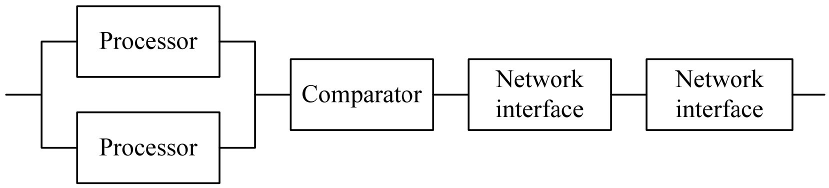
Exam in EDA122/DIT061 Fault-Tolerant Computer Systems, Oct 21, 2009.
Solutions to problems 1 to 3.

1.

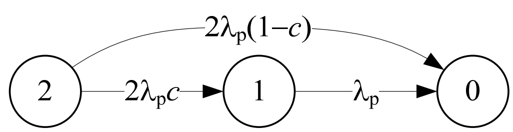
a) The system is modelled by the following reliability block diagram:



Let $R_p(t)$ denote the reliability of a processor, $R_c(t)$ the reliability of the message comparator and $R_n(t)$ the reliability of a network interface. We then obtain:

$$R_p(t) = e^{-\lambda_p t}$$
$$R_c(t) = e^{-\lambda_c t}$$
$$R_n(t) = e^{-\lambda_n t}$$
$$R_{2p} = 1 - \left(1 - R_p\right)^2 = 2R_p - R_p^2$$
$$R_{node} = R_{2p} \cdot R_c \cdot R_n^2 = (2R_p - R_p^2) \cdot R_c \cdot R_n^2$$

b) The processor subsystem is modelled using the following Markov model:



From the Markov model, we obtain the following transition rate matrix:

$$Q = \begin{bmatrix} -2\lambda_p & 2\lambda_p c & 2\lambda_p(1-c) \\ 0 & -\lambda_p & \lambda_p \\ 0 & 0 & 0 \end{bmatrix}$$

From the equation $P'(t) = P(t)Q$, we obtain the following system of differential equations:

$$\begin{cases} P_2'(t) = -2\lambda_p P_2(t) \\ P_1'(t) = 2\lambda_p c P_2(t) - \lambda_p P_1(t) \\ P_0'(t) = 2\lambda_p (1-c) P_2(t) + \lambda_p P_1(t) \end{cases}$$

We also know that $P(0) = [1 \quad 0 \quad 0]$. The equation system is solved using the Laplace transform.

$$\begin{cases} sP_2(s) - 1 = -2\lambda_p P_2(s) \\ sP_1(s) = 2\lambda_p c P_2(s) - \lambda_p P_1(s) \\ P_0(s) = 2\lambda_p (1-c) P_2(s) + \lambda_p P_1(s) \end{cases}$$

$$P_2(s) = \frac{1}{s + 2\lambda_p}$$

$$P_1(s) = \frac{2\lambda_p c}{s + \lambda_p} \cdot P_2(s) = \frac{2\lambda_p c}{s + \lambda_p} \cdot \frac{1}{s + 2\lambda_p} = 2c\left(\frac{1}{s + \lambda_p} - \frac{1}{s + 2\lambda_p}\right)$$

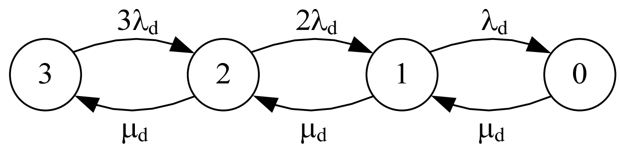
$$P_2(t) = e^{-2\lambda_p t}$$

$$P_1(t) = 2c\left(e^{-\lambda_p t} - e^{-2\lambda_p t}\right)$$

$$R_{2p}(t) = P_1(t) + P_2(t) = e^{-2\lambda_p t}(1 - 2c) + 2ce^{-\lambda_p t}$$

$$R_{node}(t) = R_{2p}(t) \cdot R_c(t) \cdot R_n^2(t)$$

2. The system is modelled by viewing the disks and the processors as two independent primary subsystems. The disk subsystem is modelled using the following Markov model:



The label of each state represents the number of working disks. Using the formulas for a general birth-death process, we obtain the following system of equations:

$$\begin{cases} \Pi_2 = \dfrac{3\lambda_d}{\mu_d}\Pi_3 \\ \Pi_1 = \dfrac{2\lambda_d}{\mu_d}\Pi_2 = \dfrac{6\lambda_d^2}{\mu_d^2}\ \Pi_3 \\ \Pi_0 = \dfrac{\lambda_d}{\mu_d}\Pi_1 = \dfrac{6\lambda_d^3}{\mu_d^3}\ \Pi_3 \\ 1 = \Pi_3 + \Pi_2 + \Pi_1 + \Pi_0 \end{cases}$$

From these equations, we obtain

$$1 = \Pi_3\left(1 + \frac{3\lambda_d}{\mu_d} + \frac{6\lambda_d^2}{\mu_d^2} + \frac{6\lambda_d^3}{\mu_d^3}\right) = \frac{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\,\mu_d + 6\lambda_d^3}{\mu_d^3} \cdot \Pi_3$$

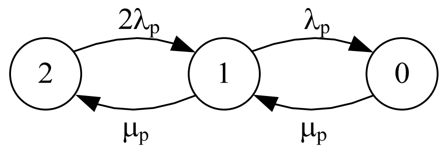
$$\Rightarrow \Pi_3 = \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\,\mu_d + 6\lambda_d^3}$$

The steady-state availability of the disk subsystem is

$$\lim_{t\to\infty} A_{disks}(t) = \Pi_3 + \Pi_2 = \Pi_3\left(1 + \frac{3\lambda_d}{\mu_d}\right) = \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}\left(1 + \frac{3\lambda_d}{\mu_d}\right)$$

$$= \frac{\mu_d^3 + 3\lambda_d\,\mu_d^2}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}$$

The processors are modelled using the following Markov-model:



We obtain the following system of equations:

$$\begin{cases} \Pi_1 = \dfrac{2\lambda_p}{\mu_p}\Pi_2 \\ \Pi_0 = \dfrac{\lambda_p}{\mu_p}\Pi_1 = \dfrac{2\lambda_p^2}{\mu_p^2}\Pi_2 \\ 1 = \Pi_2 + \Pi_1 + \Pi_0 \end{cases}$$

$$1 = \Pi_2\left(1 + \frac{2\lambda_p}{\mu_p} + \frac{2\lambda_p^2}{\mu_p^2}\right) = \frac{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2}{\mu_p^2}\cdot\Pi_2$$

$$\Rightarrow \Pi_2 = \frac{\mu_p^2}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2}$$

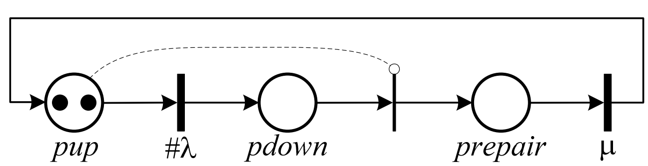
The steady-state availability of the processors is

$$\lim_{t\to\infty} A_{processors}(t) = \Pi_2 + \Pi_1 = \Pi_2\left(1 + \frac{2\lambda_p}{\mu_p}\right) = \frac{\mu_p^2}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2}\left(1 + \frac{2\lambda_p}{\mu_p}\right) = \frac{\mu_p^2 + 2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2}$$

The steady-state availability of the system is then calculated as

$$\lim_{t\to\infty} A_{system}(t) = A_{disks}(t)\cdot A_{processors}(t)$$

3. The system is described by the GSPN model below.



We represent markings of the GSPN model as (*#pup #pdown #prepair*). The following markings correspond to the event that the system is unavailable: (0 2 0), (0 0 2), (1 0 1) and (0 1 1).