Lava 4

brief info for TH exam

Q: How can we speed this up?

```
adder0 :: [(Bit, Bit)] -> ([Bit], Bit)
adder0 abs = row fullAdd (low,abs)
```

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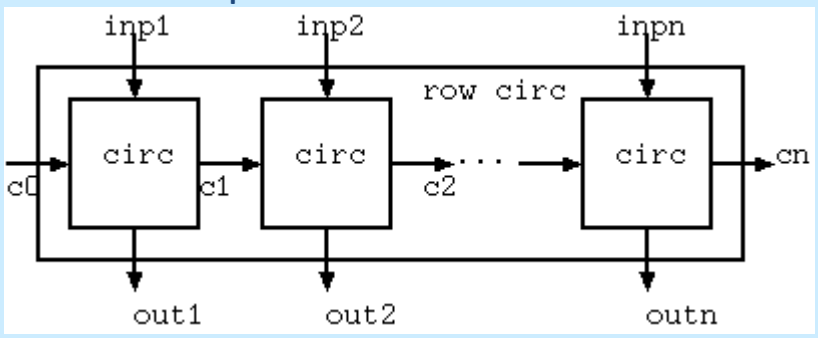
row is a connection pattern



A: Compute carries separately

will show a sequence of functions that each have same behaviour

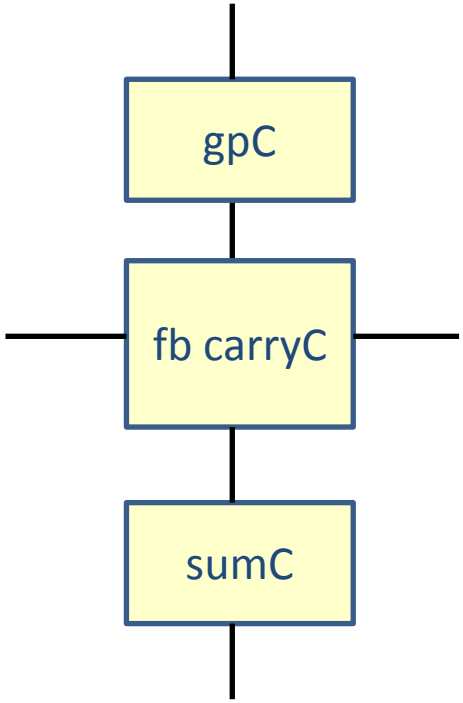
Some useful stuff

```
fsT f  = (f  -|- id)
snD f = (id -|-  f)

fb :: (a -> b) -> a -> (a,b)
fb circ a = (a, circ a)
```

another view of full adder

```
fullAdd1 :: (Bit,(Bit,Bit)) -> (Bit,Bit)
fullAdd1 = snD gpC ->- fb carryC ->- fsT sumC
```



```
gpC:: (Bit,Bit) -> (Bit,Bit)
gpC (a,b) = (a <&> b,a <#> b)

sumC :: (Bit,(Bit,Bit)) -> Bit
sumC (cin, (_,p)) = cin <#> p

carryC :: (Bit,(Bit,Bit)) -> Bit
carryC (cin, (g,p)) = g <|> (cin <&> p)
```

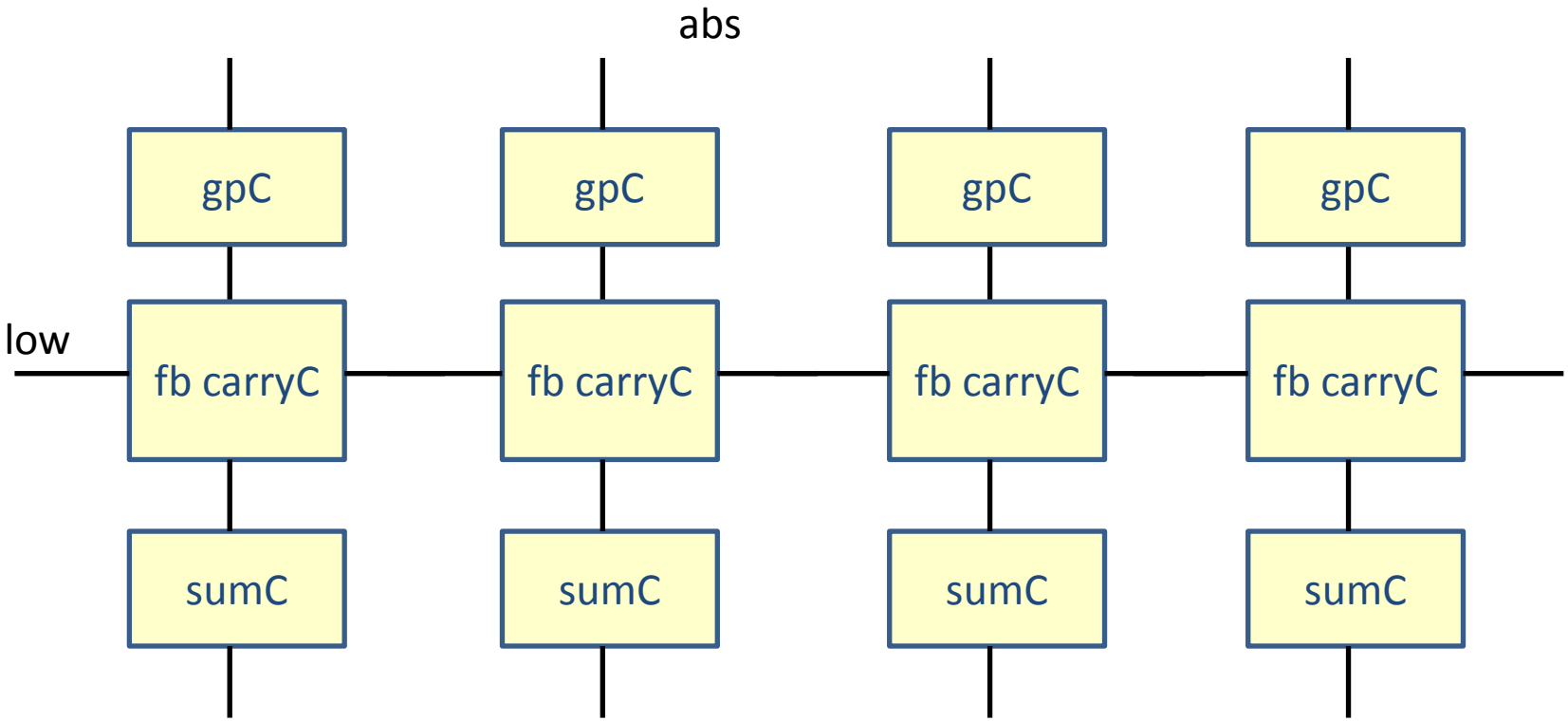
Can we rewrite this?

```
adder1 :: [(Bit, Bit)] -> ([Bit], Bit)
adder1 abs = row fullAdd1 (low,abs)
```

two maps

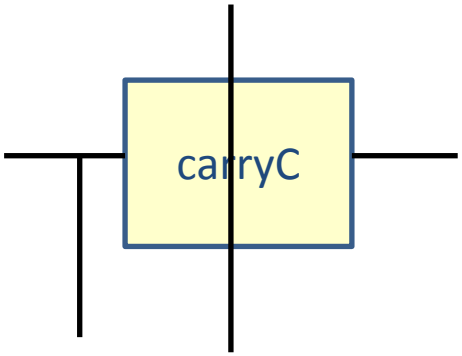
```
adder2 :: [(Bit, Bit)] -> ([Bit], Bit)
adder2 abs = (ss,cout)
  where
    gps         = map gpC abs
    (rs,cout) = row (fb carryC) (low,gps)
    ss          = map sumC rs
```

Picture


abs
low
gpC
gpC
gpC
gpC
fb carryC
fb carryC
fb carryC
fb carryC
sumC
sumC
sumC
sumC

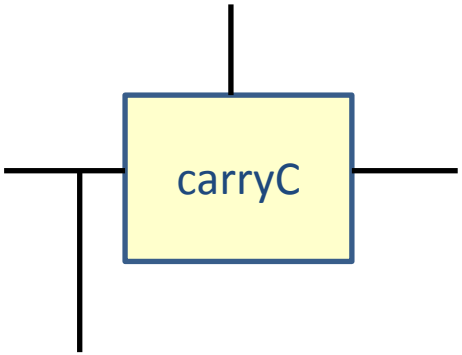
fb

```
fb :: (a -> b) -> a -> (a,b)
fb circ a = (a, circ a)
```



f1

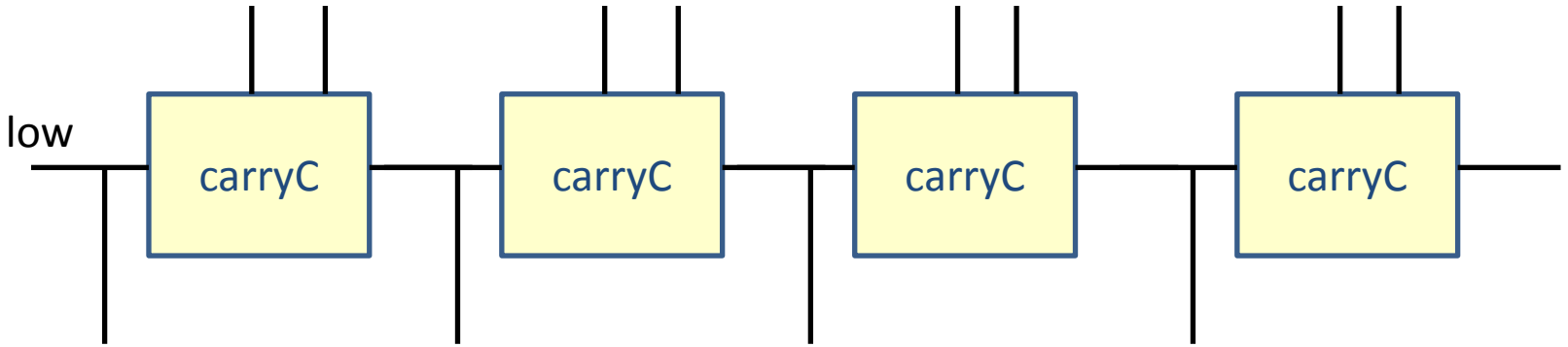
```
f1 :: ((a,b) -> c) -> (a,b) -> (a,c)
f1 circ (a,b) = (a, circ (a,b))
```



slight reorg

```
adder3 :: [(Bit, Bit)] -> ([Bit], Bit)
adder3 abs = (ss,cout)
  where
    gps         = map gpC abs
    (cs,cout) = row (f1 carryC) (low,gps)
    rs           = zip cs gps
    ss          = map sumC rs
```

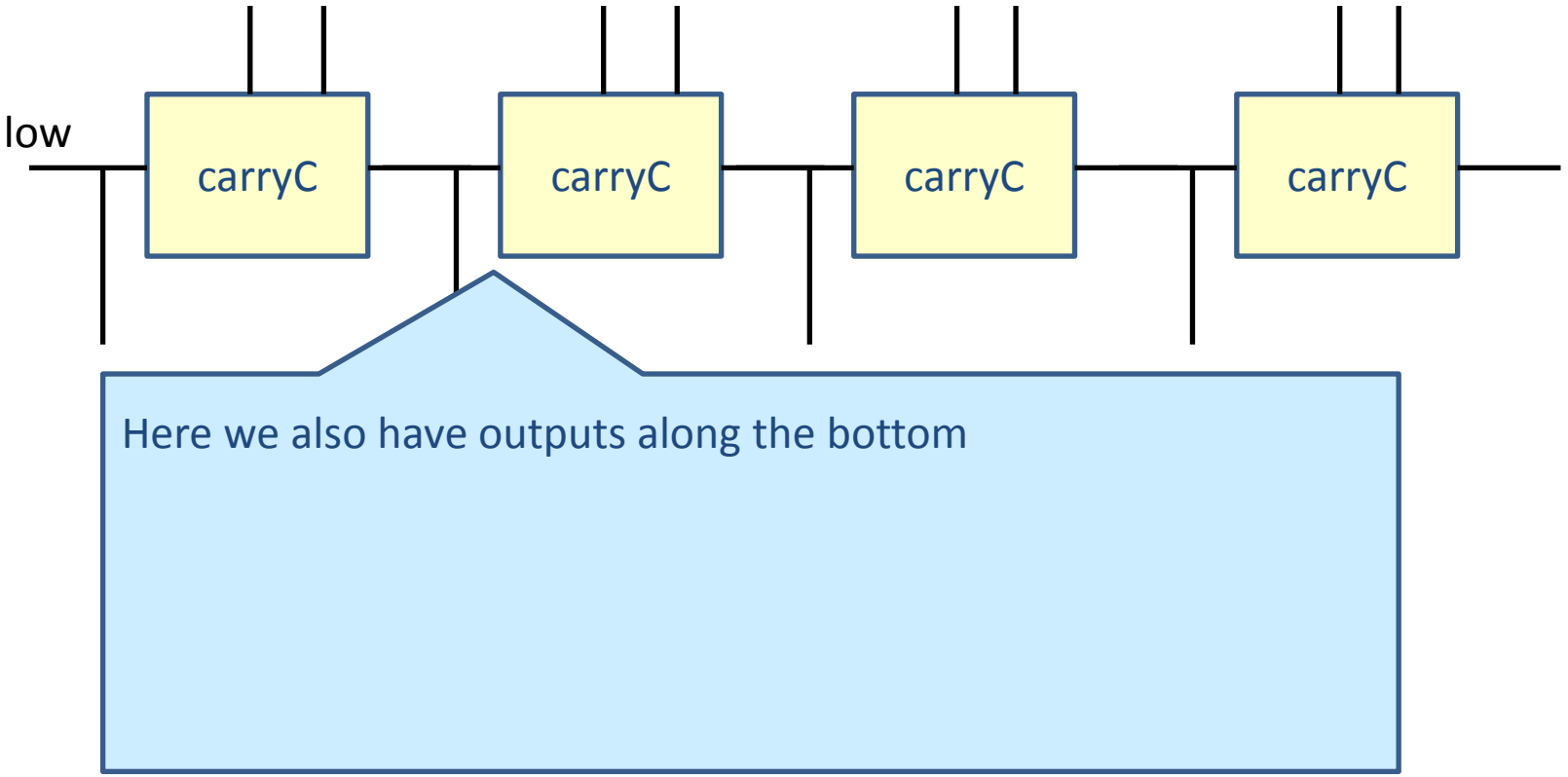
isolated the carry calculation



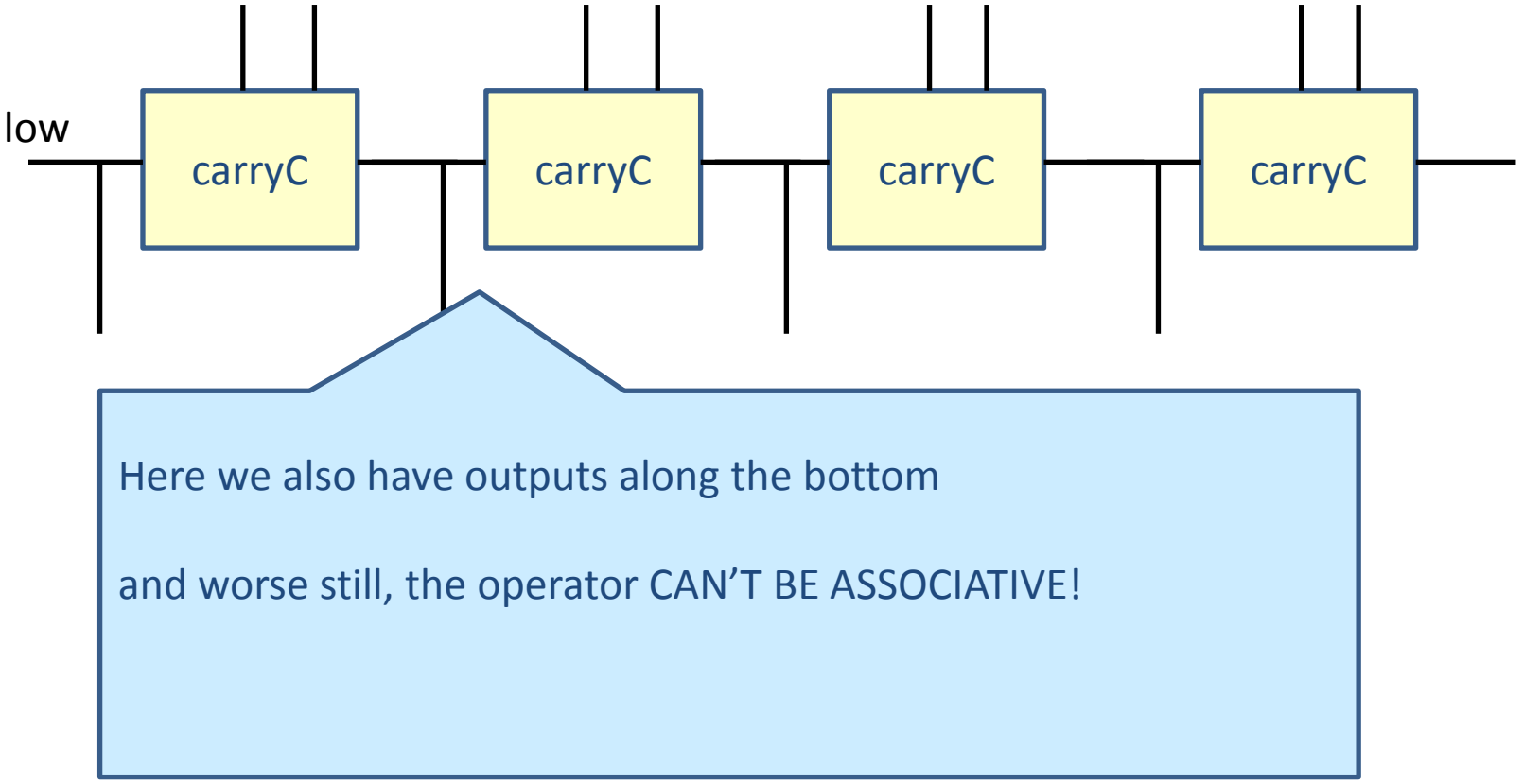
Remember

could replace linear array by binary tree for associative operator

isolated the carry calculation



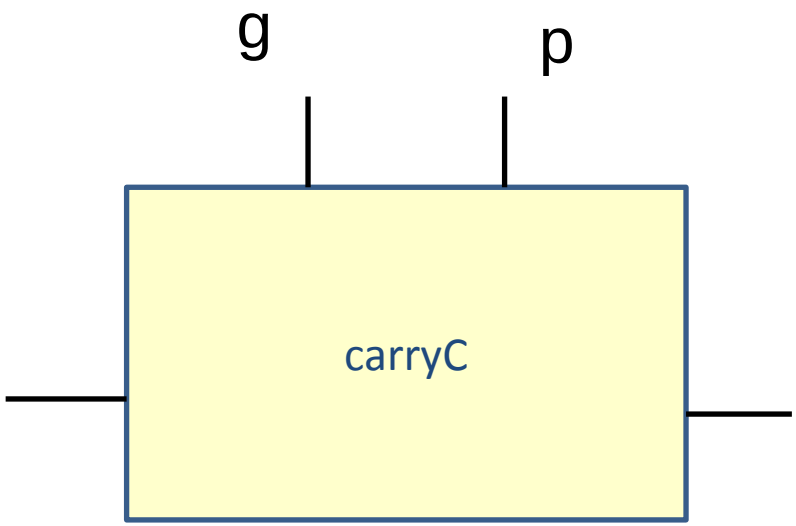
isolated the carry calculation



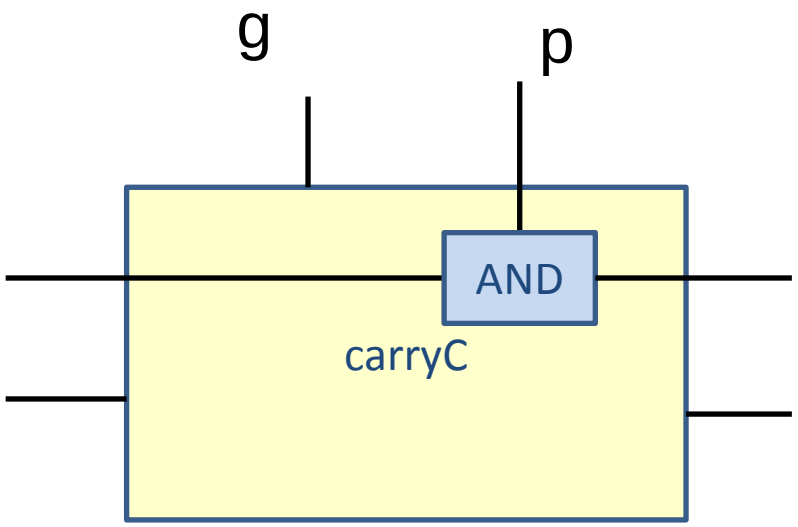
Brent and Kung's insight

If we can find an associatve operator that still computes the same thing when placed in a row, then we will be able to do much better than a linear array!

enhance carryC



enhance carryC



dotOp

enhance carryC

```
dotOp :: ((Bit,Bit), (Bit,Bit)) -> (Bit,Bit)
dotOp ((g1,p1), (g2,p2))= (carryC (g1, (g2,p2)), p1 <&> p2)
```

enhance carryC

```
dotOp :: ((Bit,Bit), (Bit,Bit)) -> (Bit,Bit)
dotOp ((g1,p1), (g2,p2))= (carryC (g1, (g2,p2)), p1 <&> p2)
```

Need also to compensate for this change so that the entire circuit retains its function

enhance carryC

```
adder4 :: [(Bit, Bit)] -> ([Bit], Bit)
adder4 abs = (ss,cout)
  where
    gps         = map gpC abs
    (cs,cout) = (row (f1 dotOp) ->- (map fst -|- fst)) ((low,high), gps)
    rs           = zip cs gps
    ss          = map sumC rs
```

enhance carryC

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adder4 :: [(Bit, Bit)] -> ([Bit], Bit)
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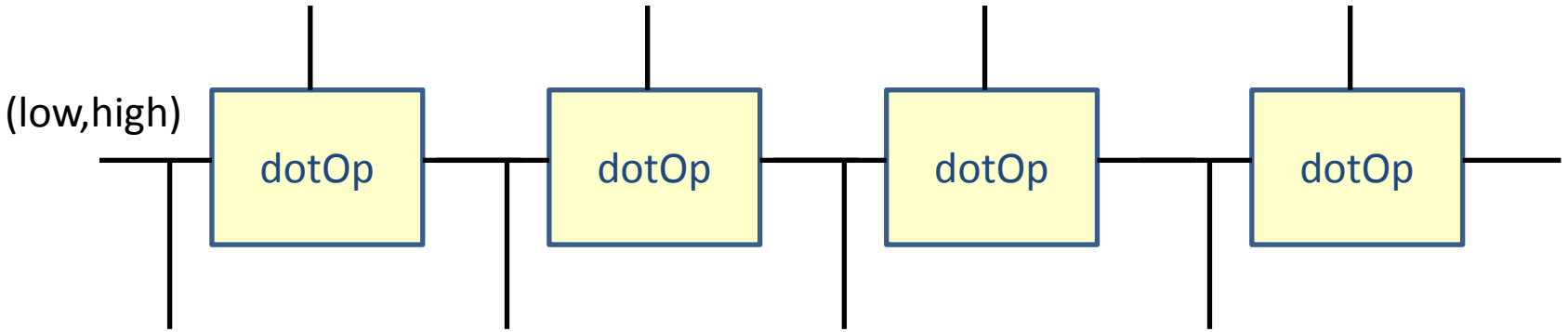
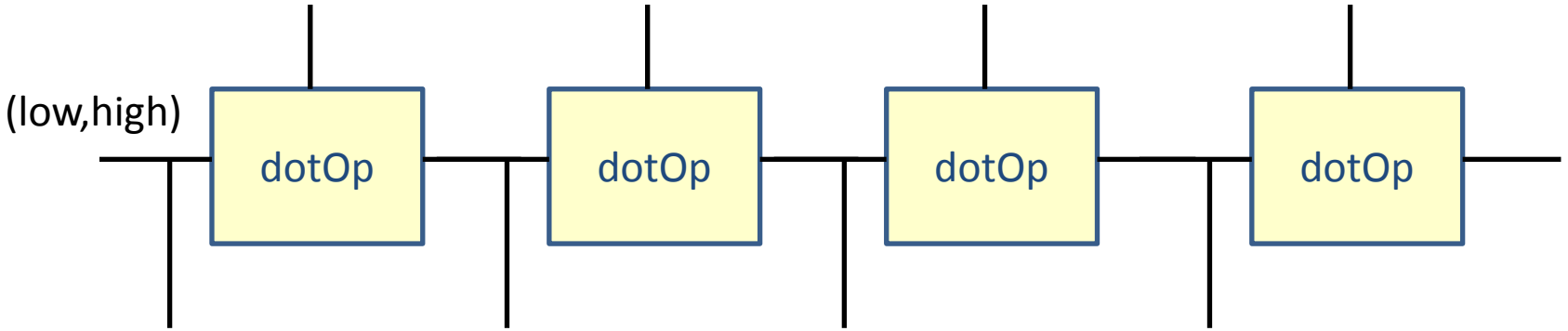
```

New situation (assoc. op.)



New situation (assoc. op.)



Matches the famous Prefix Problem!

Prefix

Given inputs x1, x2, x3 … xn

Compute x1, x1*x2, x1*x2*x3, … , x1*x2*…*xn

Where * is an arbitrary associative (but not necessarily commutative) operator

Why interesting?

Microprocessors contain LOTS of parallel prefix circuits

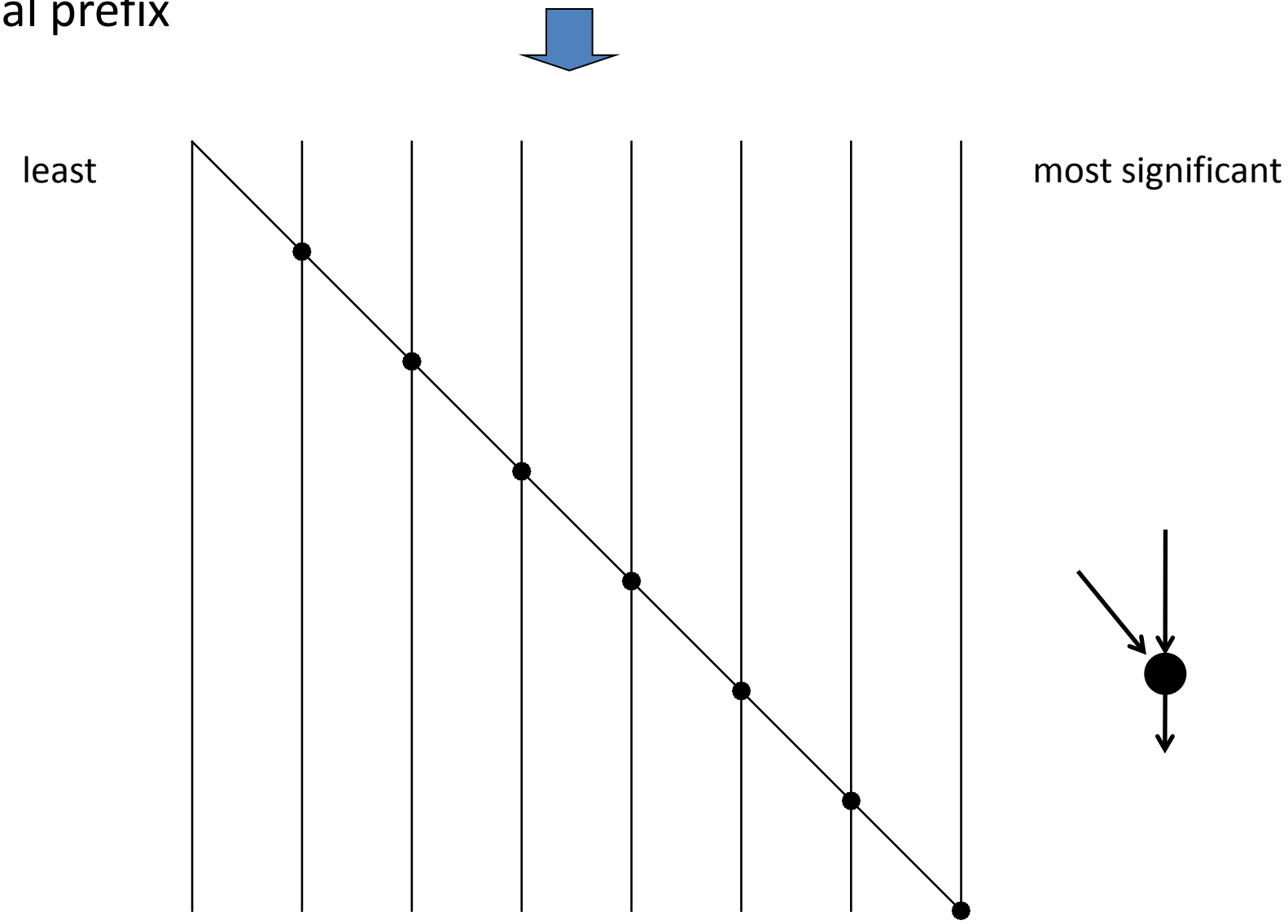
- not only binary and FP adders
- address calculation
- priority encoding etc.

Overall performance depends on making them fast

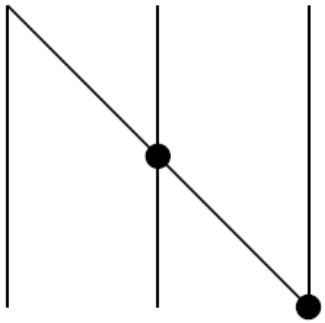
But they should also have low power consumption...

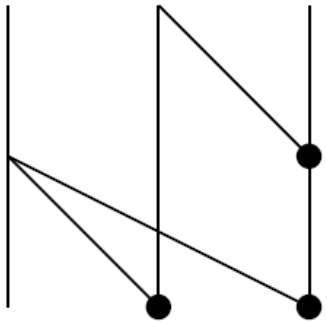
Parallel prefix is a good example of a connection pattern for which it is interesting to do better synthesis

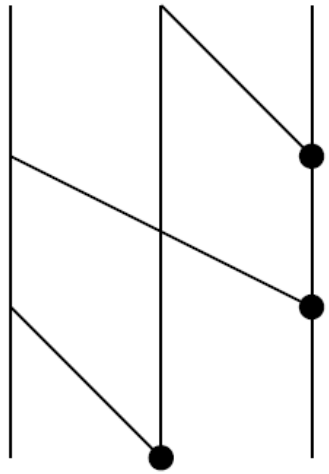
Serial prefix



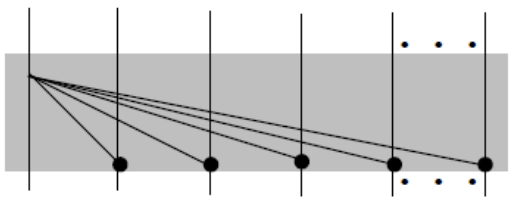
3 more

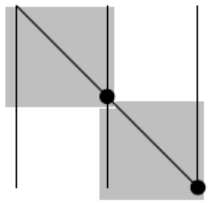


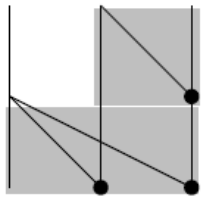


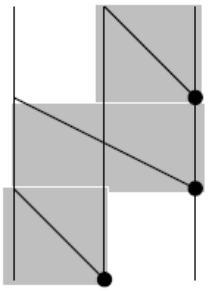


fan









```
ser3 :: PP a
ser3 f [a,b,c] = [a1,b2,c2]
  where
    [a1,b1] = f [a,b]
    [b2,c2] = f [b1,c]

f31 :: PP a
f31 f [a,b,c] = [a1,b2,c2]
  where
    [b1,c1]    = f [b,c]
    [a1,b2,c2] = f [a,b1,c1]

f32 :: PP a
f32 f [a,b,c] = [a2,b2,c2]
  where
    [b1,c1] = f [b,c]
    [a1,c2] = f [a,c1]
    [a2,b2] = f [a1,b1]
```

```
type Fan a = [a] -> [a]

type PP a = Fan a -> [a] -> [a]
```

```
mkFan :: ((a,a) -> a) -> Fan a
mkFan op (i:is) = i:[op(i,k) | k <- is]

pplus :: Fan (Signal Int)
pplus = mkFan plus

delFan ::  Fan (Signal Int)
delFan [i] = [i]
delFan is = replicate n (1 + maximum is)
  where
    n = length is
```

```
t3 = simulate (ser3 pplus) [1,2,3]
> t3
[1,3,6]
```

```
t3d = simulate (ser3 delFan) [0,0,0]
> t3d
[1,2,2]
```

serial prefix

```
ser :: PP a
ser _  [a]      = [a]
ser f  (a:b:bs) = a1:cs
  where
    [a1,a2] = f [a,b]
    cs      = ser f (a2:bs)
```

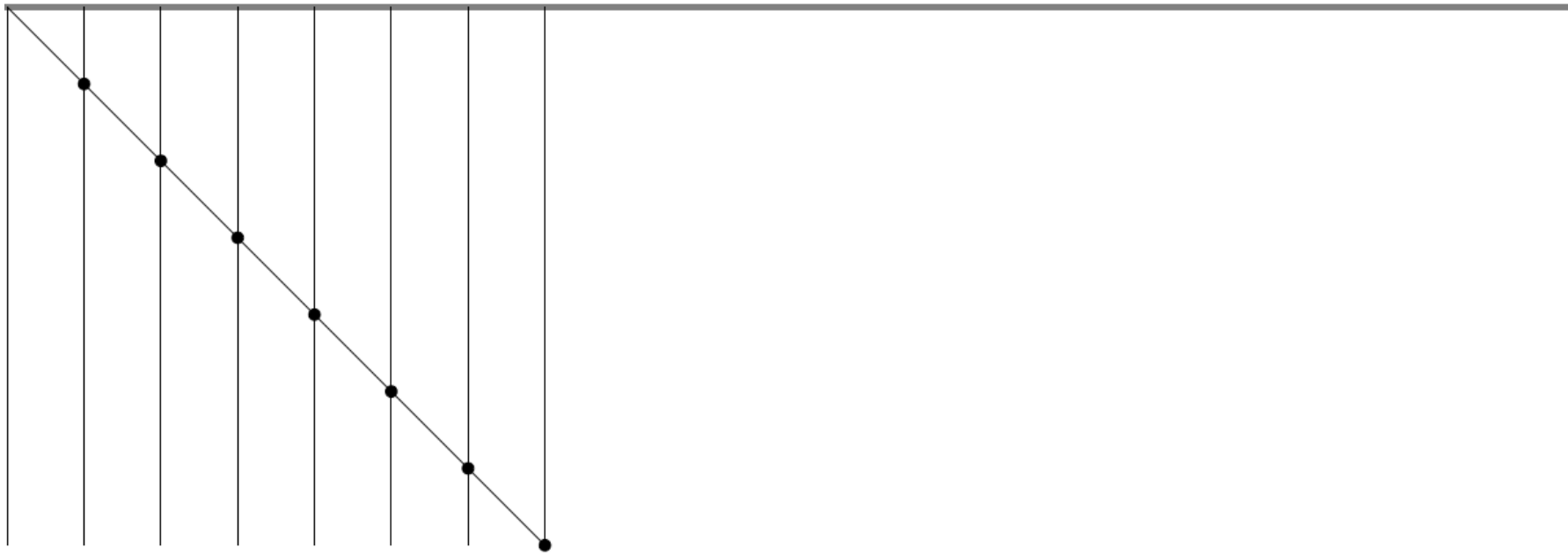
serial prefix

```
ser :: PP a
ser _  [a]      = [a]
ser f  (a:b:bs) = a1:cs
  where
    [a1,a2] = f [a,b]
    cs      = ser f (a2:bs)
```

serial prefix

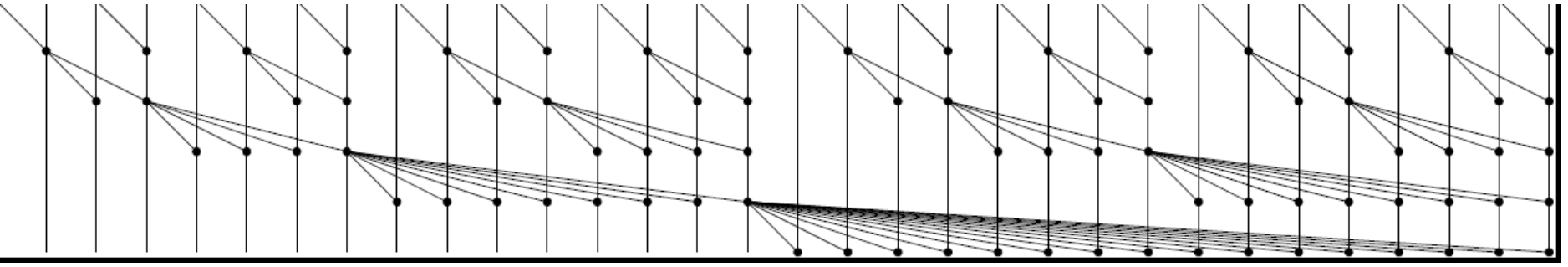
```
ser :: PP a
ser _  [a]      = [a]
ser f  (a:b:bs) = a1:cs
  where
    [a1,a2] = f [a,b]
    cs      = ser f (a2:bs)

> mdraw "ser" ser 8
```

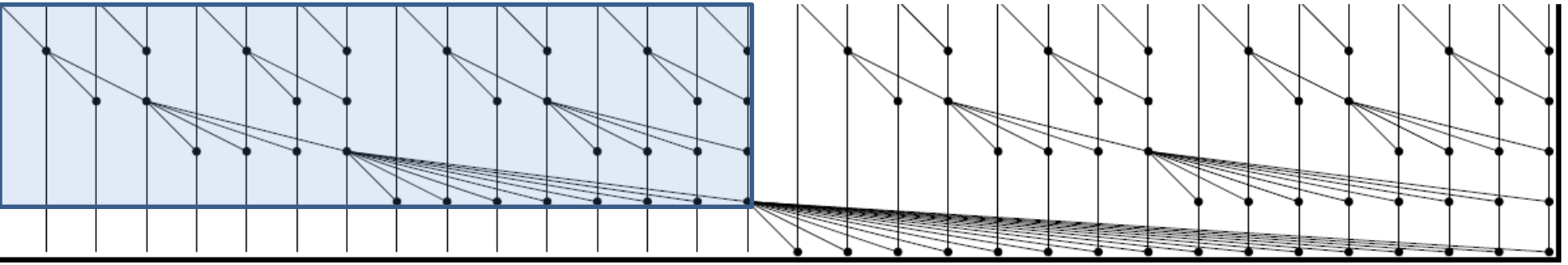


8 lines, 7 stages, 7 operators, 2 maximum fanout.

Sklansky



Sklansky



32 lines 5 stages (= minimum) 80 operators

```
skl :: PP a
skl _ [a] = [a]
skl f as  = init los ++ ros'
  where
    (los,ros) = (skl f las, skl f ras)
    ros'        = f (last los : ros)
    (las,ras) = splitAt (cnd2 (length as)) as

cnd2 n = n - n `div` 2 -- Ceiling of n/2
```

back to the adder!

```
adder4 :: [(Bit, Bit)] -> ([Bit], Bit)
adder4 abs = (ss,cout)
  where
    gps         = map gpC abs
    (cs,cout) = (row (f1 dotOp) ->- (map fst -|- fst)) ((low,high), gps)
    rs           = zip cs gps
    ss          = map sumC rs
```

if (cs,c) = row (f1 circ) (e, as)

and e is an identity of circ

then

cs ++ [c] = e : ser (mkFan circ)

back to the adder!

```
adder5 :: [(Bit, Bit)] -> ([Bit], Bit)
adder5 abs = (ss,cout)
  where
    gps        = map gpC abs
    (cs,cout) = (ser (mkFan dotOp) ->- unsnoc ->- (map fst -|- fst) ) gps
    rs          = zip (low:cs) gps
    ss          = map sumC rs
```

slight optimisation (remove low)

```
adder6 :: [(Bit, Bit)] -> ([Bit], Bit)
adder6 abs = (ss,cout)
  where
    gps             = map gpC abs
    (cs,cout)       = (ser (mkFan dotOp) ->- unsnoc ->- (map fst -|- fst) ) gps
    ((_,p) : gps') = gps
    rs              = zip cs gps'
    ss              = p : map sumC rs
```

BUT now we can use any prefix network we fancy

and there are lots to choose from!

back to the adder!

```
adder7 :: [(Bit, Bit)] -> ([Bit], Bit)
adder7 abs = (ss,cout)
  where
    gps             = map gpC abs
    (cs,cout)       = (skl (mkFan dotOp) ->- unsnoc ->- (map fst -|- fst) ) gps
    ((_,p) : gps') = gps
    rs              = zip cs gps'
    ss              = p : map sumC rs
```

back to the adder!

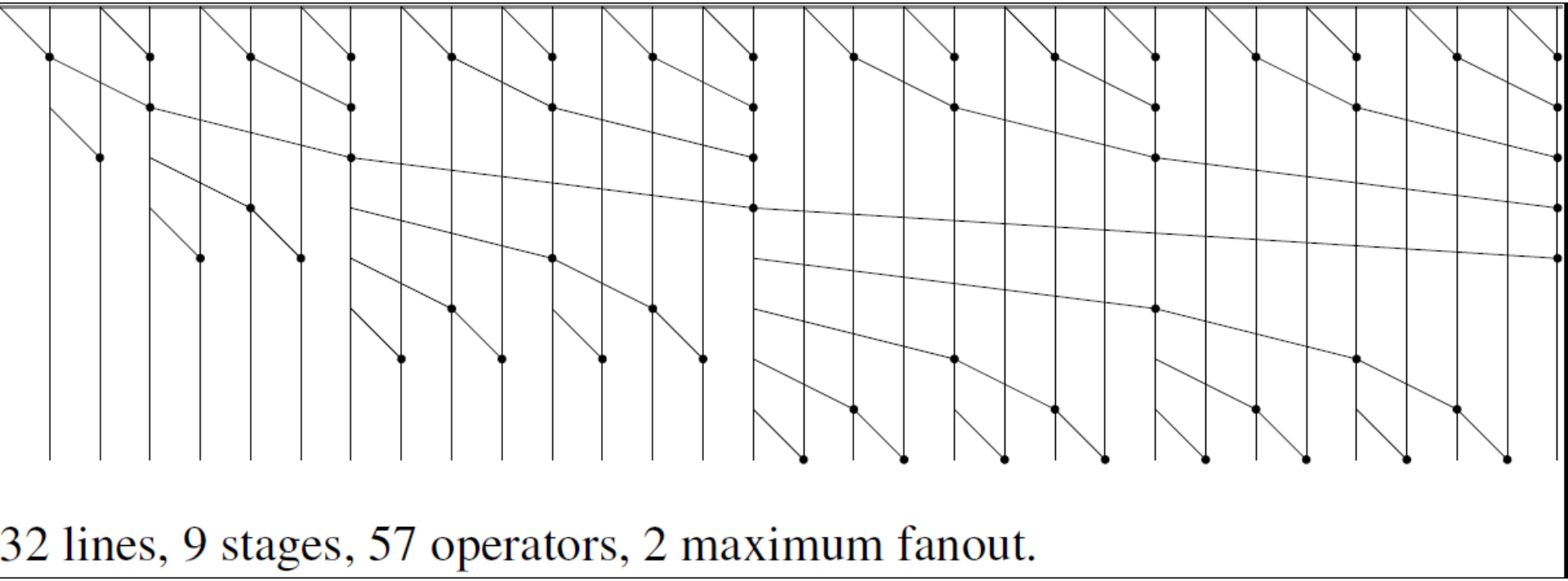
```
adder7 :: [(Bit, Bit)] -> ([Bit], Bit)
adder7 abs = (ss,cout)
  where
    gps            = map gpC abs
    (cs,cout)      = (skl (mkFan dotOp) ->- unsnoc ->- (map fst -|- fst) ) gps
    ((_,p) : gps') = gps
    rs             = zip cs gps'
    ss             = p : map sumC rs
```

Size (= power consumption) and performance completely dominated by the prefix network

Could (and should) parameterise on the pattern

Some more

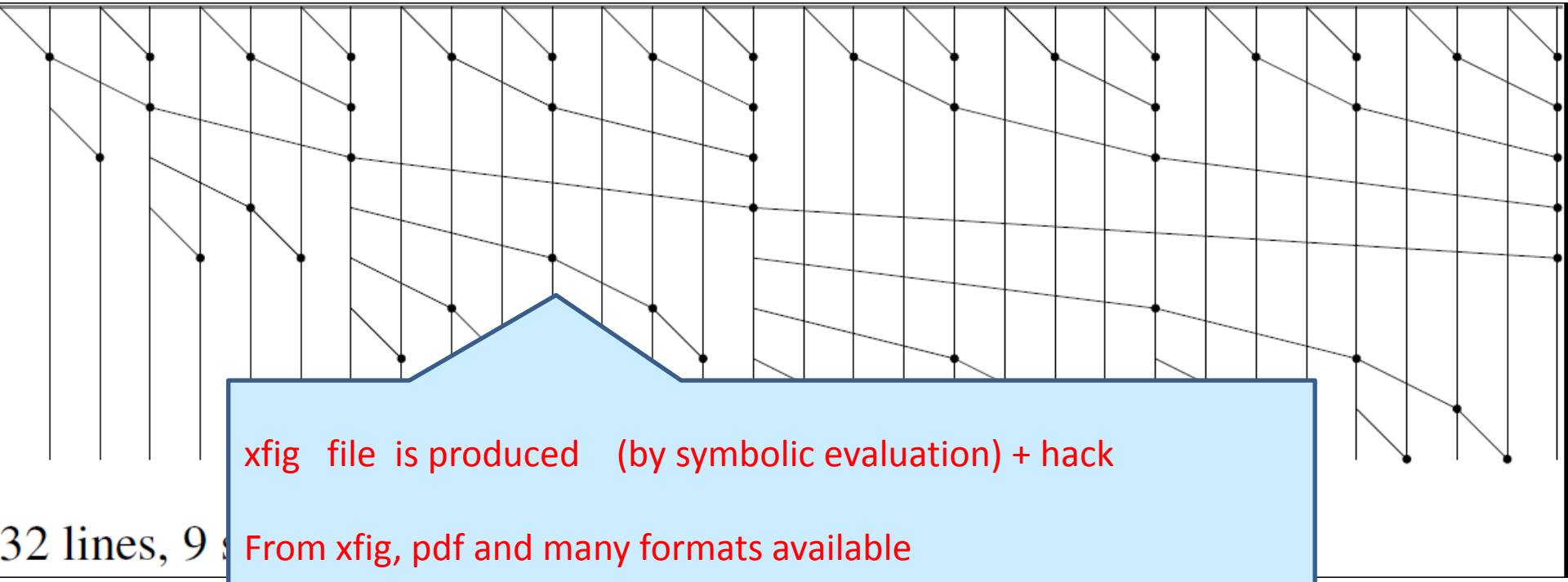
> mdraw "bK" bKung 32



32 lines, 9 stages, 57 operators, 2 maximum fanout.

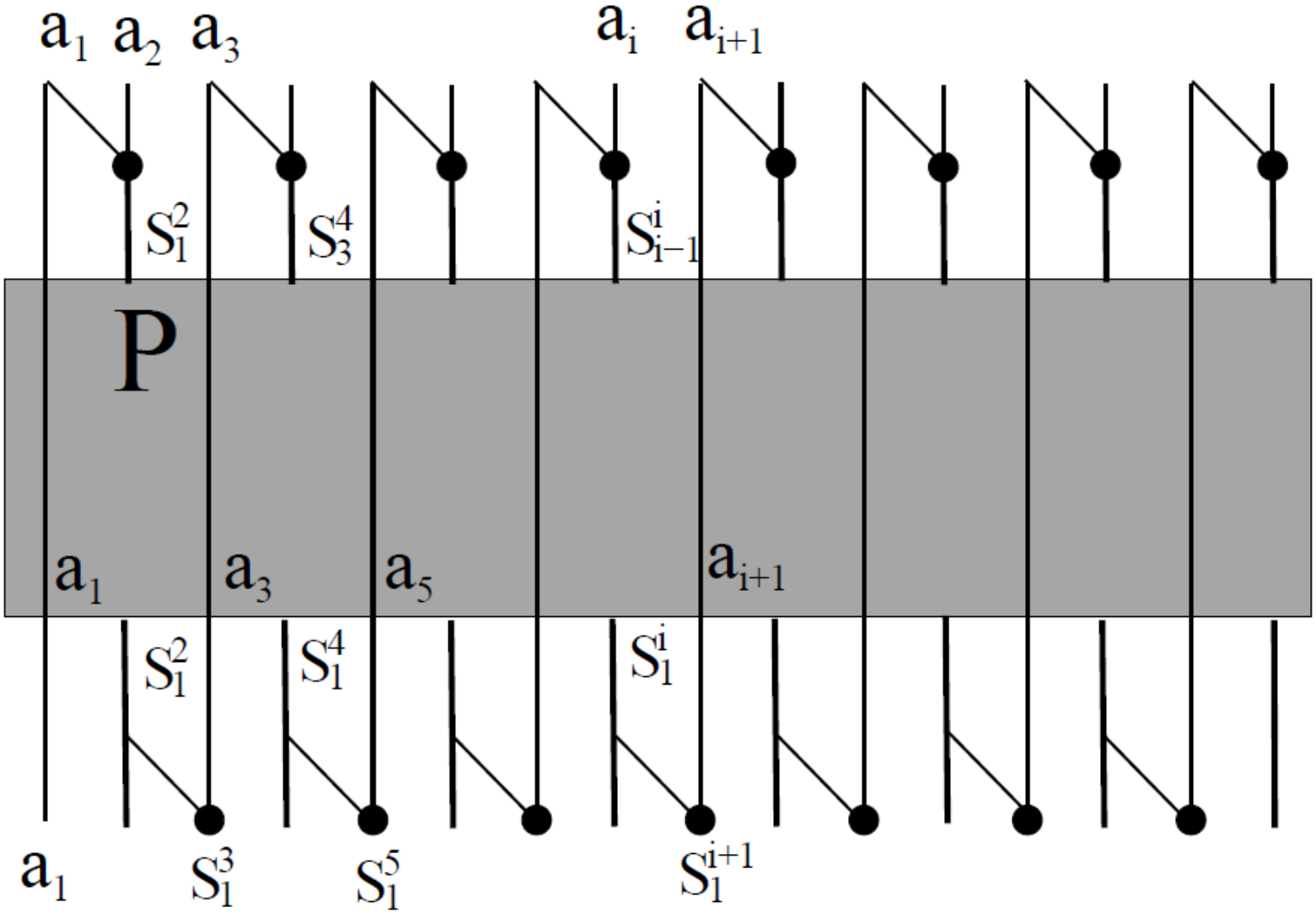
Some more

> mdraw "bK" bKung 32



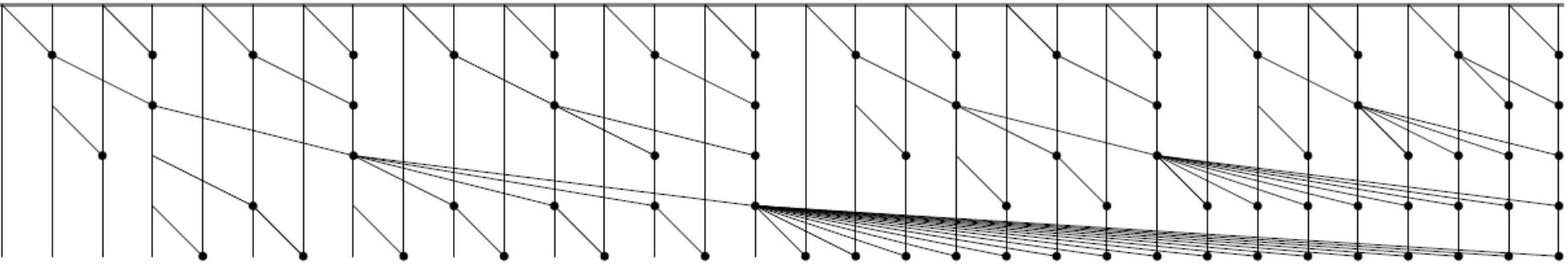
xfig file is produced (by symbolic evaluation) + hack

From xfig, pdf and many formats available


a_1 a_2 a_3
a_i a_{i+1}
S_1^2
S_3^4
S_{i-1}^i
P
a_1
a_3
a_5
a_{i+1}
S_1^2
S_1^4
S_1^i
a_1
S_1^3
S_1^5
S_1^{i+1}

Ladner Fischer min. depth

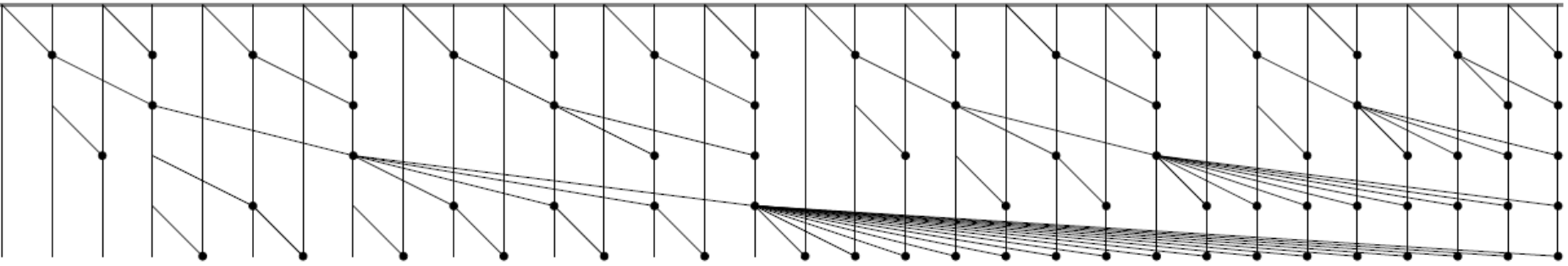
*Main> mdraw "LF0" (ladF 0) 32



32 lines, 5 stages, 74 operators, 17 maximum fanout.

Ladner Fischer min. depth

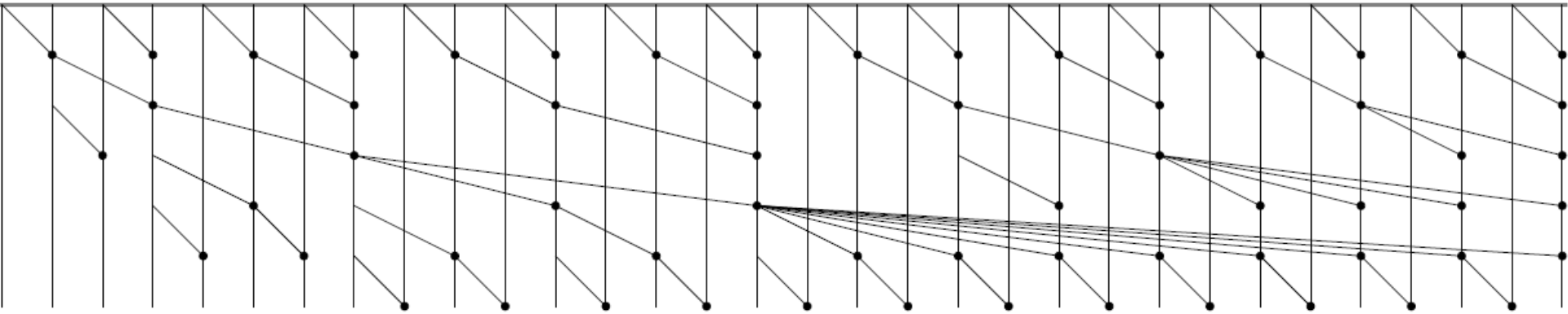
*Main> mdraw "LF0" (ladF 0) 32



32 lines, 5 stages, 74 op 17 maximum fanout.

Beware. Many papers and books are wrong about LF (and think it is the same as Sklansky). It is not!

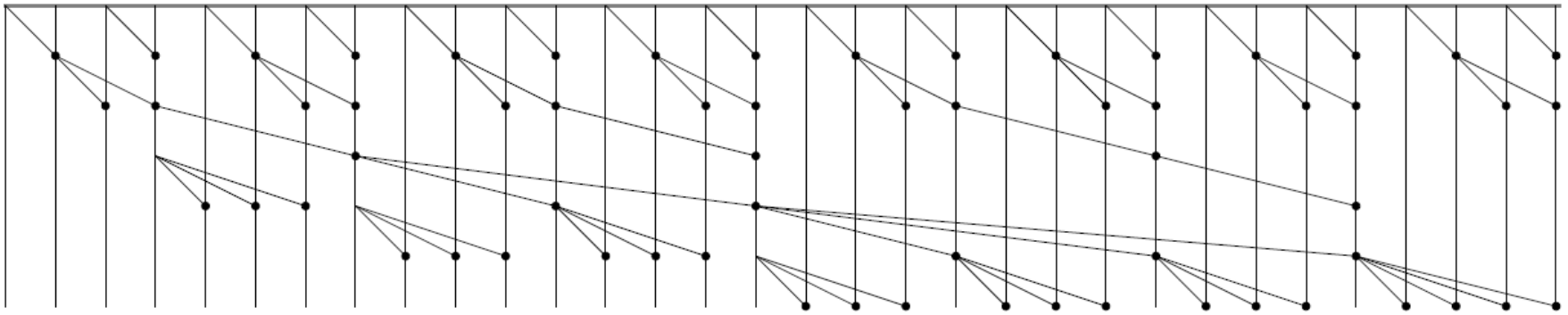
LF 1 + min depth



32 lines, 6 stages, 62 operators, 9 maximum fanout.

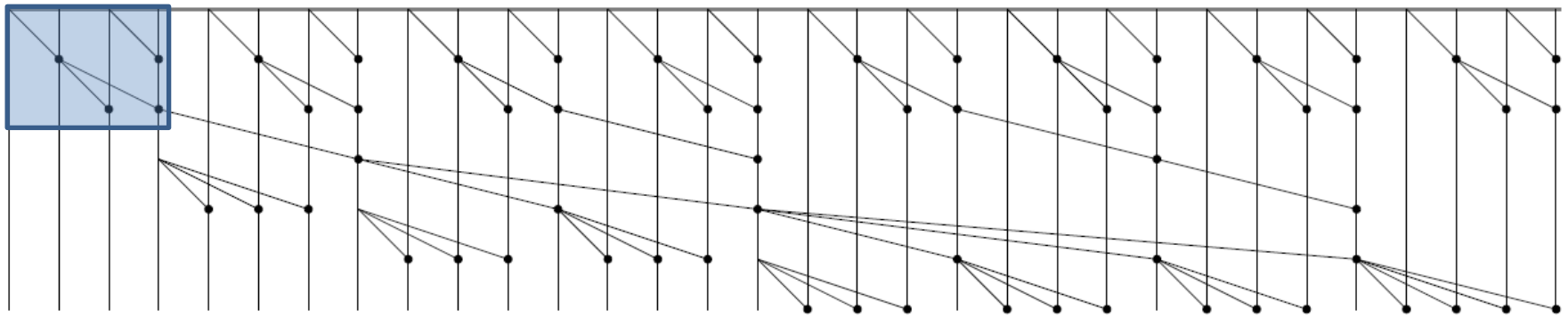
(Code for Ladner Fischer will be provided)

and more



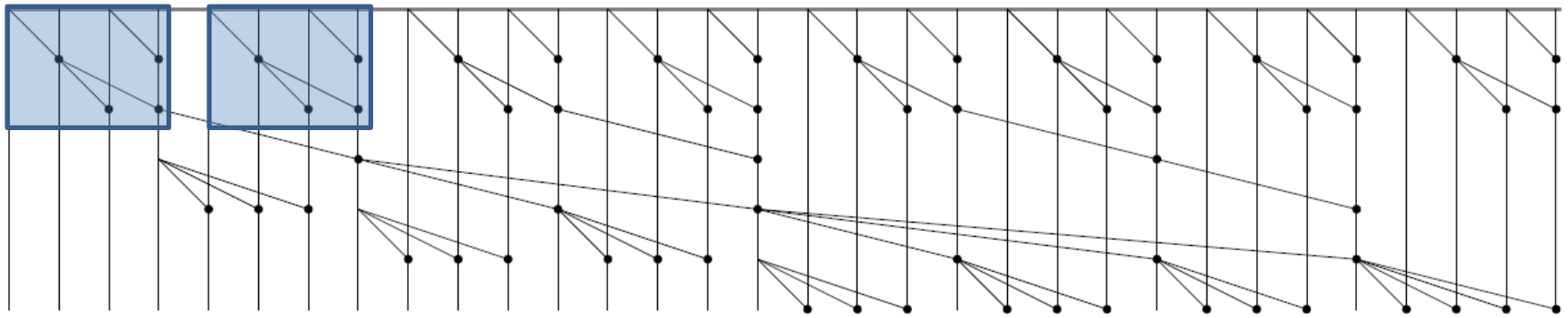
32 lines, 6 stages, 63 operators, 5 maximum fanout.

and more



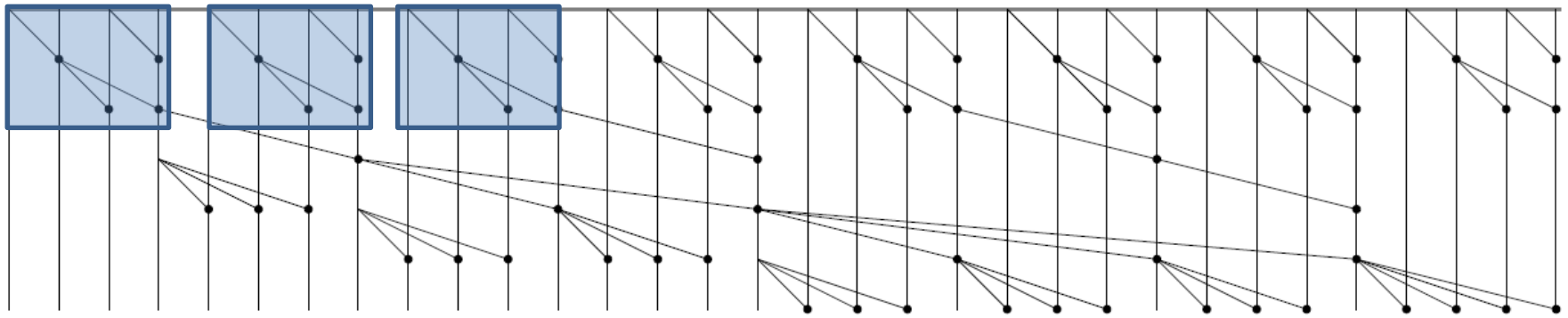
32 lines, 6 stages, 63 operators, 5 maximum fanout.

and more



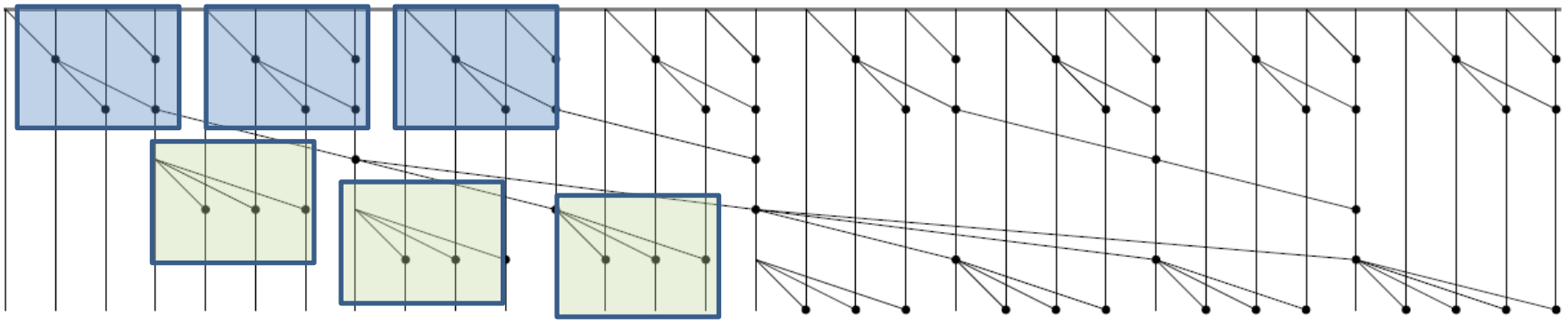
32 lines, 6 stages, 63 operators, 5 maximum fanout.

and more



32 lines, 6 stages, 63 operators, 5 maximum fanout.

and more



32 lines, 6 stages, 63 operators, 5 maximum fanout.

Problem

Find a sweet spot LAGOM

Not too big

Not too deep

Not too much fanout

Questions?