



Real-Time Systems

Exercise #4

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WCET Analysis using Shaw's Method

“The estimated WCET is the execution time of the longest structural path through the program.”

The following example is based on Problem 3 in the Exercise Compendium (Collection of Examples).

Example: WCET analysis

Problem: Consider the function `Calculate()`.

a) Using Shaw's method, estimate the WCET for function `Calculate()` in terms of 'Z' (with $Z \geq 0$).

```
int Calculate (int Z){  
    int R;  
    if(Z == 0)  
        R = 1;  
    else if (Z == 1)  
        R = 1;  
    else  
        R = Calculate(Z-1) + Calculate(Z-2);  
    return R;  
}
```

Example: WCET analysis

Problem: Consider the function `Calculate()`.

a) Using Shaw's method, estimate the WCET for function `Calculate()` in terms of 'Z' (with $Z \geq 0$).

- Each *declaration* or *assignment* statement costs 1 time unit
- Each *compare* statement costs 1 time unit
- Each *return* statement costs 1 time unit
- Each *addition* or *subtraction* operation costs 4 time units.
- A *function call* costs 2 time units plus WCET for the function in question.
- All other language constructs can be assumed to take 0 time units to execute.

Example: WCET analysis

Problem: Consider the function `Calculate()`.

- b) Function `main()` calls function `Calculate()` with parameter 5. What is the WCET of function `main()`?
- c) The deadline for executing function `main()` is 180 time units. Determine whether the deadline is met or not.

```
int main(){  
    int ans;  
    ans = Calculate(5);  
}
```

Example: WCET analysis

Problem: Now the program runs on a new processor that has a faster ALU. The execution costs of addition and subtraction are equal, but smaller than that of the older processor.

Let 'x' represent the execution time of an addition/subtraction operation. All other language constructs are assumed to have the same cost as in sub-problem a).

- d) What is the WCET for function `main()` in terms of 'x'?
- e) What is the maximum cost of an addition/subtraction operation so that the deadline of function `main()` is met?

Example: WCET analysis

a) Derive WCET for Calculate(Z)

Case 1: $Z == 0$

$$\text{WCET}(\text{Calculate}(0)) = \{\text{declare}, R\} + \{\text{compare}, Z==0\} + \\ \{\text{assign}, R\} + \{\text{return}, R\} = 1 + 1 + 1 + 1 = 4$$

Case 2: $Z == 1$

$$\text{WCET}(\text{Calculate}(1)) = \{\text{declare}, R\} + \{\text{compare}, Z==0\} + \{\text{compare}, Z==1\} + \\ \{\text{assign}, R\} + \{\text{return}, R\} = 1 + 1 + 1 + 1 + 1 = 5$$

Example: WCET analysis

Case 3: $Z > 1$

Let x represent the cost for addition and subtraction.

$$\begin{aligned}
 \text{WCET}(\text{Calculate}(Z > 1)) &= \{\text{declare}, R\} + \{\text{compare}, Z == 0\} + \{\text{compare}, Z == 1\} + \\
 &\quad \{\text{subtract}, Z - 1\} + \{\text{call}, \text{Calculate}(Z - 1)\} + \text{WCET}(\text{Calculate}(Z - 1)) + \\
 &\quad \{\text{subtract}, Z - 2\} + \{\text{call}, \text{Calculate}(Z - 2)\} + \text{WCET}(\text{Calculate}(Z - 2)) + \\
 &\quad \{\text{add}, \text{Calculate}(Z - 1) + \text{Calculate}(Z - 2)\} + \{\text{assign}, R\} + \{\text{return}, R\} = \\
 &1 + 1 + 1 + x + 2 + \text{WCET}(\text{Calculate}(Z - 1)) + \\
 &\quad x + 2 + \text{WCET}(\text{Calculate}(Z - 2)) + x + 1 + 1 = \\
 &9 + 3 * x + \text{WCET}(\text{Calculate}(Z - 1)) + \text{WCET}(\text{Calculate}(Z - 2))
 \end{aligned}$$

Example: WCET analysis

$$\text{WCET}(\text{Calculate}(2)) =$$

$$9 + 3*x + \text{WCET}(\text{Calculate}(1)) + \text{WCET}(\text{Calculate}(0))$$

$$\text{WCET}(\text{Calculate}(3)) = 9 + 3*x + \text{WCET}(\text{Calculate}(2)) + \text{WCET}(\text{Calculate}(1)) =$$

$$2(9 + 3*x) + 2*\text{WCET}(\text{Calculate}(1)) + \text{WCET}(\text{Calculate}(0))$$

$$\text{WCET}(\text{Calculate}(4)) = 9 + 3*x + \text{WCET}(\text{Calculate}(3)) + \text{WCET}(\text{Calculate}(2)) =$$

$$4(9 + 3*x) + 3*\text{WCET}(\text{Calculate}(1)) + 2*\text{WCET}(\text{Calculate}(0))$$

$$\textbf{WCET(Calculate(5))} = 9 + 3*x + \text{WCET}(\text{Calculate}(4)) + \text{WCET}(\text{Calculate}(3)) =$$

$$7(9 + 3*x) + 5*\text{WCET}(\text{Calculate}(1)) + 3*\text{WCET}(\text{Calculate}(0)) =$$

$$63 + 21*x + 5*5 + 3*4 = 63 + 21*x + 25 + 12 = \textbf{100 + 21*x}$$

Example: WCET analysis

b) Derive WCET for main()

Let $x = 4$

$$\text{WCET}(\text{Calculate}(5)) = 100 + 21 \cdot x = 100 + 84 = 184$$

$$\begin{aligned} \text{WCET}(\text{main}()) &= \{\text{declare, ans}\} + \{\text{call, Calculate}(5)\} + \\ &\quad \text{WCET}(\text{Calculate}(5)) + \{\text{assign, ans}\} = \\ &\quad 1 + 2 + \text{WCET}(\text{Calculate}(5)) + 1 = 4 + \text{WCET}(\text{Calculate}(5)) = \\ &\quad 4 + 100 + 21 \cdot x = 104 + 21 \cdot x = 104 + 84 = 188 \end{aligned}$$

c) Check deadline for main()

$\text{WCET}(\text{main}()) = 188$, which exceeds the deadline of 180

Example: WCET analysis

d) Derive WCET for main()

According to subproblem b):

$$\text{WCET}(\text{main}()) = 4 + \text{WCET}(\text{Calculate}(5)) = 4 + 100 + 21*x = 104 + 21*x$$

e) Find maximum cost x

$$\text{WCET}(\text{main}()) = 104 + 21*x \leq 180 \Rightarrow 21*x \leq 76$$

That is, $x \leq 76/21 \approx 3.6$

If x can be a non-integer number select $x = 3.6$ in order to meet deadline.

If x must be an integer number select $x = 3$ in order to meet deadline.