Software Engineering using Formal Methods

Model Checking with Temporal Logic

Wolfgang Ahrendt & Wojciech Mostowski & Richard Bubel

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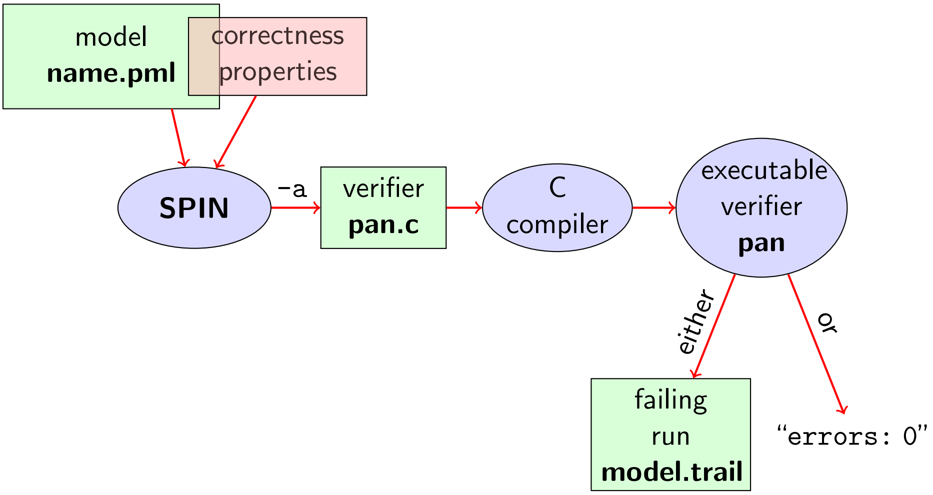
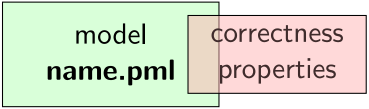
Model Checking with SPIN



Stating Correctness Properties



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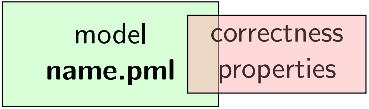

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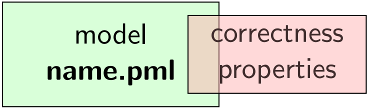

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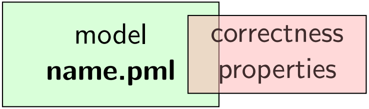

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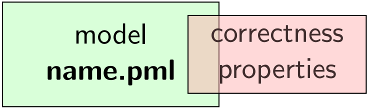

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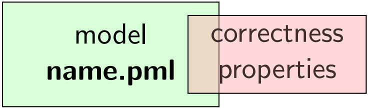

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Preliminaries

1. accept labels in PROMELA $\leftrightarrow$ Büchi automata
2. fairness

Preliminaries 1: Acceptance Cycles

Definition (Accept Location)

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A run which infinitely often passes through an accept location is called an acceptance cycle.

Acceptance cycles are mainly used in *'never claims'* (see below), to define forbidden behavior of infinite kind.

Preliminaries 2: Fairness

Does the following PROMELA model necessarily terminate?

```
byte n = 0;
bool flag = false;

active proctype P() {
  do ::  flag -> break;
     ::  else -> n = 5 - n;
  od
}
active proctype Q() {
  flag = true
}
```

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Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Model Checking of Temporal Properties

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today:

model checking of properties formulated in temporal logic

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Remark:

in this course, "temporal logic" is synonymous to *"linear temporal logic" (LTL)*

Beyond Assertions

Assertions only talk about the state ‘at their own location’ in the code.

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- no separation of concerns (model vs. correctness property)
- changing assertions is error prone (easily out of synch)
- easy to forget assertions:
  correctness property might be violated at unexpected locations
- many interesting properties not expressible via assertions

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"If some processes try to enter their critical section, eventually *one of them* does so."

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"If one process tries to enter its critical section, eventually *that process* does so."

all these are temporal properties $\Rightarrow$ *use temporal logic*

Boolean Temporal Logic

talking about numerical variables (like in `critical <= 1` or `0 <= i <= len-1`) requires variation of *propositional temporal logic* which we call Boolean temporal logic:

- Boolean expressions (over PROMELA variables), rather than *propositions*, form basic building blocks of the logic

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

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- if `P` is a process and `l` is a label in `P`, then `P@l` is $\in For_{BTL}$ (`P@l` reads "`P` is at `l`")
- if ϕ and ψ are formulas $\in For_{BTL}$, then all of

$$!\phi, \quad \phi \;\&\&\; \psi, \quad \phi \;||\; \psi, \quad \phi \;\texttt{->}\; \psi, \quad \phi \;\texttt{<->}\; \psi$$
$$[\,]\phi, \quad <>\phi, \quad \phi\, \mathtt{U}\, \psi$$

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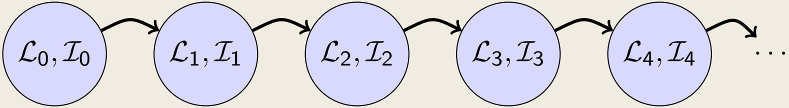
Semantics of Boolean Temporal Logic

A run σ through a PROMELA model M is a chain of states



$\mathcal{L}_j$ maps each running process to its current location counter.
From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous).
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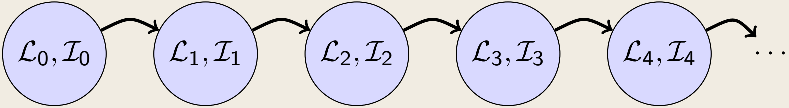

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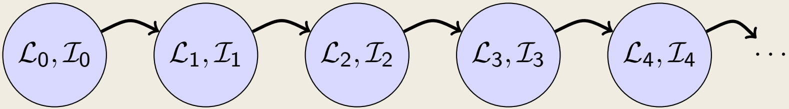

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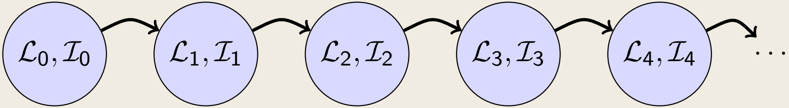

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Evaluating other formulas $\in For_{BTL}$ in runs σ: see previous lecture.

Boolean Temporal Logic Support in Spin

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instead

Boolean expressions must be abbreviated using **#define**

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example: ‘`[](critical <= 1)`’

“it is guaranteed throughout each run that at most one process visits its critical section”

or equivalently:
“more than one process visiting its critical section will never happen”

Applying Temporal Logic to Critical Section Problem

We want to verify '[](critical<=1)' as correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        critical++;
        /* critical activity */
        critical--;
        inCriticalP = false
  od
}

/* similarly for process Q */
```

Model Checking a Safety Property with jSpin

1. add ‘**#define** `mutex (critical <= 1)`’ to Promela file
2. open Promela file
3. enter `[]mutex` in LTL text field
4. select `Translate` to create a ‘never claim’, corresponding to the negation of the formula
5. ensure `Safety` is selected
6. select `Verify`
7. (if necessary) select `Stop` to terminate too long verification

Never Claims

- a never claim tries to show the user wrong
- it defines, in terms of PROMELA, all violations of correctness property
- it is semantically equivalent to the negation of correctness property
- JSPIN adds the negation for you
- using SPIN directly, you have to add the negation yourself
- accept labels in never claims mark accepting states
  in the sense of Büchi automata

Model Checking PROMELA wrt. Temporal Logic

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3. If

$$\mathcal{L}^\omega(\mathcal{M}) \cap \mathcal{L}^\omega(\mathcal{NC}_{\neg\phi}) = \emptyset$$

   then ϕ holds in $\mathcal{M}$,
   otherwise we have a counterexample.

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otherwise we have a counterexample.
4. To check $\mathcal{L}^\omega(\mathcal{M}) \cap \mathcal{L}^\omega(\mathcal{NC}_{\neg\phi})$ construct intersection automaton (both automata advance in each step) and search for accepting run.

Model Checking a Safety Property with SPIN directly

Command Line Execution

make sure ‘#define mutex (critical <= 1)’ *is in* safety1.pml

```
> spin -a -f '!([] mutex)' safety1.pml
> gcc -DSAFETY -o pan pan.c
> ./pan
```

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example: ‘<>csp’

(with csp a variable only true in the critical section of P)

“in each run, process P visits its critical section eventually”

Applying Temporal Logic to Starvation Problem

We want to verify ‘<>csp’ as correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        csp = true;
        /* critical activity */
        csp = false;
        inCriticalP = false
  od
}

/* similarly for process Q */
/* here using csq          */
```

Model Checking a Liveness Property with jSpin

1. open Promela file
2. enter `<>csp` in LTL text field
3. select `Translate` to create a 'never claim', corresponding to the negation of the formula
4. ensure that `Acceptance` is selected
   (Spin will search for *accepting* cycles through the never claim)
5. *for the moment* uncheck `Weak Fairness` (see discussion below)
6. select `Verify`

Verification Fails

Verification fails.

Why?

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Verification fails.

Why?

The liveness property on one process 'had no chance'.
Not even weak fairness was switched on!

Model Checking Liveness with Weak Fairness!

Always switch Weak Fairness on when checking for liveness!

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4. ensure that `Acceptance` is selected
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5. ensure `Weak Fairness` is checked
6. select `Verify`

Model Checking Lifeness with SPIN directly

Command Line Execution

```
> spin -a -f '!<>csp' liveness1.pml
> gcc -o pan pan.c
> ./pan -a -f
```

Verification Fails

Verification fails again.

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Why?

Weak fairness is still too weak.

Verification Fails

Verification fails again.

Why?

Weak fairness is still too weak.

Note that `!inCriticalQ` is not continuously executable!

Temporal MC Without Ghost Variables

We want to verify mutual exclusion without using ghost variables

```
#define mutex !(P@cs && Q@cs)

bool inCriticalP = false, inCriticalQ = false;

active proctype P() {
  do :: atomic {
          !inCriticalQ;
          inCriticalP = true
        }
cs:     /* critical activity */
        inCriticalP = false
  od
}
/* similarly for process Q */
/* with same label cs:     */
```

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Verify `[]mutex` with JSPIN.

Liveness again

- revisit `fair.pml`
- try to prove termination

Literature for this Lecture

Ben-Ari Chapter 5