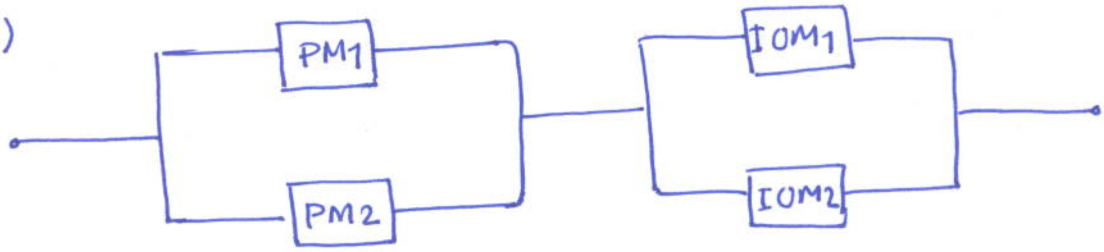
1.a) There are 8 fault containment regions:

1. PM_1 2. PM_2 3. IOM 1 4. IOM 2

5. $\text{Parallel}_{\text{Bus}}\,1$ 6. $\text{Parallel}_{\text{Bus}}\,2$

7. Serial Bus 1 8. Serial Bus 2

1.b)



$$R_{node}(t) = R_{pm}(t) \cdot R_{IO}(t)$$

$$R_{PM} = e^{-\lambda_1 t}$$

$$R_{PM_{1,2}} = 1 - (1 - R_{PM})^2$$
$$= 1 - (1 - e^{-\lambda_1 t})^2$$
$$= 2e^{-\lambda_1 t} - e^{-2\lambda_1 t}$$

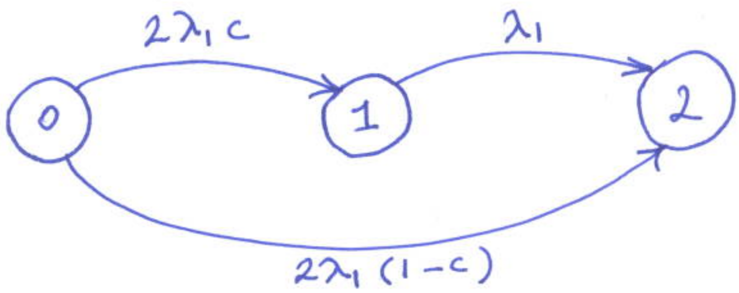
$$R_{IOM} = e^{-\lambda_2 t}$$

$$R_{I/O_{1,2}} = 1 - (1 - R_{IOM})^2$$
$$= 1 - (1 - e^{-\lambda_2 t})^2$$
$$= 2e^{-\lambda_2 t} - e^{-2\lambda_2 t}$$

$$R_{node}(t) = R_{PM_{1,2}}(t) * R_{IOM_{1,2}}(t)$$

$$= (2e^{-\lambda_2 t} - e^{-2\lambda_2 t}) *$$
$$(2e^{-\lambda_1 t} - e^{-2\lambda_1 t})$$

1.c)



Transition Matrix : $\begin{bmatrix} -2\lambda_1 & 2\lambda_1 c & 2\lambda_1(1-c) \\ 0 & -\lambda_1 & \lambda_1 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{cases} P_0'(t) = -2\lambda_1 P_0(t) \\ P_1'(t) = 2\lambda_1 c P_0(t) - \lambda_1 P_1(t) \\ P_2'(t) = 2\lambda_1(1-c) P_0(t) + \lambda_1 P_1(t) \end{cases}$$

$$sP_0(s) - 1 = -2\lambda_1 P_0(s)$$

$$\rightarrow P_0(s) = \frac{1}{s+2\lambda_1} \xrightarrow{\mathcal{L}^{-1}} P_0(t) = e^{-2\lambda_1 t}$$

$$sP_1(s) - 0 = 2\lambda_1 c P_0(s) - \lambda_1 P_1(s)$$

$$\rightarrow P_1(s) = \frac{1}{s+\lambda_1} \cdot \frac{1}{s+2\lambda_2} (2\lambda_1 c)$$

$$\xrightarrow{\mathcal{L}^{-1}} P_1(t) = 2c\,(e^{-\lambda_1 t} - e^{-2\lambda_1 t})$$

$$R(t) = P_0(t) + P_1(t)$$

$$= e^{-2\lambda_1 t} + 2c\,(e^{-\lambda_1 t} - e^{-2\lambda_1 t})$$

1.c)
cont.

$$R(t) = 2c\, e^{-\lambda_1 t} + (1-2c)\, e^{-2\lambda_1 t}$$

$$MTTF = \int_0^\infty R(t)\, dt$$

$$= \int_0^\infty \left(2c\, e^{-\lambda_1 t} + (1-2c)\, e^{-2\lambda_1 t}\right) dt$$

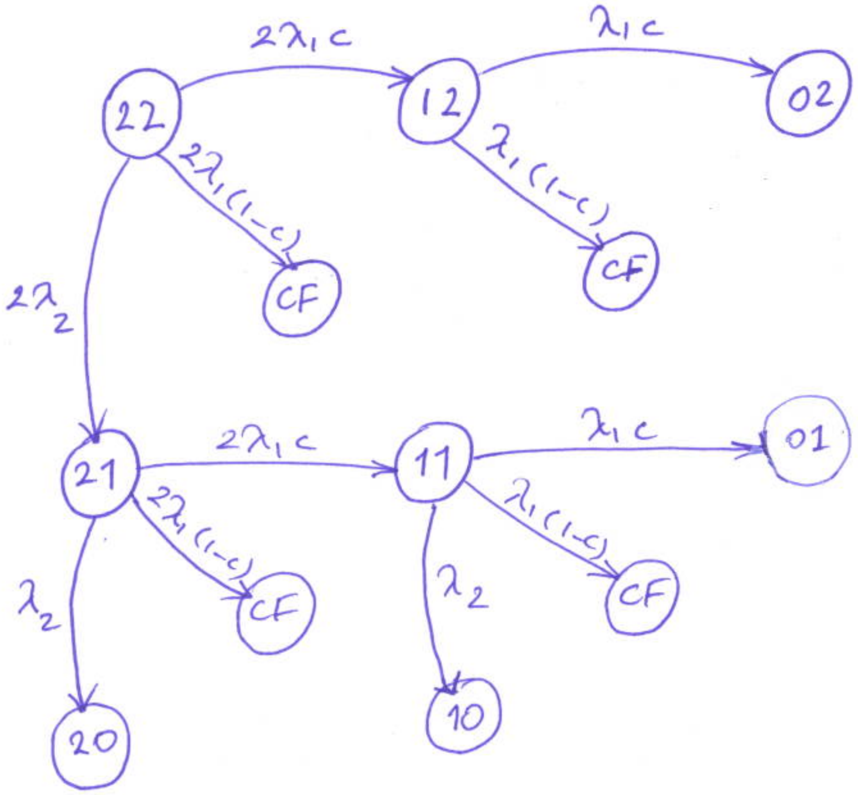
$$= \int_0^\infty 2c\, e^{-\lambda_1 t}\, dt + \int_0^\infty (1-2c)\, e^{-2\lambda_1 t}\, dt$$

$$= \left. \frac{-2c}{\lambda_1} e^{-\lambda_1 t} \right]_0^\infty + \left. \frac{1-2c}{-2\lambda_1} e^{-2\lambda_1 t} \right]_0^\infty$$

$$= 0 + \frac{2c}{\lambda_1} + 0 + \frac{1-2c}{2\lambda_1}$$

$$= \frac{4c + 1 - 2c}{2\lambda_1} = \frac{2c+1}{2\lambda_1}$$

1.d)



$$1 - S(\infty) = \frac{2\lambda_1(1-c)}{2\lambda_2 + 2\lambda_1} + \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{2\lambda_2 + \lambda_1}$$

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1(1-c)}{\lambda_2 + 2\lambda_1}$$

$$+ \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_2}{2\lambda_2 + \lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + \lambda_1}$$

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1 c}{\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + \lambda_1}$$

Steady State Safety

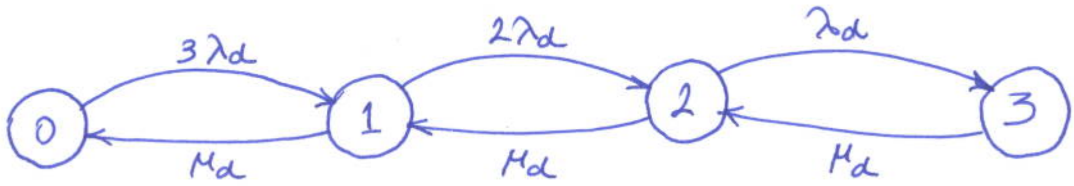
$$S(\infty) = 1 - \left(\frac{2\lambda_1(1-c)}{2\lambda_2 + 2\lambda_1} + \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{2\lambda_2 + \lambda_1} \right.$$

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1(1-c)}{\lambda_2 + 2\lambda_1}$$

$$+ \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_2}{2\lambda_2 + \lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + 2\lambda_1}$$

$$\left. + \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1 c}{\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + \lambda_1} \right)$$

2)



(Birth & Death Process)

$$\Pi_1 = \frac{3\lambda_d}{\mu_d}\Pi_0$$

$$\Pi_2 = \frac{2\lambda_d}{\mu_d}\Pi_1 = \frac{2\lambda_d}{\mu_d}\cdot\frac{3\lambda_d}{\mu_d}\Pi_0 = \frac{6\lambda_d^2}{\mu_d^2}\Pi_0$$

$$\Pi_3 = \frac{\lambda_d}{\mu_d}\Pi_2 = \frac{\lambda_d}{\mu_d}\Pi_1 = \frac{6\lambda_d^3}{\mu_d^3}\Pi_0$$

$$\Pi_0 + \Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_0\left(1 + \frac{3\lambda_d}{\mu_d} + \frac{6\lambda_d^2}{\mu_d^2} + \frac{6\lambda_d^3}{\mu_d^3}\right) = 1$$

$$\Pi_0 = \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}$$

$$A_{disks} = \Pi_0 + \Pi_1 + \Pi_2$$

$$= \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}\left(1 + \frac{3\lambda_d}{\mu_d} + \frac{6\lambda_d^2}{\mu_d^2}\right)$$

$$= \frac{\mu_d^3 + \mu_d^2\cdot 3\lambda_d + \mu_d\cdot 6\lambda_d^2}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}$$

2)
cont.

$$\begin{tikzcd} 0 \arrow[r, bend right, "\mu_p"'] & 1 \arrow[l, bend right, "2\lambda_p(1-c)"'] \arrow[r, bend left, "2\lambda_p c"] & 2 \arrow[l, bend left, "2\mu_p"] \arrow[r, bend left, "\lambda_p"] & 3 \arrow[l, bend left, "2\mu_p"] \end{tikzcd}$$

Birth and Death Process

$$\Pi_1 = \frac{\mu_p}{2\lambda_p(1-c)}\Pi_0$$

$$\Pi_2 = \frac{2\lambda_p \cdot c}{2\mu_p}\Pi_1 = \frac{\lambda_p \cdot c}{\mu_p} \cdot \frac{\mu_p}{2\lambda_p(1-c)}\Pi_0 = \frac{c}{2(1-c)}\Pi_0$$

$$\Pi_3 = \frac{\lambda_p}{2\mu_p}\Pi_2 = \frac{\lambda_p}{2\mu_p} \cdot \frac{c}{2(1-c)}\Pi_0 = \frac{c\lambda_p}{4\mu_p(1-c)}$$

$$\sum_{i=0}^{3} \Pi_i = 1$$

$$\Pi_0 \left(1 + \frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)} + \frac{c\lambda_p}{4\mu_p(1-c)} \right) = 1$$

$$\Pi_0 \left(\frac{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2}{4\mu_p\lambda_p(1-c)} \right) = 1$$

$$\Pi_0 = \frac{4\mu_p\lambda_p(1-c)}{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2}$$

$$A_{\text{2 processors}} = \Pi_1 + \Pi_2$$

$$= \Pi_0 \left(\frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)} \right)$$

$$= \frac{4\mu_p\lambda_p(1-c)}{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2} \cdot \left(\frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)} \right)$$

2)
cont.

$$A_\infty \text{ system} = A_\infty \text{ disks} * A_\infty \text{ processors}$$

$$= \frac{\mu_d^3 + \mu_d^2 3\lambda_d + \mu_d \cdot 6\lambda_d^2}{\mu_d^3 + 3\lambda_d \mu_d^2 + 6\lambda_d^2 \mu_d + 6\lambda_d^3}$$

$$* \frac{4\mu_p \lambda_p (1-c)}{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2}$$

$$* \left(\frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)} \right)$$

$$= \frac{\mu_d^3 + \mu_d^2 3\lambda_d + \mu_d \cdot 6\lambda_d^2}{\mu_d^3 + 3\lambda_d \mu_d^2 + 6\lambda_d^2 \mu_d + 6\lambda_d^3}$$

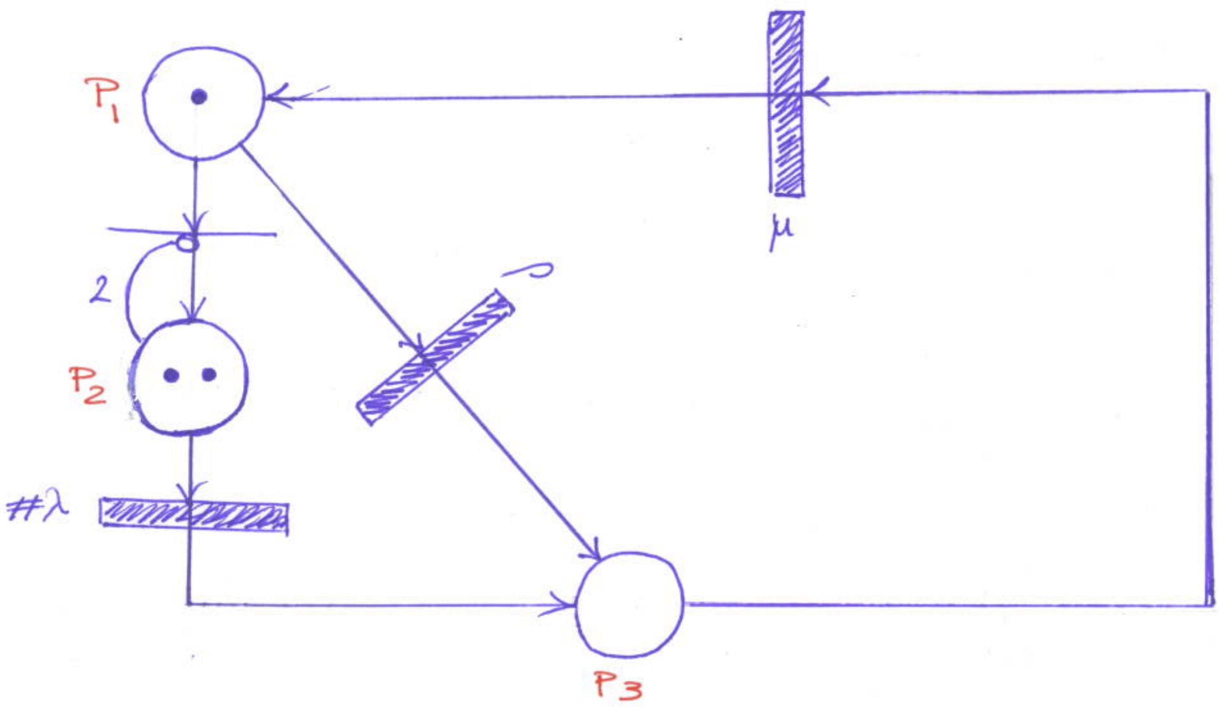
$$* \frac{4\mu_p \lambda_p (1-c)}{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2}$$

$$* \frac{\mu_p + c\lambda_p}{2\lambda_p(1-c)}$$

$$= \frac{\mu_d^3 + \mu_d^2 3\lambda_d + \mu_d 6\lambda_d^2}{\mu_d^3 + 3\lambda_d \mu_d^2 + 6\lambda_d^2 \mu_d + 6\lambda_d^3}$$

$$* \frac{2\mu_p^2 + 2\mu_p\lambda_p \cdot c}{4\mu_p\lambda_p(1-c) + 2\mu_p^2 + 2\mu_p\lambda_p c + c\lambda_p^2}$$

3)



Extended Reachability Graph