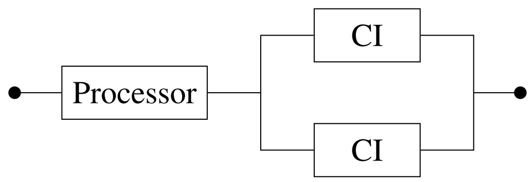
1.

a)

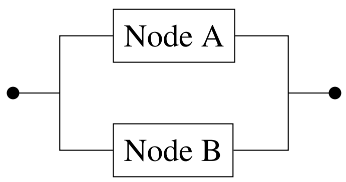
There are six error containment regions. Each processor and each communication interface must constitute an error containment region to achieve maximum reliability of the FTU.

b)

We can model a single node in the FTU using the following reliability block diagram:



The FTU, which consists of two nodes, is modelled by the following reliability block diagram:



Let R_p denote the reliability of a processor, R_c the reliability of a communication interface, and R_{cc} the reliability of two communication interfaces. We then obtain:

$$
\begin{aligned}
R_p(t) &= e^{-\lambda_p t} \\
R_c(t) &= e^{-\lambda_c t} \\
R_{cc}(t) &= 1 - (1 - R_c)^2 = 2R_c - R_c^2 = 2e^{-\lambda_c t} - e^{-2\lambda_c t} \\
R_{node}(t) &= R_p \cdot R_{cc} = e^{-\lambda_p t}\left(2e^{-\lambda_c t} - e^{-2\lambda_c t}\right) \\
R_{FTU}(t) &= 1 - (1 - R_{node})^2 = 2R_{node} - R_{node}^2.
\end{aligned}
$$

c)

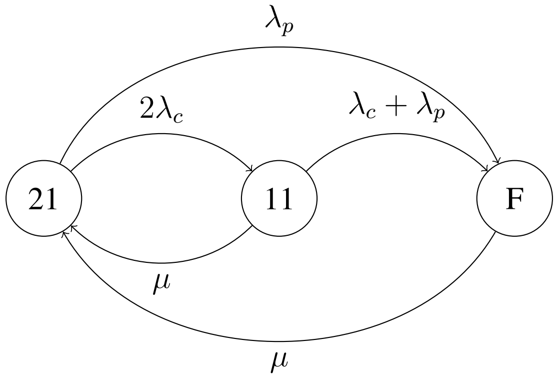
For a single communication interface, the rate for a fail-silence violation is $(1 - fsc)\lambda_c$. Let F denote the expected time from the start of four communication interfaces to the first fail-silence violation.

$$F = \int_0^\infty e^{-4(1-fsc)\lambda_c t} dt = \frac{1}{4(1-fsc)\lambda_c}$$

2.

a)

A single node in the FTU can be modelled by the following Markov model:



State label = # operational communication interfaces, # operational processors; F = failure.

We obtain the following transition rate matrix

$$Q = \begin{bmatrix} -(2\lambda_c + \lambda_p) & 2\lambda_c & \lambda_p \\ \mu & -(\lambda_c + \lambda_p + \mu) & \lambda_c + \lambda_p \\ \mu & 0 & -\mu \end{bmatrix}$$

and the following system of differential equations

$$\begin{cases} P'_{21}(t) &= -(2\lambda_c + \lambda_p)P_{21}(t) + \mu P_{11}(t) + \mu P_F(t) \\ P'_{11}(t) &= 2\lambda_c P_{21}(t) - (\lambda_c + \lambda_p + \mu)P_{11}(t) \\ P'_F(t) &= \lambda_p P_{21}(t) + (\lambda_c + \lambda_p)P_{11}(t) - \mu P_F(t). \end{cases}$$

Let $t \to \infty$, and denote $\lim_{t\to\infty} P_i(t)$ as Π_i. We obtain

$$\begin{cases} 0 = -(2\lambda_c + \lambda_p)\Pi_{21} + \mu\Pi_{11} + \mu\Pi_F & (1) \\ 0 = 2\lambda_c\Pi_{21} - (\lambda_c + \lambda_p + \mu)\Pi_{11} & (2) \\ 0 = \lambda_p\Pi_{21} + (\lambda_c + \lambda_p)\Pi_{11} - \mu\Pi_F & (3) \end{cases}$$

We also know that

$$\Pi_{21} + \Pi_{11} + \Pi_F = 1. \quad (4)$$

From (2), we obtain

$$\Pi_{11} = \frac{2\lambda_c}{\lambda_c + \lambda_p + \mu}\Pi_{21}. \quad (5)$$

From (3) and (5), we obtain

$$\begin{aligned}\Pi_F &= \frac{\lambda_p}{\mu}\Pi_{21} + \frac{\lambda_c+\lambda_p}{\mu}\Pi_{11} = \frac{\lambda_p}{\mu}\Pi_{21} + \frac{\lambda_c+\lambda_p}{\mu}\frac{2\lambda_c}{\lambda_c+\lambda_p+\mu}\Pi_{21} \\ &= \frac{\lambda_p(\lambda_c+\lambda_p+\mu)+2\lambda_c(\lambda_c+\lambda_p)}{\mu(\lambda_c+\lambda_p+\mu)}\Pi_{21}. \qquad (6)\end{aligned}$$

From (4), (5), and (6), we obtain

$$\begin{aligned}1 &= \Pi_{21}\left(1+\frac{2\lambda_c}{\lambda_c+\lambda_p+\mu}+\frac{\lambda_p(\lambda_c+\lambda_p+\mu)+2\lambda_c(\lambda_c+\lambda_p)}{\mu(\lambda_c+\lambda_p+\mu)}\right) \\ &= \frac{\mu(\lambda_c+\lambda_p+\mu)+2\lambda_c\mu+\lambda_p(\lambda_c+\lambda_p+\mu)+2\lambda_c(\lambda_c+\lambda_p)}{\mu(\lambda_c+\lambda_p+\mu)}\Pi_{21} \\ &= \frac{(\mu+\lambda_p+2\lambda_c)(\lambda_c+\lambda_p+\mu)}{\mu(\lambda_c+\lambda_p+\mu)}\Pi_{21} = \frac{\mu+\lambda_p+2\lambda_c}{\mu}\Pi_{21} \\ \Rightarrow \Pi_{21} &= \frac{\mu}{\mu+\lambda_p+2\lambda_c}\end{aligned}$$

The steady-state availability of a single node is

$$\begin{aligned}\lim_{t\to\infty} A_{node}(t) &= \Pi_{21}+\Pi_{11} = \frac{\mu}{\mu+\lambda_p+2\lambda_c}\left(1+\frac{2\lambda_c}{\lambda_c+\lambda_p+\mu}\right) \\ &= \frac{\mu(3\lambda_c+\lambda_p+\mu)}{(\mu+\lambda_p+2\lambda_c)(\lambda_c+\lambda_p+\mu)}.\end{aligned}$$

In this problem, we are interested of the steady-state unavailability of a node:

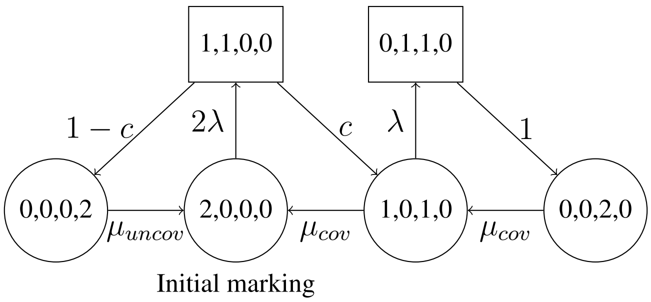
$$\begin{aligned}\lim_{t\to\infty} F_{node}(t) &= \Pi_F = \frac{\lambda_p(\lambda_c+\lambda_p+\mu)+2\lambda_c(\lambda_c+\lambda_p)}{\mu(\lambda_c+\lambda_p+\mu)}\frac{\mu}{\mu+\lambda_p+2\lambda_c} \\ &= \frac{3\lambda_p\lambda_c+\lambda_p^2+\lambda_p\mu+2\lambda_c^2}{(\lambda_c+\lambda_p+\mu)(\mu+\lambda_p+2\lambda_c)}.\end{aligned}$$

Using the expression for the unavailability of a node, we calculate the steady-state availability of the FTU as

$$\lim_{t\to\infty} A_{FTU}(t) = 1-\Pi_F^2.$$

3.

We denote the place left of the transition *tfail* as *pup*, the place right of *tfail* as *pfail*, the place right of *tcov* as *pcov*, and the place right of *tuncov* as *puncov*. We represent possible markings of the GPSN using the quadruple (*#pup*, *#pfail*, *#pcov*, *#puncov*), and obtain the extended reachability graph below.



The effects of vanishing markings are then accounted for by modifying the transition rates, resulting in the following reachability graph: