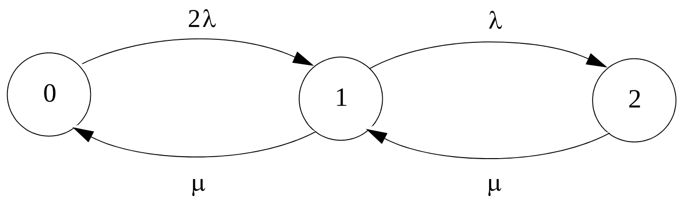
Blackboard examples, lecture 5

Availability for a hot-stand-by system with one spare module.

(See slide 19)

We obtain the following Markov chain modell



Since this is a birth-death process we know that

$$\Pi_1 = \frac{\lambda_0}{\mu_1} \cdot \Pi_0$$

$$\Pi_2 = \frac{\lambda_1 \cdot \lambda_0}{\mu_2 \cdot \mu_1} \cdot \Pi_0$$

where

$$\lambda_0 = 2\lambda \ , \ \lambda_1 = \lambda \text{ and } \mu_1 = \mu_2 = \mu$$

We thus obtain:

$$\Pi_1 = \frac{2\lambda}{\mu} \cdot \Pi_0$$

$$\Pi_2 = \frac{2\lambda^2}{\mu^2} \cdot \Pi_0$$

Using $\Pi_0 + \Pi_1 + \Pi_2 = 1$ we obtain the following expression for Π_0:

$$\Pi_0 + \frac{2\lambda}{\mu} \cdot \Pi_0 + \frac{2\lambda}{\mu^2} \cdot \Pi_0 = 1$$

$$\Pi_0 = \frac{1}{1 + \frac{2\lambda}{\mu} + \frac{2\lambda^2}{\mu^2}} = \frac{\mu^2}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

We obtain the following expression for Π_1

$$\Pi_1 = \frac{2\lambda}{\mu} \cdot \Pi_0 = \frac{2\lambda}{\mu} \cdot \frac{\mu^2}{\mu^2 + 2\lambda\mu + 2\lambda^2} = \frac{2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

The steady-state availability of the system is

$$\lim_{t \to \infty} A(t) = \Pi_0 + \Pi_1 = \frac{\mu^2 + 2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$