Greedy:

Choices to make.

$\underbrace{\text{Short-sighted}}_{\text{barely any time spent making next choice}}$ $\underbrace{\text{greed}}_{\text{seems to maximise "benefit"}}$

- Doesn't always work...
- When it does, yields efficient alg.

Correctness arguments:

Two general kinds:

1. Staying ahead argument
   (pick as much as possible)
   1. Greedy stays ahead,
   2. Staying ahead is better.
2. Exchange argument
   (pick all, in best order)
   - No solution can be both
     - different
     - better

   than valid greedy solution

   (Fixing inversion brings you to a ~~bett~~ ~~er~~ solution)

Example

[KT SE4.1]

Exam 2009 greedy tape storage

[KT SE4.1]

d max hike distance/day

l length of trail

$x_1 \le \dots \le x_n$ potential stopping points

$x_{i+1} - x_i \le d$, $\forall i$.

Algorithm: "hike as far as possible each day".

$S = \{x_{s_1}, \dots, x_{s_k}\}$ stop points by algorithm.

Valid since • you don't hike more than d each day,
• you stop at valid stop points.

$O = \{x_{o_1}, \dots, x_{o_m}\}$ any valid stop points

$P(i) =$ " $x_{o_i} \le x_{s_i}$ ".

Greedy stays ahead: $\forall i.\ P(i)$.

proof: ind. in i.

$i=1$: $x_{s_1} = \max\limits_{x_j \le d} \{x_j\}$.

$x_{o_1} > x_{s_1} \Rightarrow x_{o_1} > d \Rightarrow O$ invalid.

$i+1$ given i: IH $= P(i)$ holds.

To show: $P(i+1)$ holds

$x_{o_{i+1}} - x_{o_i} \le d$ (by validity of O)

$x_{o_{i+1}} - x_{s_i} \le x_{o_{i+1}} - x_{o_i}$ (by $P(i)$: $x_{o_i} \le x_{s_i}$)

$\Rightarrow x_{o_{i+1}} - x_{s_i} \le d$ (combine)

$\Rightarrow x_{s_{i+1}} \ge x_{o_{i+1}}$ (as $x_{s_{i+1}} = \max\limits_{x_j;\, x_j - x_{s_i} \le d} \{x_j\}$). ∎

Staying ahead is optimal: S is optimal.

proof: Assume $m < k$.

$x_{s_m} < l - d$ (else no need to stop at $x_{s_{m+1}}$)

$x_{o_m} \le x_{s_m}$ ($P(m)$)

$\Rightarrow x_{o_m} < l - d$

since $<$, and x_{o_m} last stop in O, O cannot be valid. ∎