Exam in EDA122(Chalmers) and DIT062(GU), Wednesday, October 20,2010, 14:00 - 18:00

Problem 1

1.a

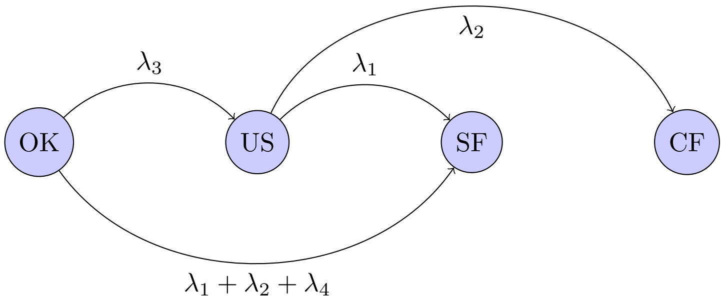

Figure 1: Markov chain

1.b

An expression for the steady-state safety of the network interface

$$S(\infty) = \frac{\lambda_1 + \lambda_2 + \lambda_4}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} + \frac{\lambda_1}{\lambda_1 + \lambda_2} * \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}$$

1.c

Derive an expression for the reliability of the network interface.

$$\begin{aligned} P'(t) &= P(t)Q \\ P(0) &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\ Q &= \begin{bmatrix} -\lambda_3 - \lambda_1 - \lambda_2 - \lambda_4 & \lambda_3 & \lambda_1 + \lambda_2 + \lambda_4 & 0 \\ 0 & -\lambda_1 - \lambda_2 & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$Q = \begin{bmatrix} -2\lambda_1 - 2\lambda_2 & \lambda_1 & \lambda_1 + 2\lambda_2 & 0 \\ 0 & -\lambda_1 - \lambda_2 & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Laplace transform:

$$\mathcal{L}\{P'(t) = P(t)Q\} \Rightarrow sP(s) - P(0) = P(s)Q$$

$$\begin{cases} sP_{OK} - 1 & = & -(2\lambda_1 + 2\lambda_2)P_{OK} \\ sP_{US} & = & \lambda_1 P_{OK} - (\lambda_1 + \lambda_2)P_{US} \\ sP_{SF} & = & (\lambda_1 + 2\lambda_2)P_{OK} - \lambda_1 P_{US} \\ sP_{CF} & = & \lambda_2 P_{US} \end{cases}$$

$$\begin{aligned} P_{OK}(s) &= \frac{1}{s + 2\lambda_1 + 2\lambda_2} \\ sP_{US} &= \frac{\lambda_1}{s + 2\lambda_1 + 2\lambda_2} - (\lambda_1 + \lambda_2)P_{US} \\ P_{US} &= \frac{\lambda_1}{s + 2\lambda_1 + 2\lambda_2}\frac{1}{s + \lambda_1 + \lambda_2} \end{aligned}$$

Inverse Laplace transform (see, e.g., Mathematics Handbook, pp. 332, L22.):

$$\mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\} = t^n e^{-at}$$

$$\begin{aligned} P_{OK}(t) &= \mathcal{L}^{-1}\left\{\frac{1}{s + 2\lambda_1 + 2\lambda_2}\right\} = e^{-(2\lambda_1 + 2\lambda_2)t} \\ P_{US}(t) &= \mathcal{L}^{-1}\left\{\frac{\lambda_1}{(\lambda_1 + \lambda_2)}(\frac{1}{s + \lambda_1 + \lambda_2} - \frac{1}{s + 2\lambda_1 + 2\lambda_2})\right\} \\ P_{US}(t) &= \frac{\lambda_1}{(\lambda_1 + \lambda_2)}(e^{-(\lambda_1 + \lambda_2)t} - e^{-(2\lambda_1 + 2\lambda_2)t}) \\ \Rightarrow R(t) &= P_{OK}(t) + P_{US}(t) \\ R(t) &= (1 - \frac{\lambda_1}{\lambda_1 + \lambda_2}) * e^{-2(\lambda_1 + \lambda_2)*t} + (\frac{\lambda_1}{\lambda_1 + \lambda_2}) * e^{-(\lambda_1 + \lambda_2)*t} \end{aligned}$$

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Problem 2

2.a

There are 6 fault containment regions:
1.Processor 1
2.Processor 2
3.Disk 1
4.Disk 2
5.Bus 1
6.Bus 2

2.b

Availability for the processors

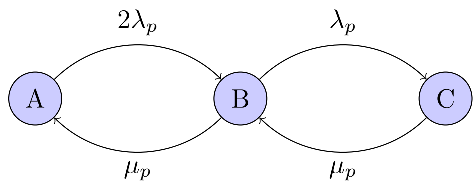

Figure 1: Processors – Markov chain for $A_p(t)$

This is a birth and death process (see, e.g., Mathematics Handbook, pp. 440-441).

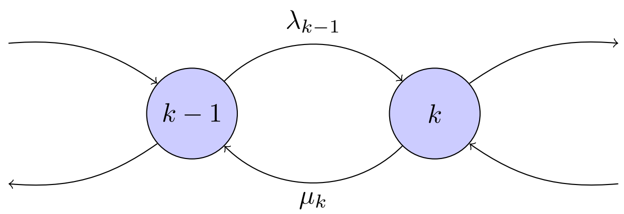

Figure 2: General birth and death process

$$
\begin{aligned}
Pi_k &= \frac{\lambda_{k-1}}{\mu_k}\Pi_{k-1} \\
\sum_k \Pi_k &= 1
\end{aligned}
$$

Using the formulas for a birth and death process, we get:

$$\begin{cases} \Pi_B & = \frac{2\lambda_p}{\mu_p}\Pi_A \\ \Pi_C & = \frac{\lambda_p}{\mu_p}\Pi_B \\ \Pi_A + \Pi_B + \Pi_C & = 1 \end{cases}$$

$$\begin{aligned} \Pi_A &= \frac{\mu_p^2}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \\ \Pi_B &= \frac{2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \\ A_p(t) &= \Pi_A + \Pi_B \\ A_p(t) &= \frac{\mu_p^2 + 2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \end{aligned}$$

Availability for the Disks

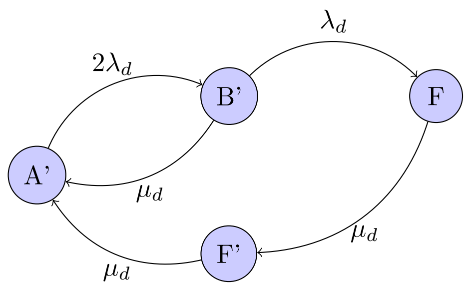

Figure 3: Markov chain for $A_d(t)$

$$\begin{aligned} A_d(t) &= P_{A'}(t) + P_{B'}(t) \\ \mathbf{P}'(t) &= \mathbf{P}(t)\mathbf{Q} \qquad \mathbf{Q} \text{ transition rate matrix} \\ \mathbf{P}(t) &= \begin{bmatrix} P_{A'}(t) & P_{B'}(t) & P_{F'}(t) & P_F(t) \end{bmatrix} \end{aligned}$$

Transition rate matrix:

$$\mathbf{Q} = \begin{bmatrix} -2\lambda_d & 2\lambda_d & 0 & 0 \\ \mu_d & -\lambda_d - \mu_d & 0 & \lambda_d \\ \mu_d & 0 & -\mu_d & 0 \\ 0 & 0 & \mu_d & -\mu_d \end{bmatrix}$$

Steady-state solution $\Rightarrow \mathbf{P}' = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$.
Let

$$\Pi_i = \lim_{t\to\infty} P_i(t)$$

We now have

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \Pi_{A'} & \Pi_{B'} & \Pi_{F'} & \Pi_{F} \end{bmatrix} \begin{bmatrix} -2\lambda_d & 2\lambda_d & 0 & 0 \\ \mu_d & -\lambda_d - \mu_d & 0 & \lambda_d \\ \mu_d & 0 & -\mu_d & 0 \\ 0 & 0 & \mu_d & -\mu_d \end{bmatrix}$$

We also now that

$$\sum_i \Pi_i = 1$$

We get the following equations:

$$\begin{cases} 0 & = & -2\lambda_d\Pi_{A'} + \mu_d\Pi_{B'} + \mu_d\Pi_{F'} \\ 0 & = & 2\lambda_d\Pi_{A'} - (\lambda_d + \mu_d)\Pi_{B'} \\ 0 & = & -\mu_d\Pi_{F'} + \mu_d\Pi_F \\ 0 & = & \lambda_d\Pi_{B'} - \mu_d\Pi_F \\ \Pi_{A'} + \Pi_{B'} + \Pi_{F'} + \Pi_F & = & 1 \end{cases}$$

$$\begin{aligned} \Rightarrow \Pi_{B'} &= \frac{2\lambda_d\mu_d}{3\lambda_d\mu_d + \mu_d^2 + 4\lambda_d^2} \\ \Pi_{A'} &= \frac{\lambda_d + \mu_d}{2\lambda_d}\Pi_{B'} \\ \Rightarrow \Pi_{A'} &= \frac{\mu_d^2 + \lambda_d\mu_d}{3\lambda_d\mu_d + \mu_d^2 + 4\lambda_d^2} \end{aligned}$$

$$\begin{aligned} \lim_{t\to\infty} A_d(t) &= \lim_{t\to\infty} P_{A'}(t) + P_{B'}(t) = \Pi_{A'} + \Pi_{B'} \\ \Pi_{A'} + \Pi_{B'} &= \frac{3\lambda_d\mu_d + \mu_d^2}{3\lambda_d\mu_d + \mu_d^2 + 4\lambda_d^2} \end{aligned}$$

Availability of the whole system:

$$\begin{aligned} A(t) &= A_d(t)A_p(t) \\ A(t) &= \tfrac{3\lambda_d\mu_d + \mu_d^2}{3\lambda_d\mu_d + \mu_d^2 + 4\lambda_d^2} \tfrac{\mu_p^2 + 2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \end{aligned}$$

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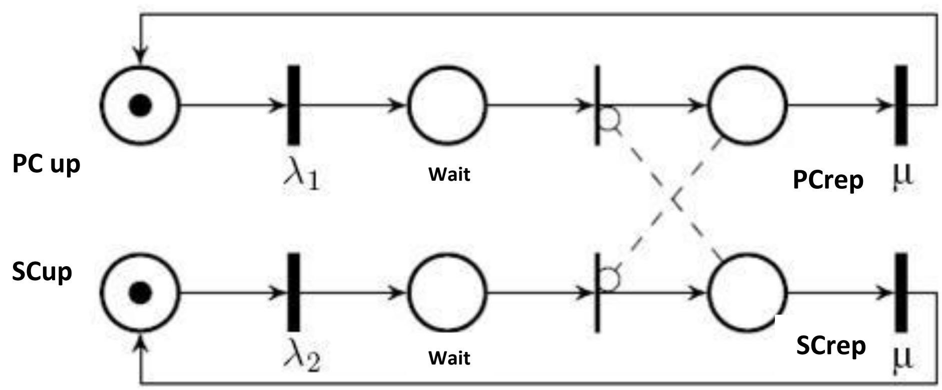
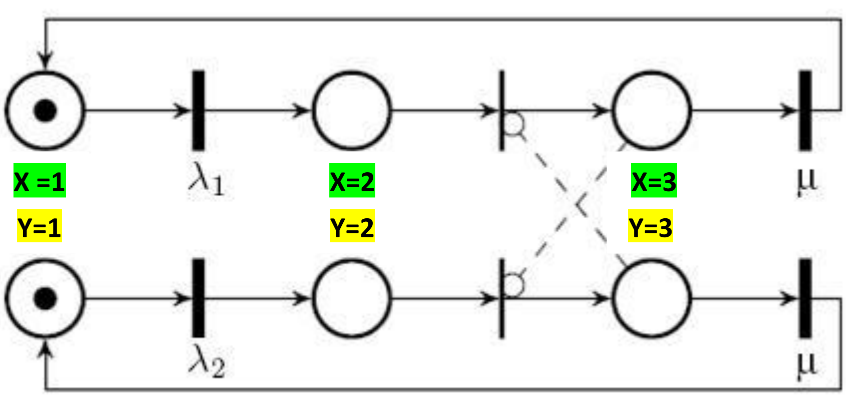
Problem 3

GSPN



Labeling X and Y



Reachability Graph: