu_d + \mu_d^2}{\mu_d^2 + 2\lambda_d\mu_d + 2\lambda_d^2} * \frac{\mu_c}{\mu_c + 2\lambda_c}$$

$$= \frac{2\lambda_d\mu_d\mu_c + \mu_d^2\mu_c}{\mu_d^2\mu_c + 2\lambda_d\mu_d\mu_c + 2\lambda_d^2\mu_c + 2\lambda_c\mu_d^2 + 4\lambda_d\lambda_c\mu_d + 4\lambda_d^2\lambda_c}$$

3.a) GSPN



3.b) Extended Reachability Graph