

Quicksort

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 kapitel 8, avsnitt 9

Quicksort

Quicksort väljer ett specifikt värde (kallat *pivot*), och delar upp resten av fältet i två delar:

- alla element som är $\leq$ pivot läggs i vänstra delen
- alla element som är $>$ pivot läggs i högra delen
- pivoten placeras mittemellan
- sortera vänstra och högra delen var för sig, rekursivt

Trace of Quicksort

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

Trace of Quicksort (cont.)

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

Arbitrarily select
the first element
as the pivot

Trace of Quicksort (cont.)

55	75	23	43	44	12	64	77	33
----	----	----	----	----	----	----	----	----

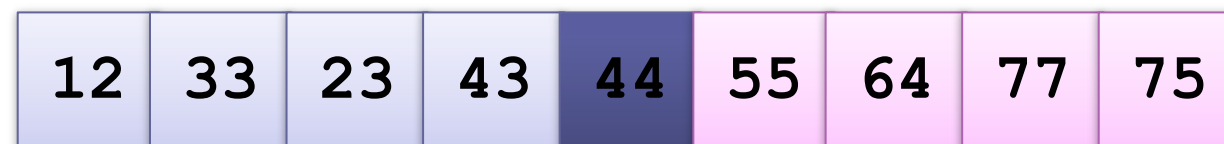
Swap the pivot with
the element in the
middle

Trace of Quicksort (cont.)

55	75	23	43	44	12	64	77	33
----	----	----	----	----	----	----	----	----

Partition the elements
so that all values less
than or equal to the
pivot are to the left,
and all values greater
than the pivot are to
the right

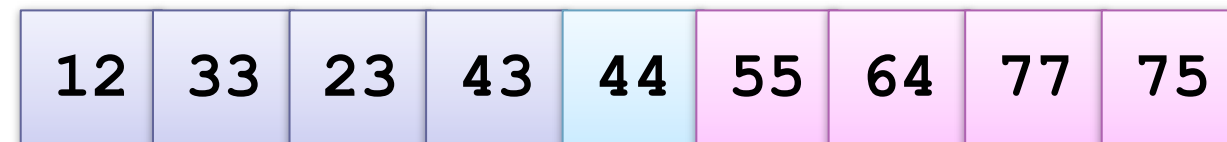
Trace of Quicksort (cont.)



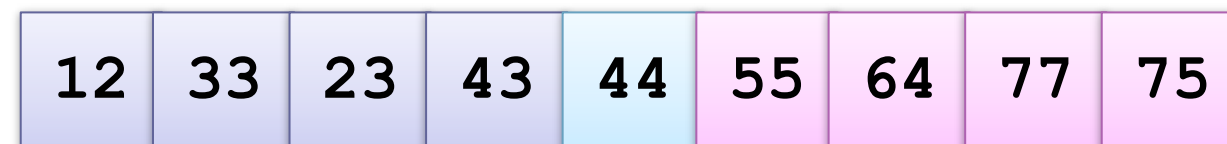
Partition the elements
so that all values less
than or equal to the
pivot are to the left,
and all values greater
than the pivot are to
the right

Quicksort Example(cont.)

44 is now in its
correct position



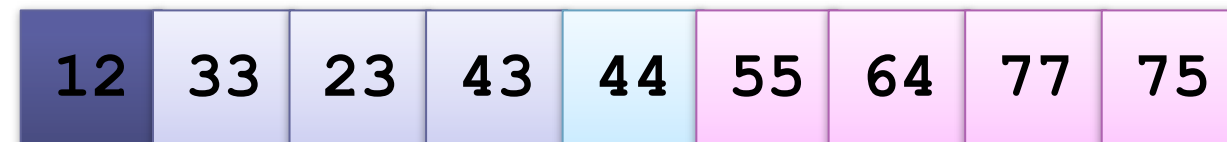
Trace of Quicksort (cont.)



Now apply
quicksort
recursively to the
two subarrays

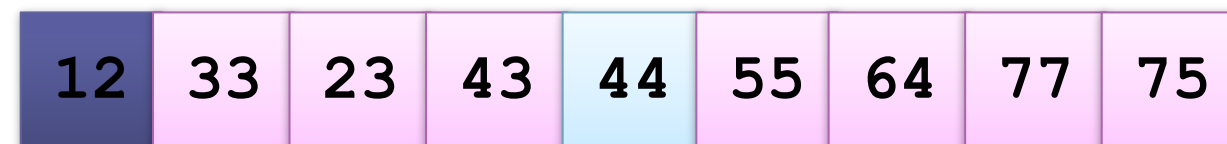
Trace of Quicksort (cont.)

Pivot value = 12



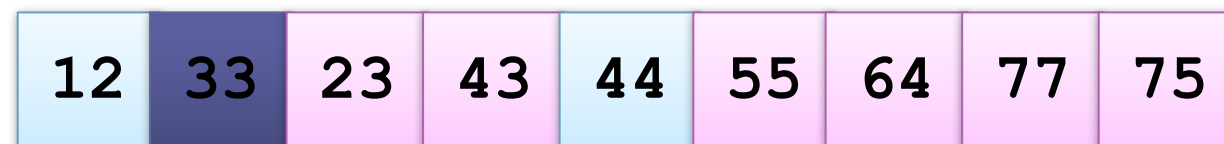
Trace of Quicksort (cont.)

Pivot value = 12



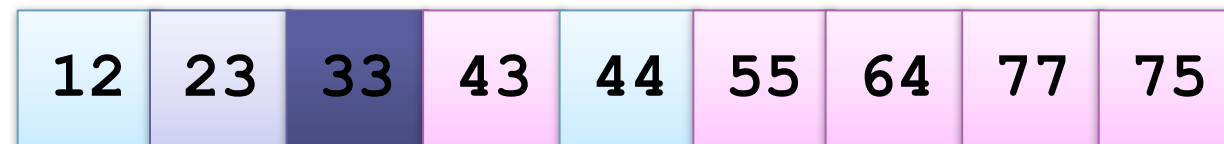
Trace of Quicksort (cont.)

Pivot value = 33



Trace of Quicksort (cont.)

Pivot value = 33



Trace of Quicksort (cont.)

Pivot value = 33

12	23	33	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

Trace of Quicksort (cont.)

Pivot value = 33

12	23	33	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

Left and right
subarrays have
single values; they
are sorted

Trace of Quicksort (cont.)

Pivot value = 33

12	23	33	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

Left and right
subarrays have
single values; they
are sorted

Trace of Quicksort (cont.)

Pivot value = 55



Trace of Quicksort (cont.)

Pivot value = 64



Trace of Quicksort (cont.)

Pivot value = 77

12	23	33	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

Trace of Quicksort (cont.)

Pivot value = 77



Trace of Quicksort (cont.)

Pivot value = 77

12	23	33	43	44	55	64	75	77
----	----	----	----	----	----	----	----	----

Trace of Quicksort (cont.)

12	23	33	43	44	55	64	75	77
----	----	----	----	----	----	----	----	----

Left subarray
has single value;
it is sorted

Trace of Quicksort (cont.)

12	23	33	43	44	55	64	75	77
----	----	----	----	----	----	----	----	----

Analys av Quicksort

Om pivoten är ett slumpmässigt värde från det aktuella fältet,

- så kommer statistiskt sett pivoten att ligga i mitten av fältet
- vänster och höger halva kommer (statistiskt sett) att bli lika stora
- dvs, varje rekursivt anrop kommer att sortera ett hälften så stort fält (statistiskt sett)
- dvs, samma resonemang som för Mergesort
- fast Quicksort är in-place

Algorithm for Partitioning

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

If the array is randomly ordered, it does not matter which element is the pivot.

For simplicity we pick the element with subscript `first`

Trace of Partitioning (cont.)

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

If the array is randomly ordered, it does not matter which element is the pivot.

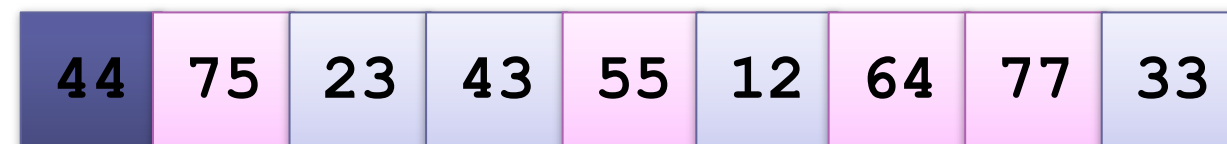
For simplicity we pick the element with subscript `first`

Trace of Partitioning (cont.)



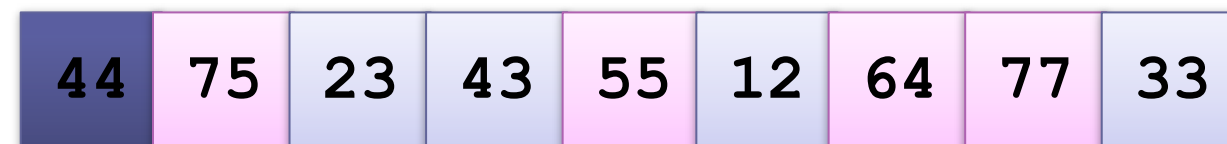
For visualization purposes,
items less than or equal to the
pivot will be colored blue;
items greater than the pivot
will be colored light purple

Trace of Partitioning (cont.)



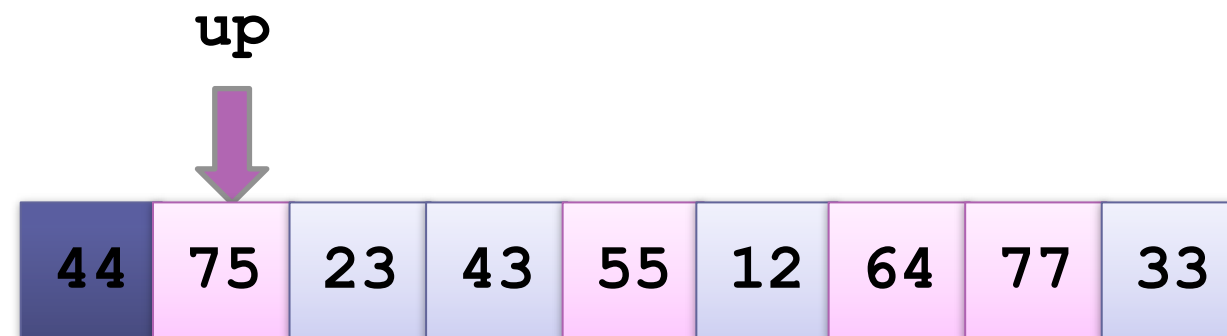
For visualization purposes,
items less than or equal to the
pivot will be colored blue;
items greater than the pivot
will be colored light purple

Trace of Partitioning (cont.)



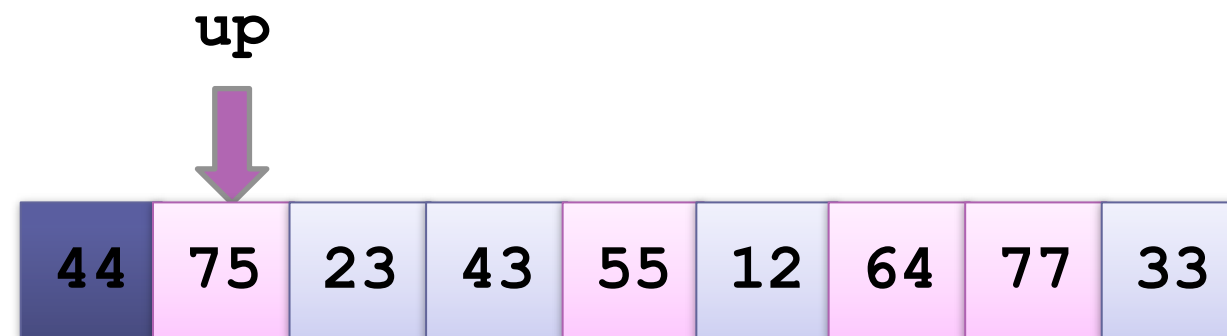
Search for the first value at the left end of the array that is greater than the pivot value

Trace of Partitioning (cont.)



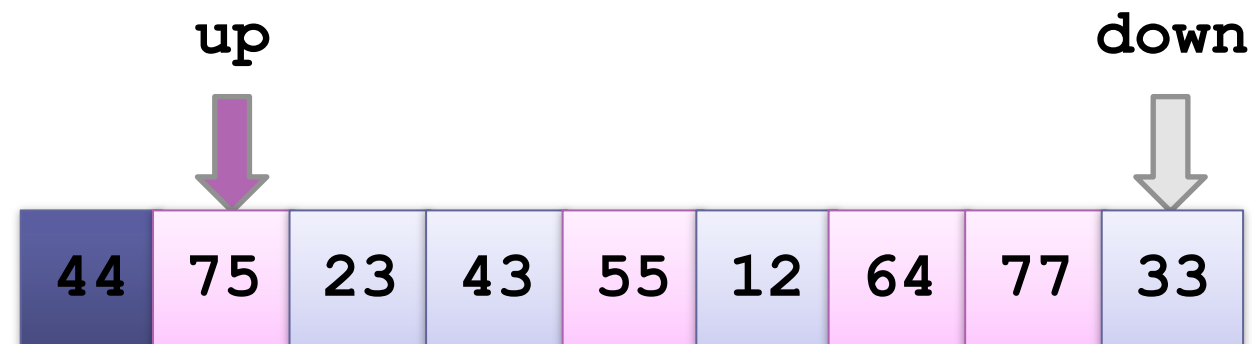
Search for the first value at the left end of the array that is greater than the pivot value

Trace of Partitioning (cont.)



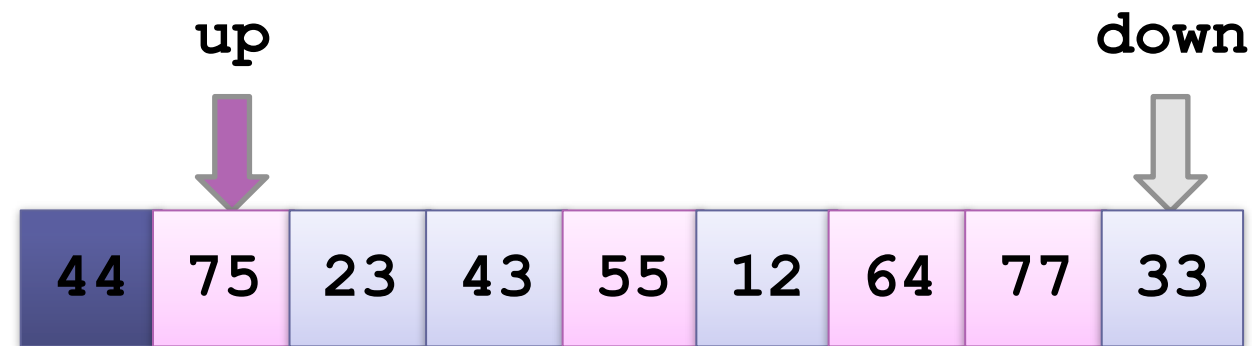
Then search for the first value
at the right end of the array
that is less than or equal to the
pivot value

Trace of Partitioning (cont.)



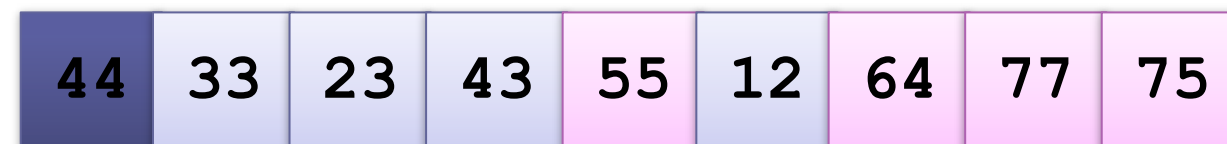
Then search for the first value
at the right end of the array
that is less than or equal to the
pivot value

Trace of Partitioning (cont.)



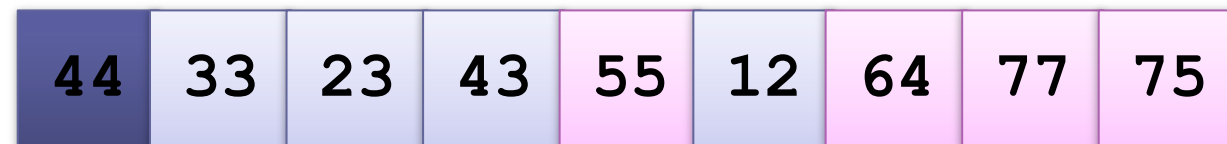
Exchange these values

Trace of Partitioning (cont.)



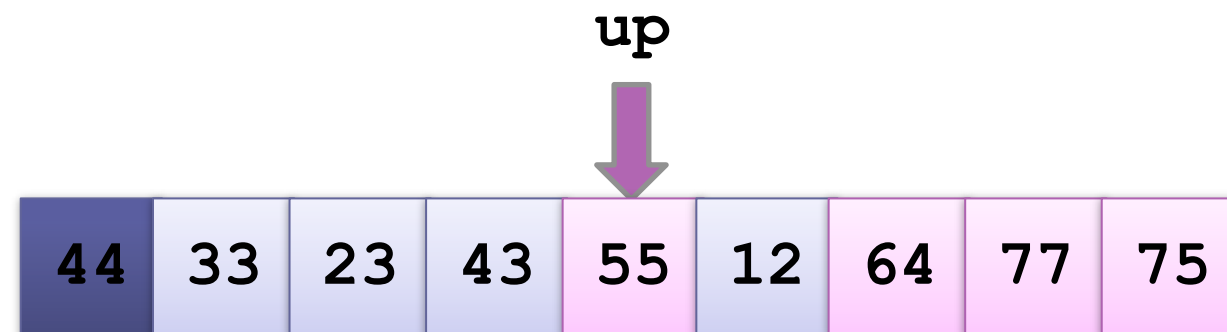
Exchange these values

Trace of Partitioning (cont.)



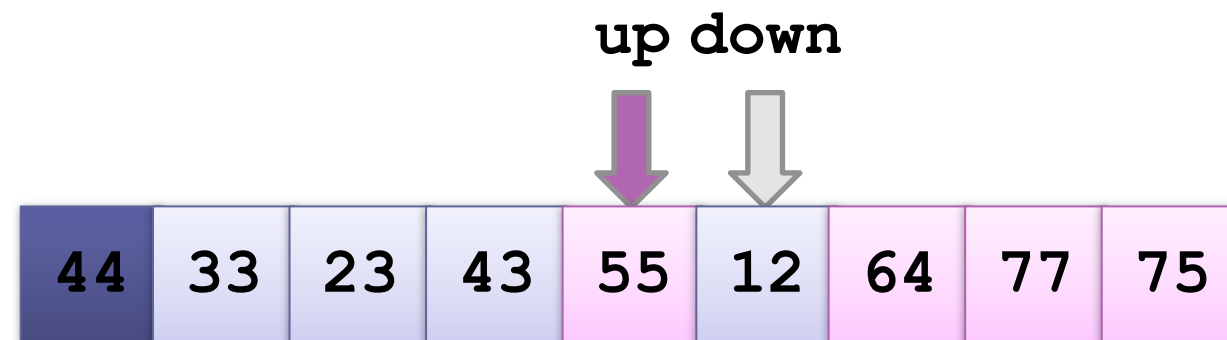
Repeat

Trace of Partitioning (cont.)



Find first value at left end
greater than pivot

Trace of Partitioning (cont.)



Find first value at right end
less than or equal to pivot

Trace of Partitioning (cont.)



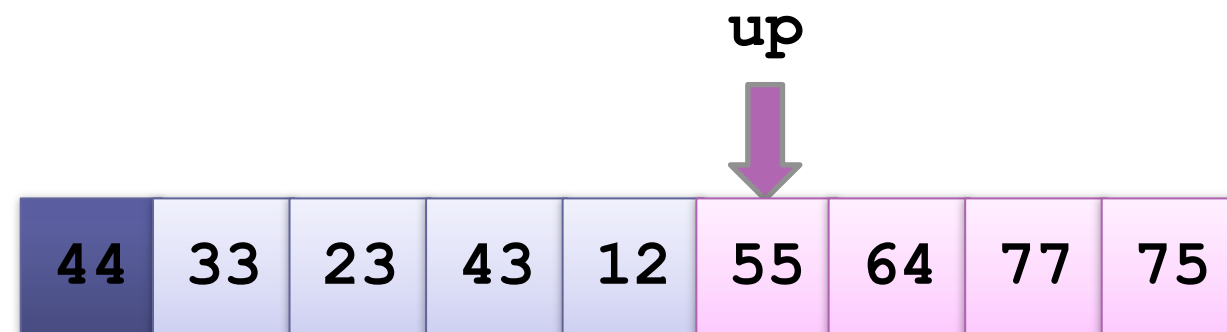
Exchange

Trace of Partitioning (cont.)



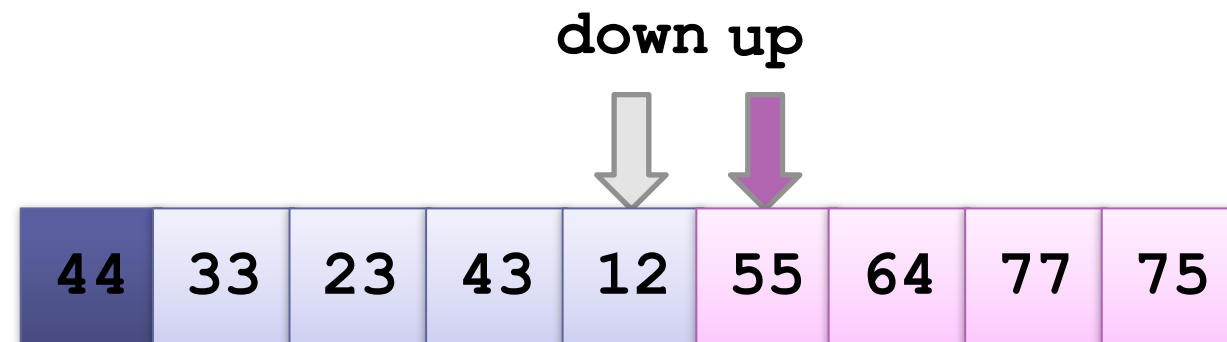
Repeat

Trace of Partitioning (cont.)



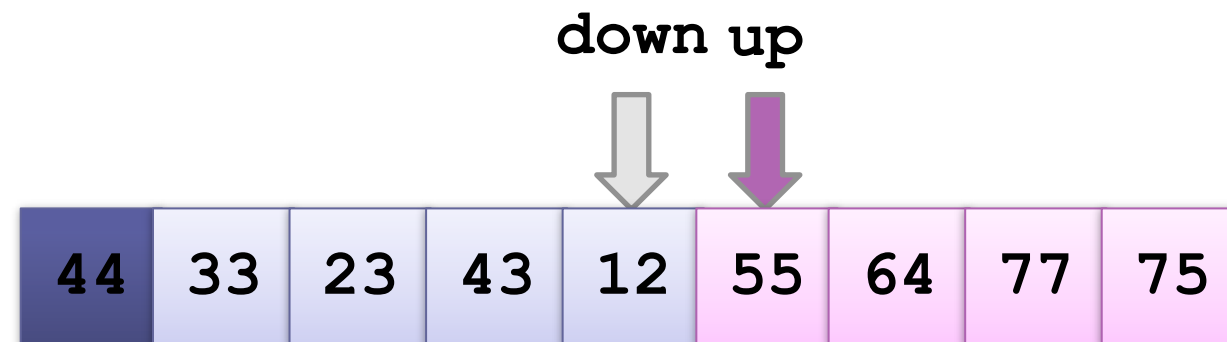
Find first element at left end
greater than pivot

Trace of Partitioning (cont.)



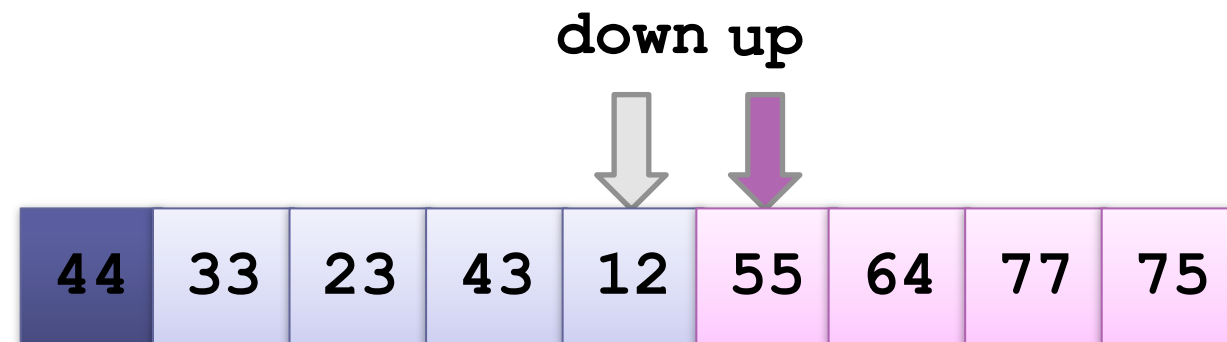
Find first element at right end
less than or equal to pivot

Trace of Partitioning (cont.)



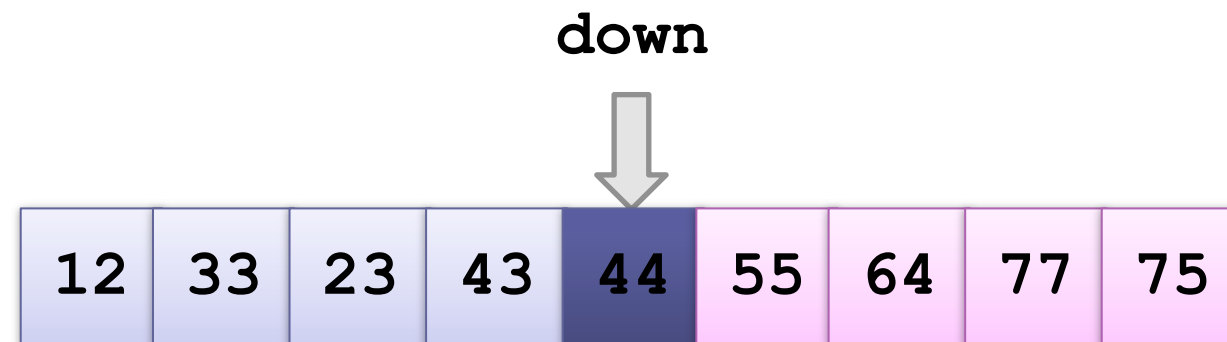
Since down has "passed" up, do not exchange

Trace of Partitioning (cont.)



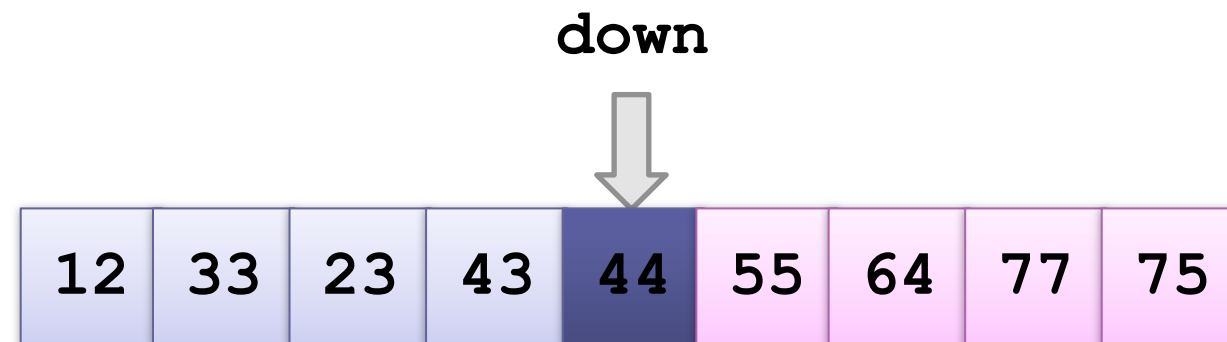
Exchange the pivot value with
the value at down

Trace of Partitioning (cont.)



Exchange the pivot value with
the value at down

Trace of Partitioning (cont.)



The pivot value is in the correct position; return the value of `down` and assign it to the pivot index `pivIndex`

Partition algorithm

Algorithm for partition Method

1. Define the pivot value as the contents of `table[first]`.
2. Initialize `up` to `first` and `down` to `last`.
3. **do**
4. Increment `up` until `up` selects the first element greater than the pivot value or `up` has reached `last`.
5. Decrement `down` until `down` selects the first element less than or equal to the pivot value or `down` has reached `first`.
6. **if** `up < down` **then**
7. Exchange `table[up]` and `table[down]`.
8. **while** `up` is to the left of `down`
9. Exchange `table[first]` and `table[down]`.
10. Return the value of `down` to `pivIndex`.

Analys av Quicksort

Om pivoten väljs dumt så kan Quicksort bli ineffektiv:

- om det ena delfältet alltid råkar bli tomt
- t.ex., om man alltid tar första värdet som pivot, och listan redan är sorterad
- i det fallet blir komplexiteten $O(n^2)$, precis som insättningssortering

Bättre är att välja pivoten smartare:

- t.ex., utgå från tre värden — första, mittersta och sista elementet
- välj medianen av dessa som pivot
- detta minskar risken för dålig delning

Sorteringsalgoritmer

	Number of Comparisons		
	Best	Average	Worst
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Bubble sort	$O(n)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Heapsort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quicksort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$