Computing reflection and refraction rays

Slide 2 is important

Slides 3-8 are bonus material

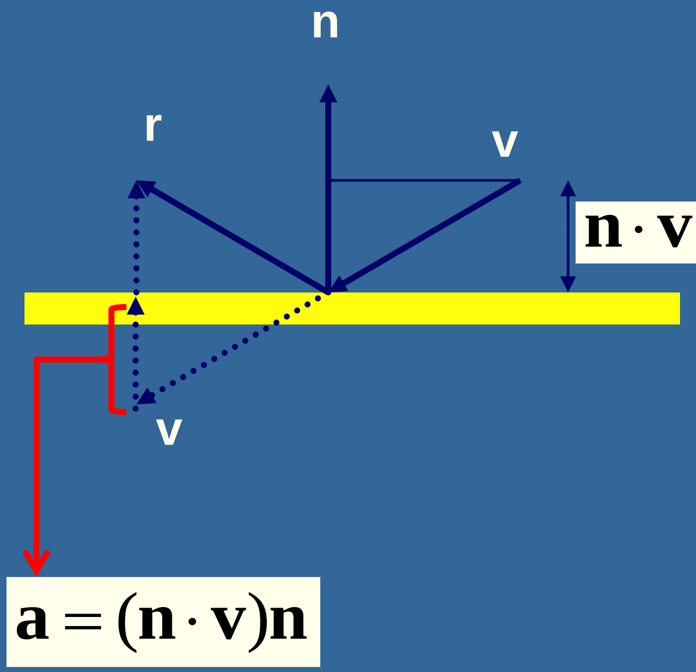
Important

Reflection vector (recap)

- Reflecting the incoming ray **v** around **n:**
- **N**ote that the incoming ray is sometimes called –**v** depending on the direction of the vector.
- **r** can be computed as **v**+(2**a**). I.e.,

$$\mathbf{r} = \mathbf{v} + 2(\mathbf{n} \cdot \mathbf{v})\mathbf{n}$$

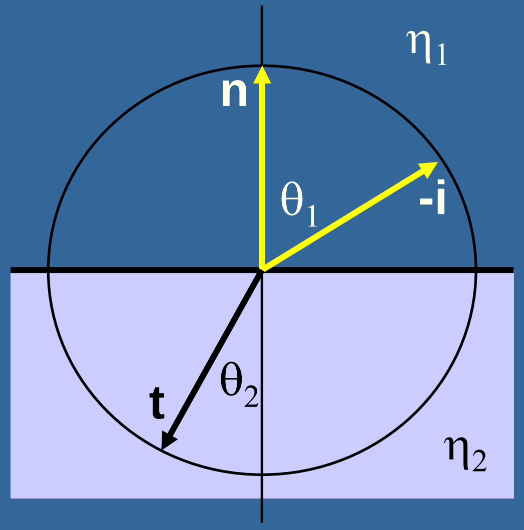
where n is unit vector

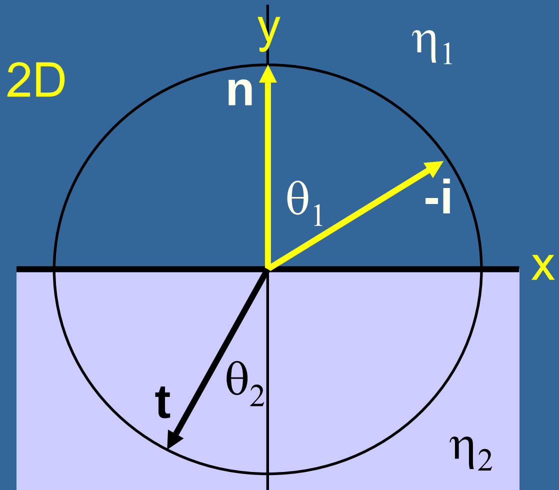


Important

Refraction: Need a transmission direction vector, t

- **n**, **i**, **t** are unit vectors
- η_1 & η_2 are refraction indices
- Snell's law says that:
  - $\sin(\theta_2)/\sin(\theta_1) = \eta_1/\eta_2 = \eta$, where η is relative refraction index.
- How can we compute the refraction vector **t** ?
- This would be easy in 2D:
  - $t_x = -\sin(\theta_2)$
  - $t_y = -\cos(\theta_2)$
  - I.e., $\mathbf{t} = -\sin(\theta_2)\hat{\mathbf{x}} - \cos(\theta_2)\hat{\mathbf{y}}$





Known as Heckbert's method

Important

Refraction:

- **But we are in 3D, not in 2D!**
- So, the solution will look like:

$$\mathbf{t} = -\sin(\theta_2)\hat{\mathbf{v}}_1 - \cos(\theta_2)\hat{\mathbf{v}}_2$$

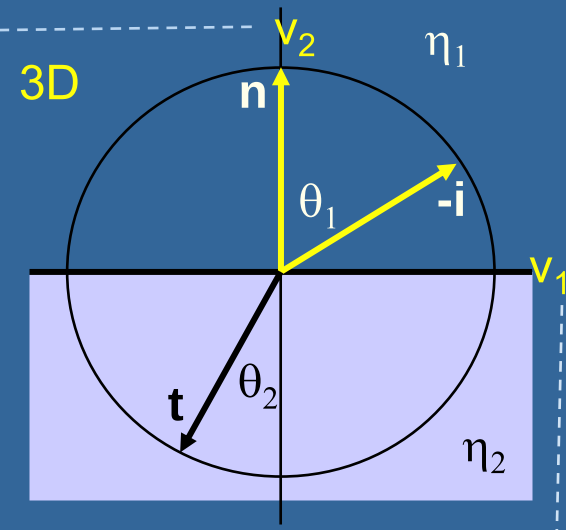
instead of $\mathbf{t} = -\sin(\theta_2)\hat{\mathbf{x}} - \cos(\theta_2)\hat{\mathbf{y}}$

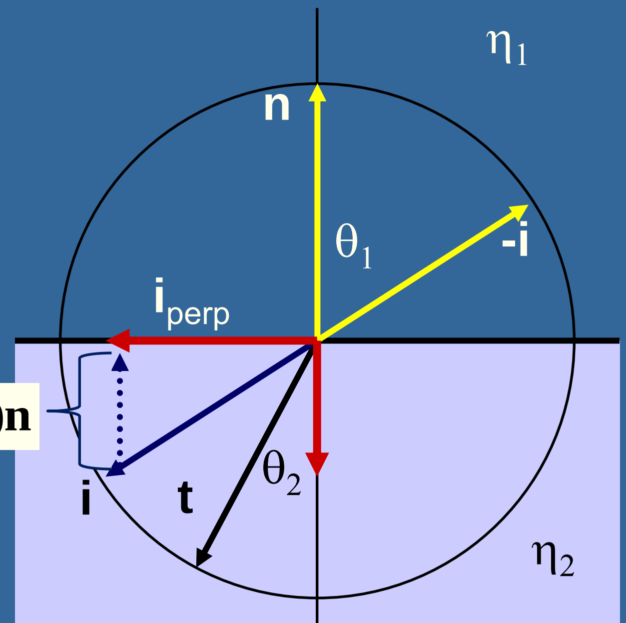
where $\mathbf{v}_2$=**n** and $\mathbf{v}_1$ is perpendicular to **n** and lies in the reflection plane.

- Similar to how we computed the reflection ray, such a vector is

$$\mathbf{i}_{perp} = \mathbf{i} + \cos(\theta_1)\mathbf{n}$$

$$(-\mathbf{i}\cdot\mathbf{n})\mathbf{n} = \cos(\theta_1)\mathbf{n}$$





Known as Heckbert's method

Important

Refraction:

- We also need to normalize the vectors to get unit vectors:
  - $\mathbf{v}_2 = \mathbf{n}$, because **n** already normalised
  - $\mathbf{v}_1 = \mathbf{i}_{perp} / \| \mathbf{i}_{perp} \|$, where $\|\mathbf{i}\| = \sqrt{i_x^2 + i_y^2 + i_z^2}$
- Snell's law gives:
  - $\sin(\theta_2) = \eta \sin(\theta_1)$
- We now have everything to compute

$$\mathbf{t} = -\sin(\theta_2)\hat{\mathbf{v}}_1 - \cos(\theta_2)\hat{\mathbf{v}}_2$$

So we could concider us done

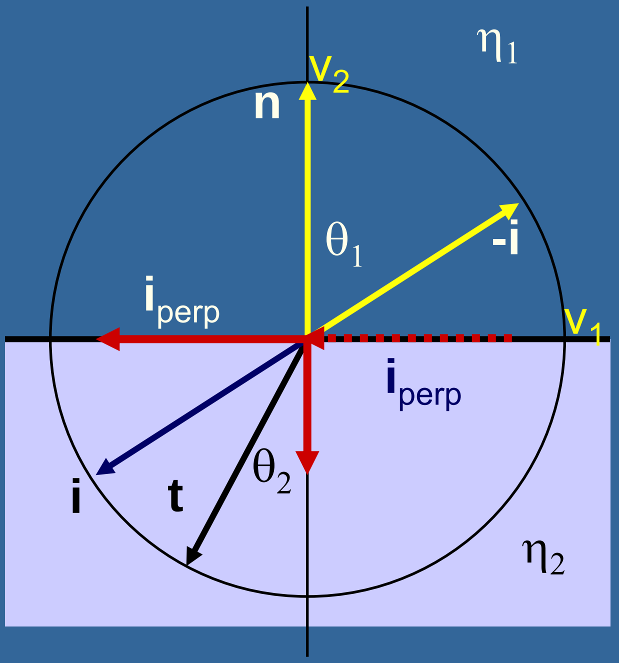
- But let us continue simplifying…

Known as Heckbert's method

Bonus

Refraction:

- If we are smart, we realize from the trigonometric laws (see figure) that the length of $\mathbf{i}_{perp} = \sin(\theta_1)$
  I.e., $\| \mathbf{i}_{perp} \| = \sin(\theta_1)$



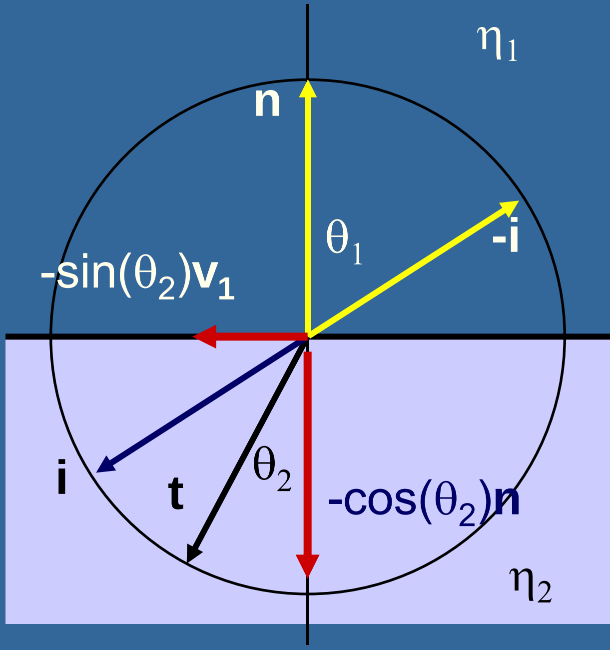
- We already know that

$$\mathbf{t} = -\sin(\theta_2)\hat{\mathbf{v}}_1 - \cos(\theta_2)\hat{\mathbf{v}}_2$$

$\mathbf{v}_1 = \mathbf{i}_{perp} / \| \mathbf{i}_{perp} \| = (\mathbf{i}+\cos(\theta_1)\mathbf{n}) / \sin(\theta_1)$

$\mathbf{v}_2 = \mathbf{n}$ (assuming that $\mathbf{n}$ is normalized)

Thus: $t = -\sin(\theta_2)(\mathbf{i}+\cos(\theta_1)\mathbf{n}) / \sin(\theta_1) - \cos(\theta_2)\mathbf{n}$

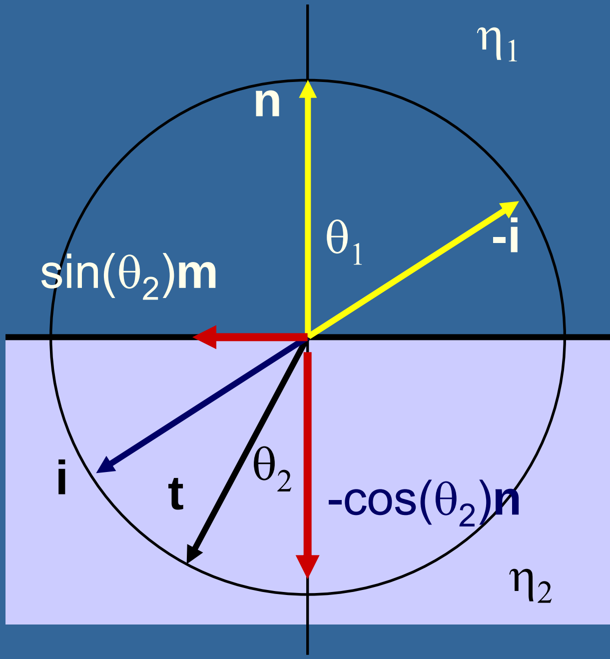


Known as Heckbert's method

Modified by Ulf Assarsson, 2004

Bonus

Refraction:



- **t** = $\sin(\theta_2)$ (**i**+$\cos(\theta_1)$**n**)/$\sin(\theta_1)$ - $\cos(\theta_2)$**n**
- Use Snell's law:
  - $\sin(\theta_2)/\sin(\theta_1) = \eta_1/\eta_2 = \eta$
- **i.e., t** = η**i** + ($\eta\cos(\theta_1)$ - $\cos(\theta_2)$) **n**

  $\cos(\theta_2)$ is still expensive to compute
  since: $\cos(\theta_2)=\cos(\arcsin(\eta\sin(\theta_1))$

  So we continue simplifying…
- Simplify: $\cos(\theta_2) = \mathrm{sqrt}[\,1 - \eta^2(1-(\cos(\theta_1))^2)\,]$, since
  1. Pythagorean theorem: $\cos(\theta_2)^2 = 1^2 - \sin(\theta_2)^2$
  2. Snell's Law: $\sin(\theta_2) = \eta \sin(\theta_1)$
  3. (1) + (2)^2 gives: $\sin(\theta_2)^2 = \eta^2 (1-\cos(\theta_1)^2)$
  4. (3) + (1) gives: $1^2 - \cos(\theta_2)^2 = \eta^2 (1-\cos(\theta_1)^2)$

Known as Heckbert's method

Refraction

- Thus:

  $\mathbf{t} = \eta\mathbf{i} + (\eta\cos(\theta_1) - \mathrm{sqrt}[\ 1 - \eta^2(1-(\cos(\theta_1))^2)\ ])\mathbf{n}$

  This is fast to compute since

  $\cos(\theta_1) = -\mathbf{n}\cdot\mathbf{i}$

  which only requires a simple dot product