

1a)

The FTU can be viewed as a parallel system with three computer nodes:

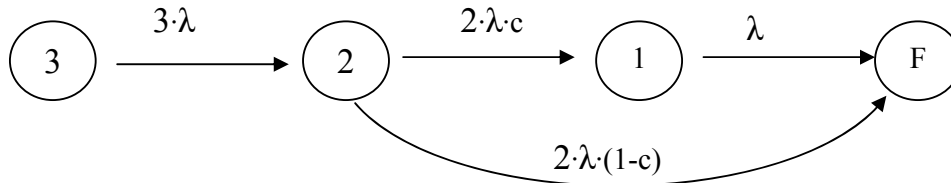
$$R_{\text{sys}} = 1 - \prod_{i=1}^3 (1 - R_{\text{CM}}) = 1 - (1 - R_{\text{CM}})^3 = 3R_{\text{CM}} - 3R_{\text{CM}}^2 + R_{\text{CM}}^3$$

$$R_{\text{CM}}(t) = e^{-\lambda t} \Rightarrow R_{\text{sys}}(t) = 3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t}$$

1b)

$$\begin{aligned} \text{MTTF} &= \int_0^{\infty} R_{\text{sys}}(t) dt = \int_0^{\infty} 3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t} dt = \left[-\frac{3}{\lambda} e^{-\lambda t} + \frac{3}{2\lambda} e^{-2\lambda t} - \frac{1}{3\lambda} e^{-3\lambda t} \right]_0^{\infty} = \\ &= \frac{3}{\lambda} - \frac{3}{2\lambda} + \frac{1}{3\lambda} = \frac{18 - 9 + 2}{6\lambda} = \frac{11}{6\lambda} \end{aligned}$$

1c)



States:

- 3: Three computer nodes OK
- 2: Two computer nodes OK, one computer node broken
- 1: One computer node OK, two computer nodes broken
- F: All computer nodes broken

This gives the following equation system:

$$\bar{P}'(t) = \bar{P}(t) \cdot \bar{Q}$$

$$\bar{P}(t) = [P_3(t) \ P_2(t) \ P_1(t) \ P_F(t)]$$

$$\bar{Q} = \begin{bmatrix} -3\lambda & 3\lambda & 0 & 0 \\ 0 & -2\lambda & 2\lambda c & 2\lambda(1-c) \\ 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We obtain:

$$P_3'(t) = -3\lambda \cdot P_3(t)$$

$$P_2'(t) = 3\lambda \cdot P_3(t) - 2\lambda \cdot P_2(t)$$

$$P_1'(t) = 2\lambda c \cdot P_2(t) - \lambda \cdot P_1(t)$$

$$P_F'(t) = 2\lambda(1-c) \cdot P_2(t) + \lambda \cdot P_1(t)$$

We solve the equation system using Laplace transform:

$$s \cdot \tilde{P}_3(s) - 1 = -3\lambda \cdot \tilde{P}_3(s) \quad (1)$$

$$s \cdot \tilde{P}_2(s) = 3\lambda \cdot \tilde{P}_3(s) - 2\lambda \cdot \tilde{P}_2(s) \quad (2)$$

$$s \cdot \tilde{P}_1(s) = 2\lambda c \cdot \tilde{P}_2(s) - \lambda \cdot \tilde{P}_1(s) \quad (3)$$

From (1) , (2) and (3) we obtain:

$$\tilde{P}_3(s) = \frac{1}{s + 3\lambda}$$

$$\tilde{P}_2(s) = \frac{3\lambda}{(s + 2\lambda) \cdot (s + 3\lambda)} = 3\lambda \left[\frac{A}{s + 2\lambda} + \frac{B}{s + 3\lambda} \right] = \left\{ A = \frac{1}{\lambda}, B = -\frac{1}{\lambda} \right\} = \frac{3}{s + 2\lambda} - \frac{3}{s + 3\lambda}$$

$$\begin{aligned} \tilde{P}_1(s) &= \frac{2\lambda c}{s + \lambda} \cdot \tilde{P}_2(s) = \frac{6\lambda c}{(s + \lambda) \cdot (s + 2\lambda)} - \frac{6\lambda c}{(s + \lambda) \cdot (s + 3\lambda)} \\ &= 6\lambda c \cdot \left[\frac{A}{s + \lambda} + \frac{B}{s + 2\lambda} - \left(\frac{C}{s + \lambda} + \frac{D}{s + 3\lambda} \right) \right] = \left\{ A = \frac{1}{\lambda}, B = -\frac{1}{\lambda}, C = \frac{1}{2\lambda}, D = -\frac{1}{2\lambda} \right\} \\ &= 6c \cdot \left[\frac{1}{s + \lambda} - \frac{1}{s + 2\lambda} - \frac{1}{2(s + \lambda)} + \frac{1}{2(s + 3\lambda)} \right] = \frac{3c}{s + \lambda} - \frac{6c}{s + 2\lambda} + \frac{3c}{s + 3\lambda} \end{aligned}$$

The inverse Laplace transform give:

$$P_3(t) = e^{-3\lambda t}$$

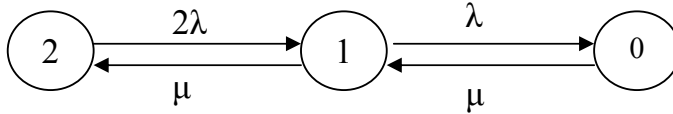
$$P_2(t) = 3e^{-2\lambda t} - 3e^{-3\lambda t}$$

$$P_1(t) = 3c \cdot e^{-3\lambda t} - 6c \cdot e^{-2\lambda t} + 3c \cdot e^{-\lambda t}$$

The reliability is:

$$\begin{aligned} R(t) &= P_3(t) + P_2(t) + P_1(t) = e^{-3\lambda t} + 3e^{-2\lambda t} - 3e^{-3\lambda t} + 3c \cdot e^{-3\lambda t} - 6c \cdot e^{-2\lambda t} + 3c \cdot e^{-\lambda t} \\ &= 3c \cdot e^{-\lambda t} + 3(1 - 2c) \cdot e^{-2\lambda t} + (3c - 2) \cdot e^{-3\lambda t} \end{aligned}$$

2a)



States:

2: Two computer nodes OK

1: One computer node OK, one computer node broken

0: Two computer nodes broken

Birth-and-death process:

$$\Pi_1 = \frac{2\lambda}{\mu} \cdot \Pi_2 \quad (1)$$

$$\Pi_0 = \frac{\lambda}{\mu} \cdot \Pi_1 = \frac{2\lambda^2}{\mu^2} \cdot \Pi_2 \quad (2)$$

$$1 = \Pi_2 + \Pi_1 + \Pi_0 \quad (3)$$

From (1), (2) and (3), we obtain:

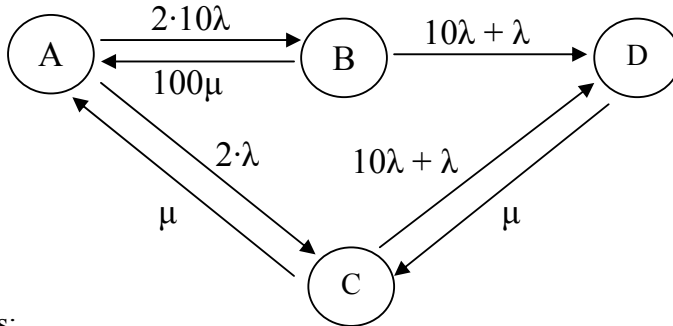
$$\Pi_2 \cdot \left(1 + \frac{2\lambda}{\mu} + \frac{2\lambda^2}{\mu^2} \right) = 1 \Rightarrow \Pi_2 = \frac{\mu^2}{\mu^2 + 2\lambda\mu + 2\lambda^2} \quad (4)$$

From (1) and (4), we obtain:

$$\Pi_1 = \frac{2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

$$A_{(\infty)} = \Pi_2 + \Pi_1 = \frac{\mu^2 + 2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

2b)



States:

- A: Two nodes OK
- B: One node OK, one node broken due to transient fault
- C: One node OK, one node broken
- D: Two nodes broken

This gives the following equation system:

$$\begin{aligned} \bar{P}'(t) &= \bar{P}(t) \cdot \bar{Q} \\ \bar{P}(t) &= [P_A \ P_2 \ P_1 \ P_0] \\ \bar{Q} &= \begin{bmatrix} -22\lambda & 20\lambda & 2\lambda & 0 \\ 100\mu & -(11\lambda + 100\mu) & 0 & 11\lambda \\ \mu & 0 & -(11\lambda + \mu) & 11\lambda \\ 0 & 0 & \mu & -\mu \end{bmatrix} \end{aligned}$$

Let $t \rightarrow \infty \Rightarrow P'_i(t) = 0, P_i(t) = \Pi_i :$

$$0 = -22\lambda \cdot \Pi_A + 100\mu \cdot \Pi_B + \mu \cdot \Pi_C \quad (1)$$

$$0 = 20\lambda \cdot \Pi_A - (11\lambda + 100\mu) \cdot \Pi_B \quad (2)$$

$$0 = 2\lambda \cdot \Pi_A - (11\lambda + \mu) \cdot \Pi_1 + \mu \cdot \Pi_D \quad (3)$$

$$0 = 11\lambda \cdot \Pi_B + 11\lambda \cdot \Pi_C - \mu \cdot \Pi_D \quad (4)$$

$$1 = \Pi_A + \Pi_B + \Pi_C + \Pi_D \quad (5)$$

From (2), we get:

$$\Pi_A = \frac{11\lambda + 100\mu}{20\lambda} \cdot \Pi_B \quad (6)$$

From (1) and (6), we get:

$$\mu \cdot \Pi_c = \frac{22(11\lambda + 100\mu)}{20} \cdot \Pi_B - 100\mu \cdot \Pi_B \Rightarrow \Pi_c = \frac{121\lambda + 100\mu}{10\mu} \cdot \Pi_B \quad (7)$$

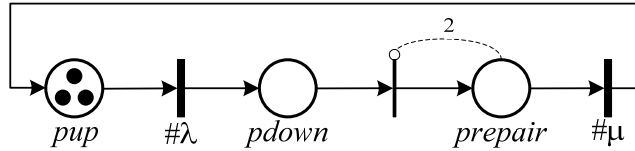
From (4) and (7), we get:

$$\Pi_D = \frac{11\lambda}{\mu} \cdot \Pi_B + \frac{11\lambda}{\mu} \cdot \left(\frac{121\lambda + 100\mu}{10\mu} \right) \cdot \Pi_B = \frac{1331\lambda^2 + 1210\lambda\mu}{10\mu^2} \cdot \Pi_B \quad (8)$$

From (5), (6), (7) and (8), we obtain:

$$\begin{aligned} 1 &= \left(\frac{11\lambda + 100\mu}{20\lambda} + 1 + \frac{121\lambda + 100\mu}{10\mu} + \frac{1331\lambda^2 + 1210\lambda\mu}{10\mu^2} \right) \cdot \Pi_B = \\ &= \frac{11\lambda\mu^2 + 100\mu^3 + 20\lambda\mu^2 + 242\lambda^2\mu + 200\lambda\mu^2 + 2662\lambda^3 + 2420\lambda^2\mu}{20\lambda\mu^2} \cdot \Pi_B \Rightarrow \\ \Pi_B &= \frac{20\lambda\mu^2}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3} \\ A_{(\infty)} &= \Pi_A + \Pi_B + \Pi_C = \frac{(11\lambda + 100\mu) \cdot \mu^2 + 20\lambda\mu^2 + (121\lambda + 100\mu) \cdot 2\lambda\mu}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3} = \\ &= \frac{100\mu^3 + 231\lambda\mu^2 + 242\lambda^2\mu}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3} \end{aligned}$$

3. The system is described by the following GSPN model:



Markings of the GSPN model are represented as ($\#pup \#pdown \#prepair$). The marking (0 1 2) corresponds to the event that the system is unavailable.