

# Spatial Data Structures and Speed-Up Techniques

Ulf Assarsson

Department of Computer Science and  
Engineering

Chalmers University of Technology

# Have you done your "homework" ;-)?

## Exercises

- Create a function (by writing code on paper) that tests for intersection between:
  - two spheres
  - a ray and a sphere
  - view frustum and a sphere
  - Ray and triangle (e.g. use formulas from last lecture)
- Make sure you understand matrices:
  - Give a scaling matrix, translation matrix, rotation matrix and simple orthogonal projection matrix

# ...e.g., the ray/sphere test

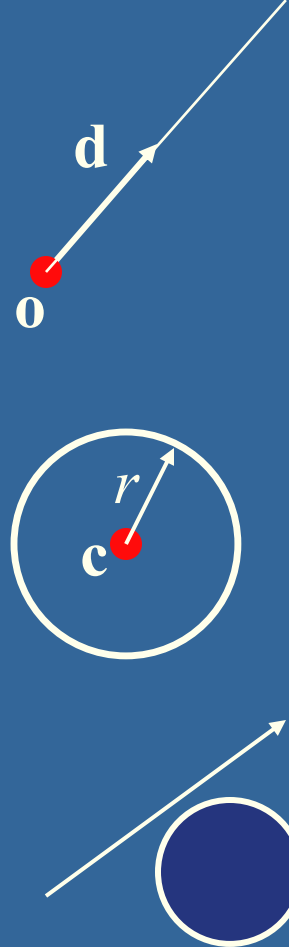
- Ray:  $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$
- Sphere center:  $\mathbf{c}$ , and radius  $r$
- Sphere formula:  $\|\mathbf{p} - \mathbf{c}\| = r$
- Replace  $\mathbf{p}$  by  $\mathbf{r}(t)$ , and square it:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) - r^2 = 0$$

$$t^2 + 2((\mathbf{o} - \mathbf{c}) \cdot \mathbf{d})t + (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - r^2 = 0$$

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

```
Bool raySphereIntersect(vec3f o, d, c, float r, Vec3f &hitPt) {  
    float a = d.dot(d);  
    float b = 2.0f*((o-c).dot(d)); // dot is implemented in class Vec3f  
    float c = (o-c).dot(o-c);  
    if(b*b/4.0f < c) return false;  
    float t = -b/(2.0f*a) - sqrt(b*b/4.0f-c); // intersection for smallest t  
    if (t < 0) t = -b/(2.0f*a) + sqrt(b*b/4.0f-c); // larger t  
    if (t < 0) return false; else hitPt = o + d*t; // where * is an operator for vector multiplication  
    return true;  
}
```



# Misc

- Half Time wrapup slides are available in “Schedule” on home page
  - Including 3 old exams
- There is an Advanced Computer Graphics Seminar Course in sp 4, 7.5p
  - One seminar every week
    - Advanced CG techniques
  - Do a project of your choice.
  - Register to the course

# Spatial data structures

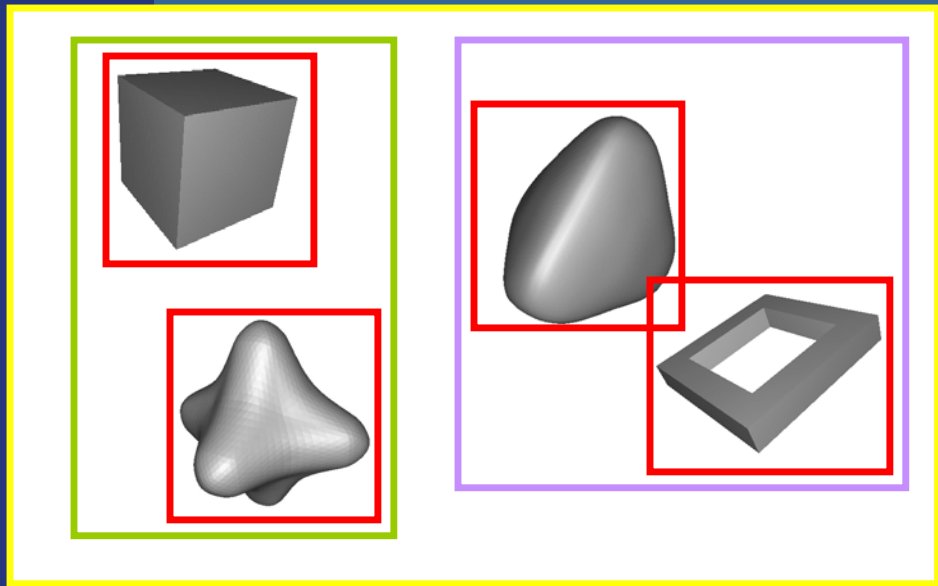
- What is it?
  - Data structure that organizes geometry in 2D or 3D or higher
  - The goal is faster processing
  - Needed for most "speed-up techniques"
    - Faster real-time rendering
    - Faster intersection testing
    - Faster collision detection
    - Faster ray tracing and global illumination
- Games & Movie production tools use them extensively

# Bounding-Volume Hierarchy

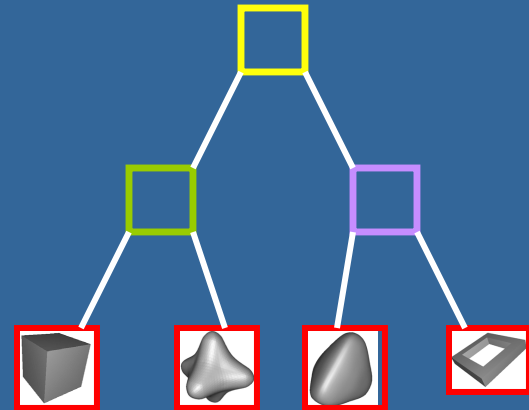
– BOTTOM-UP construction:

- Organizes geometry in some hierarchy

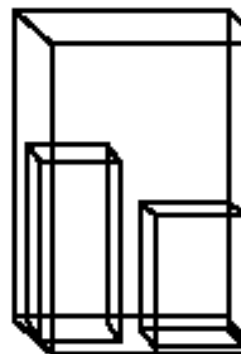
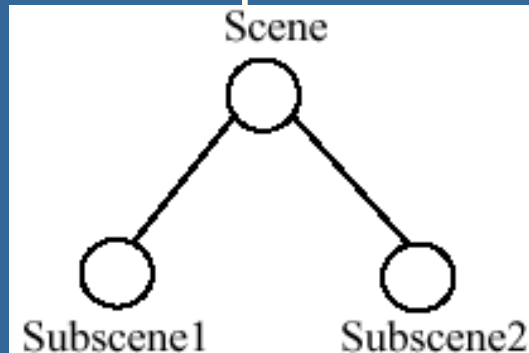
In 2D space



Data structure



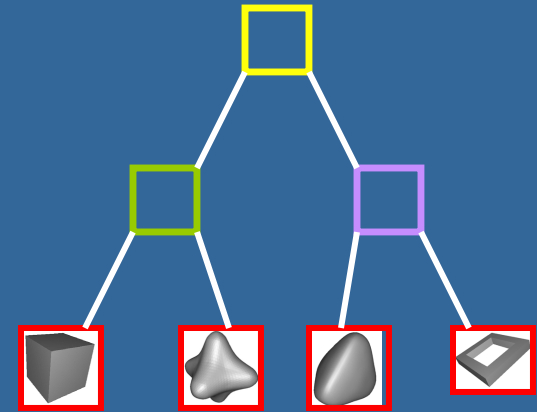
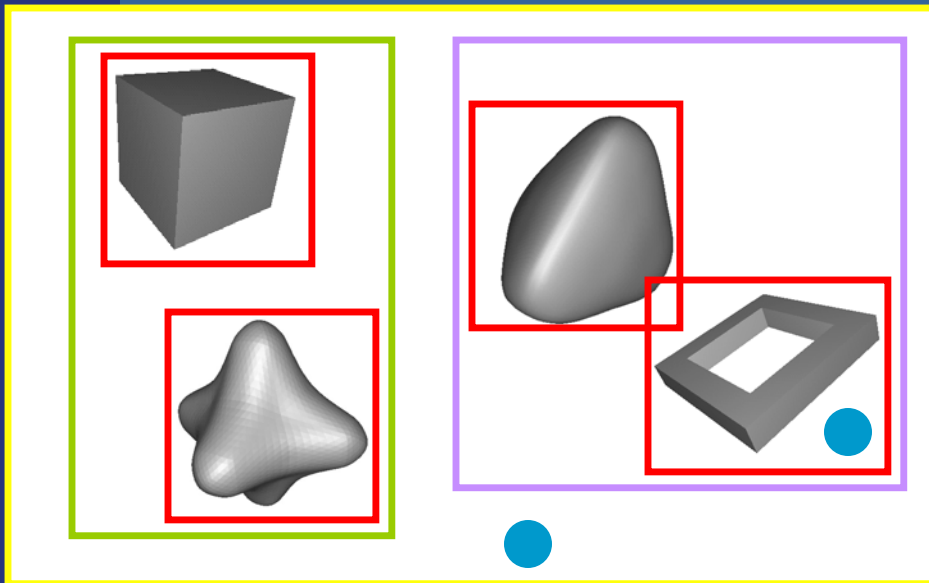
In 3D space:



# What's the point with hierarchies?

## An example

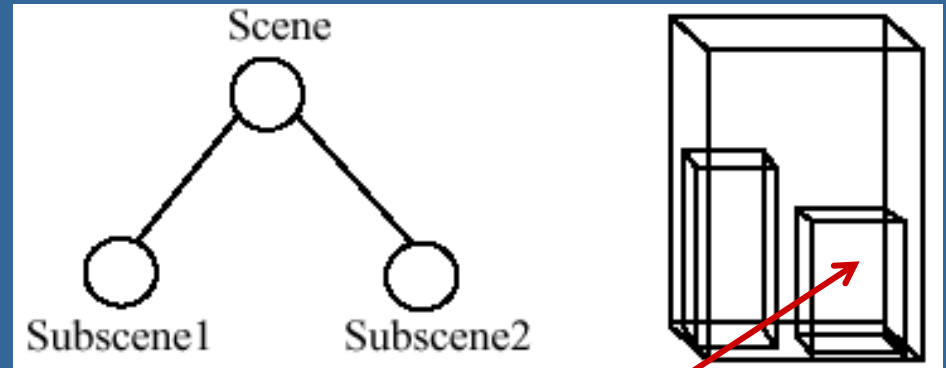
- Assume we click on screen, and want to find which object we clicked on



•  
click!

- 1) Test the root first
  - 2) Descend recursively as needed
  - 3) Terminate traversal when possible
- In general: get  $O(\log n)$  instead of  $O(n)$

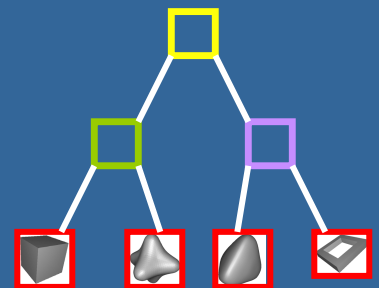
# 3D example





# Bounding Volume Hierarchy (BVH)

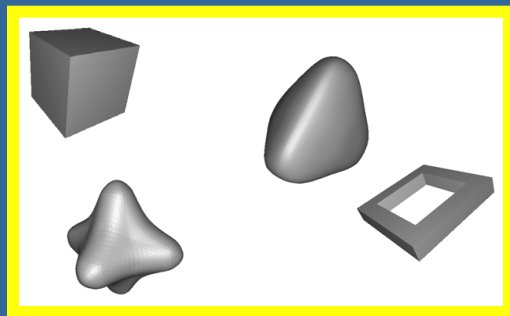
- Most common bounding volumes (BVs):
  - Axis-Aligned Bounding Boxes (AABB)
  - But can also use spheres and Oriented Bounding Boxes (OBBs)
  - AABB hierarchies are used by the NVIDIA RTX chip
- The BV does not contribute to the rendered image -- rather, encloses an object
- The data structure is a **tree**
  - Leaves hold geometry
  - Internal nodes hold BVs that enclose all geometry in its subtree



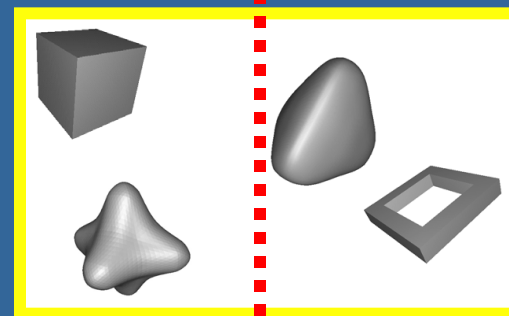
# Bounding-Volume Hierarchy

## – TOP-DOWN construction:

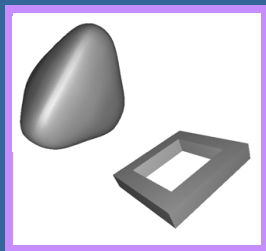
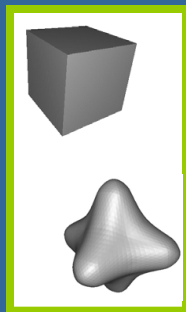
- Find minimal box, then split along longest axis



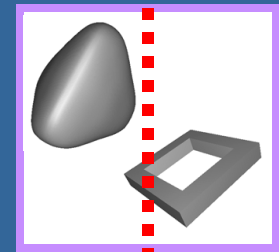
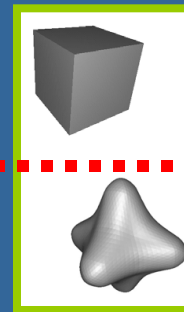
x is longest



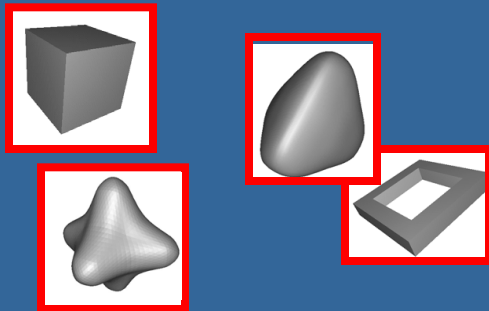
Find minimal boxes



Split along longest axis



Find minimal boxes



Called TOP-DOWN method  
Works similarly for other BVs

# Example

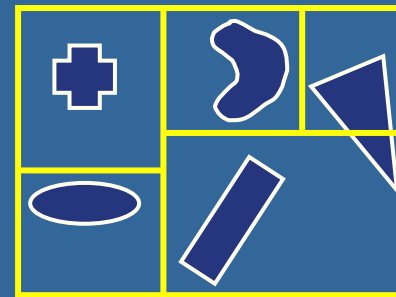
Killzone (2004-PS2) used kd-tree / AABB-tree based system for the collision detection



Kd-tree = Axis Aligned BSP tree

# Binary Space Partitioning (BSP) Trees

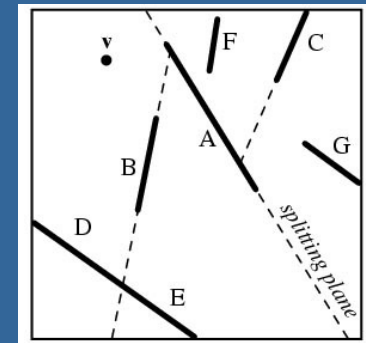
- Two different types:
  - Axis-aligned BSP
  - Polygon-aligned BSP



Axis-aligned

- General idea:

- Split space with a plane
- Divide geometry into the sub space it belongs
- Repeat recursively



Polygon-aligned

- If traversed in a certain way, we can get the geometry sorted back-to-front or front-to-back in w.r.t. any camera position, in **constant time!**
  - Exact for polygon-aligned
  - Approximately for axis-aligned

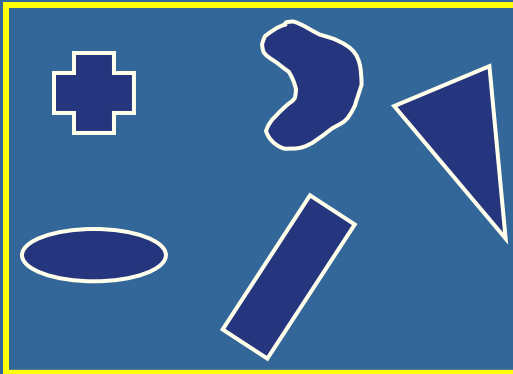
# Axis-Aligned BSP tree

## – TOP-DOWN construction

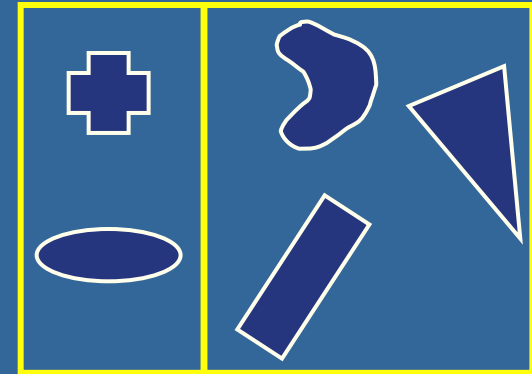
- Split space with a plane
- Divide geometry into the space it belongs
- Done recursively

- Axis-aligned => Can only choose a splitting plane along x,y, or z

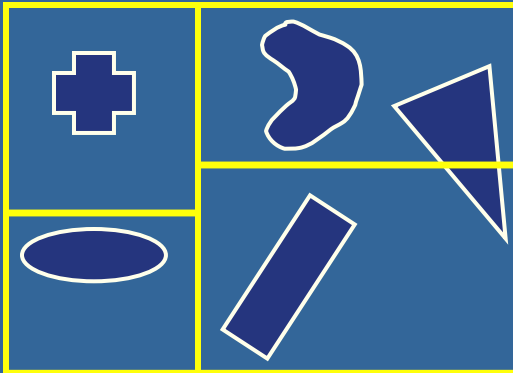
Minimal box



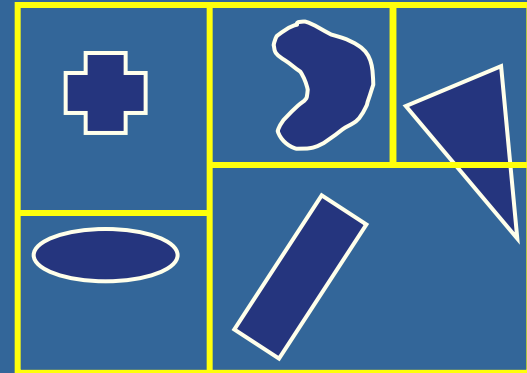
Split along plane



Split along plane

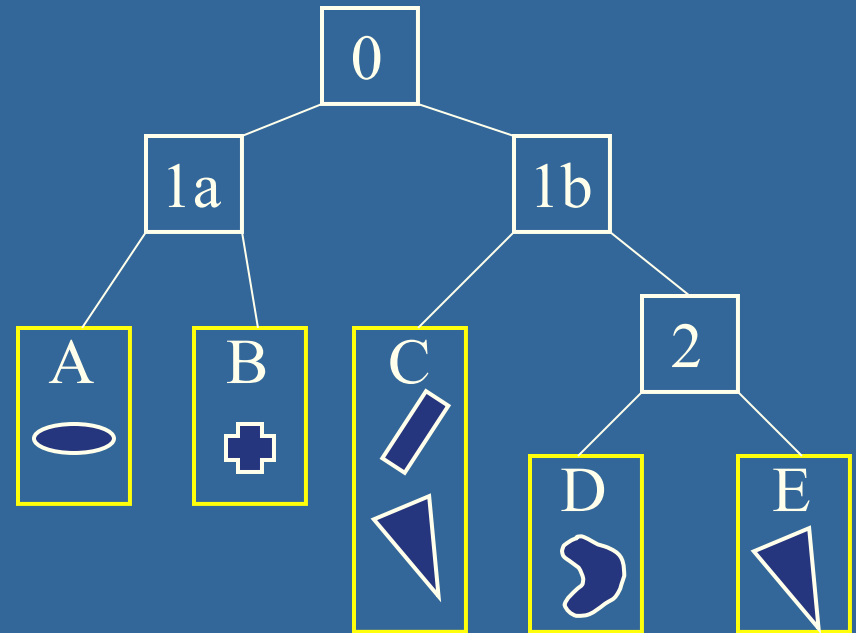
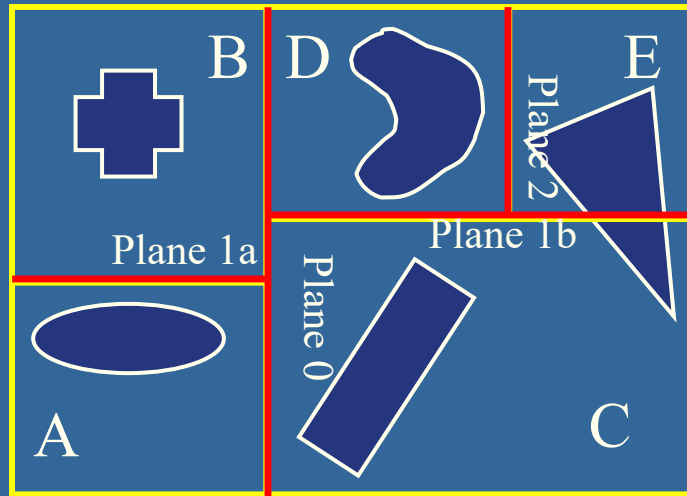


Split along plane



# Axis-Aligned BSP tree

## – tree structure

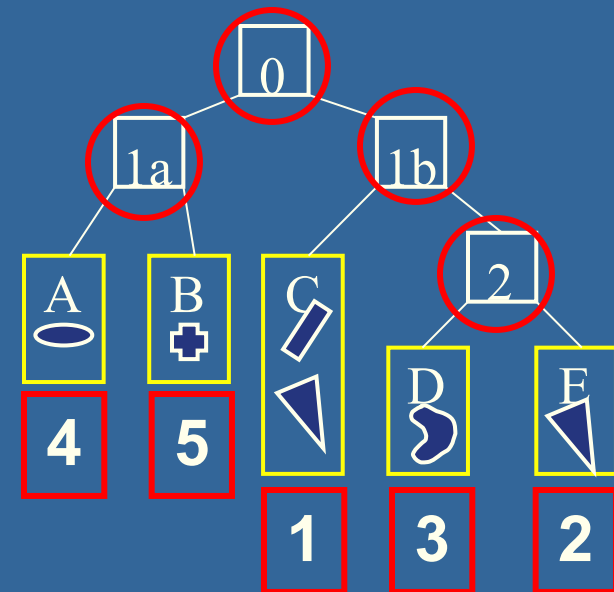
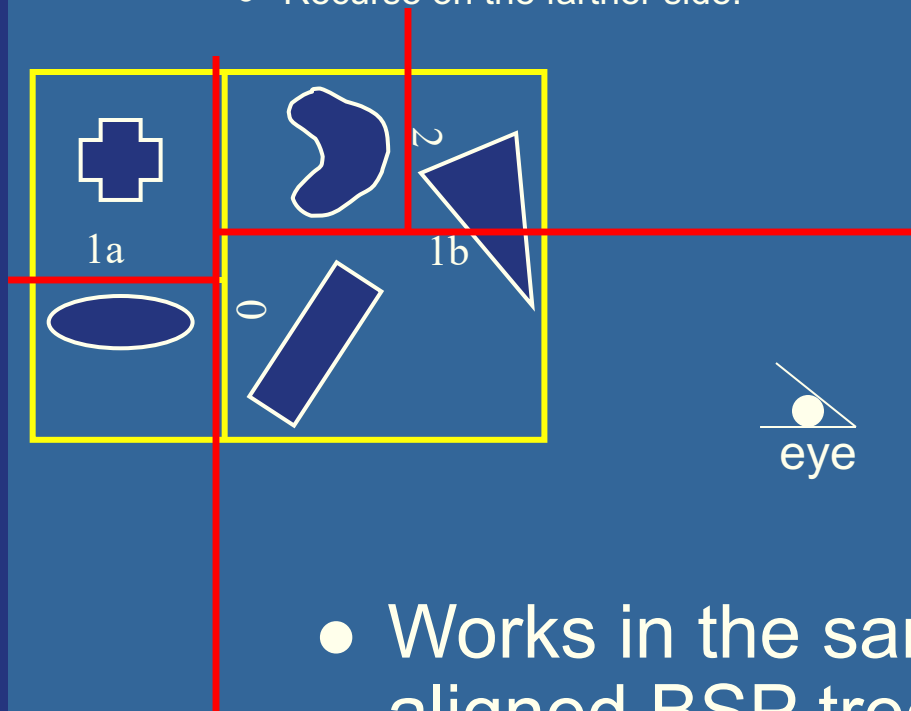


- Each internal node holds a divider plane
- Leaves hold geometry
- Differences compared to BVH
  - BSP tree encloses entire space and provides sorting
  - The BV hierarchy can have spatially overlapping nodes(no sort)
  - BVHs can use any desirable type of BV

# Axis-aligned BSP tree

## – Rough sorting front-to-back w.r.t camera

- Test the planes, recursively from root, against the point of view. For each traversed node (for front-to-back rendering):
  - If node is leaf, draw the node's geometry
  - else
    - Recurse on the "hither" side with respect to the eye (to sort front to back)
    - Recurse on the farther side.

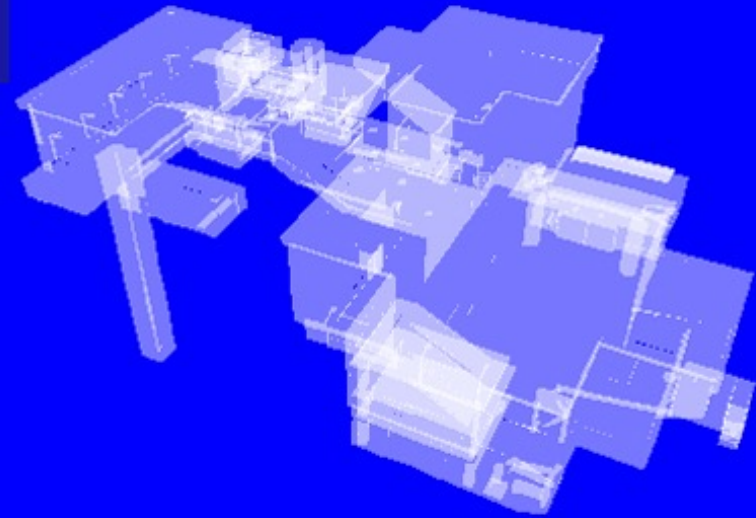


- Works in the same way for polygon-aligned BSP trees --- but that gives exact sorting

# Polygon Aligned BSP tree – Quake 2



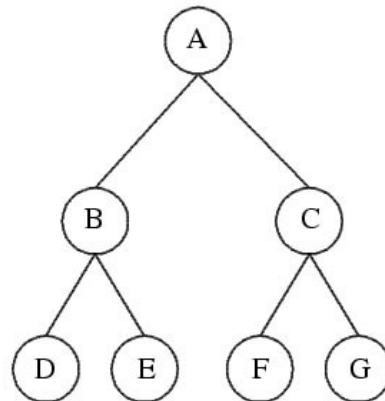
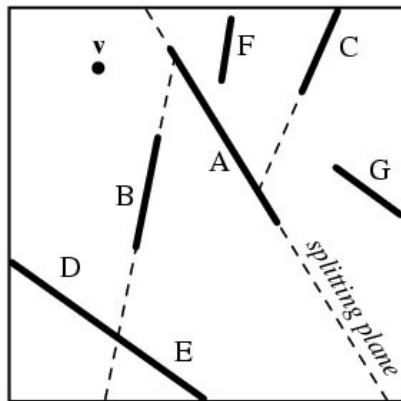
FPS: 25 / 24  
MS: 44  
MEM: 91,539  
MAX: 91,539





# Polygon-aligned BSP tree

- Allows exact sorting from camera
  - Since planes clip intersecting triangles
- Very similar to axis-aligned BSP tree
  - But the triangle planes are used as the splitting planes.



```
Drawing Back-to-Front {  
    Recurse on farther side of P;  
    Draw P;  
    Recurse on hither side of P;  
}  
//Where hither and  
farther are with respect  
to viewpoint  $v$ 
```

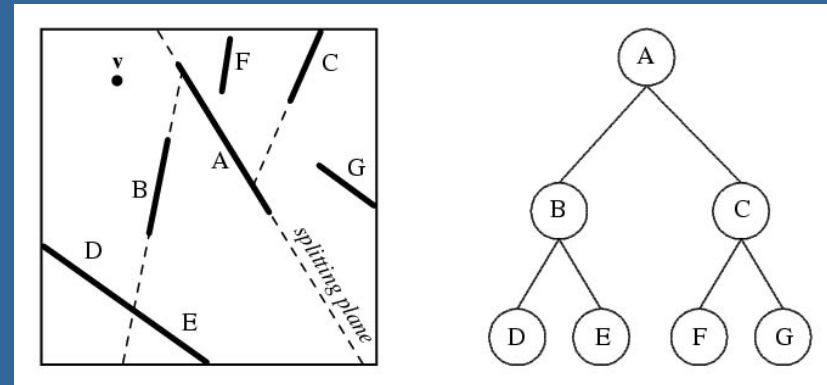
# Polygon-aligned BSP tree

```
class BSPtree:  
    Polygon P;  
    BSPtree behindP;  
    BSPtree frontOfP;
```

```
Tree CreateBSP(PolygonList L) {  
    If L empty, return empty tree;  
    Else:  
        T->P = arbitrary polygon in L.  
        T->behindP = CreateBSP(polygons behind P)  
        T->frontOfP = CreateBSP(polygons in front of P)  
    Return T.  
}
```

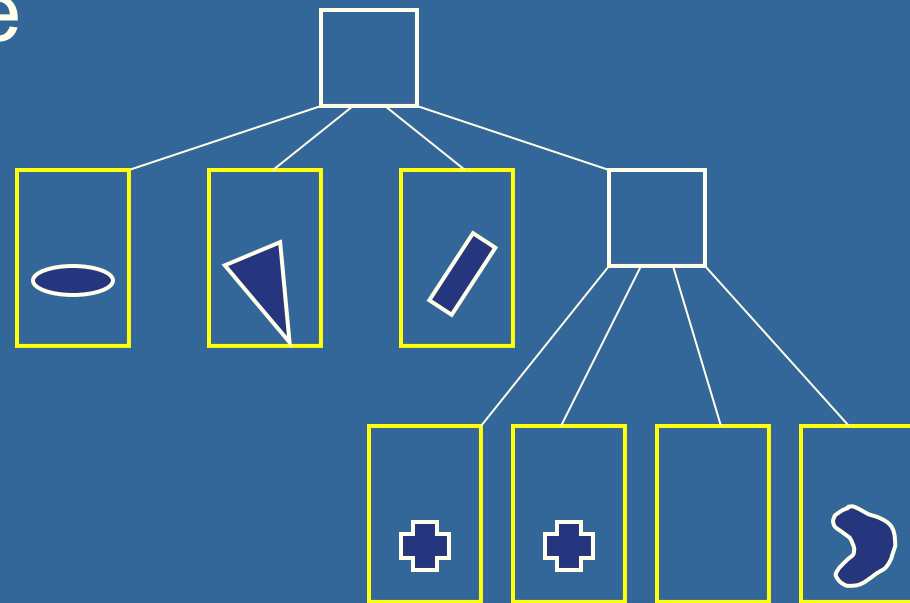
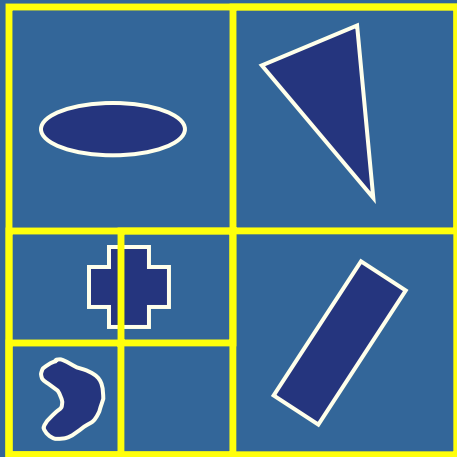
```
Drawing Back-to-Front {  
    recurse on farther* side of P;  
    Draw P;  
    Recurse on hither* side of P;  
}
```

\*With respect to viewpoint v



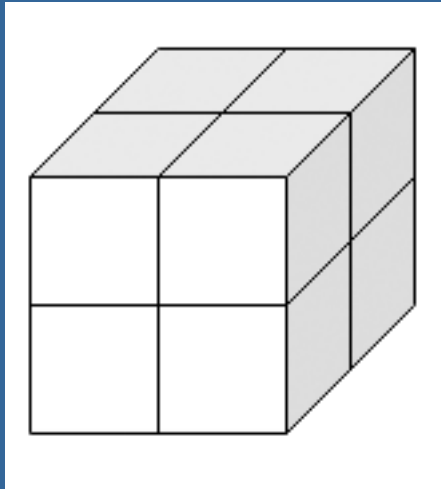
# Octrees (1)

- A bit similar to axis-aligned BSP trees
- Will explain the quadtree, which is the 2D variant of an octree

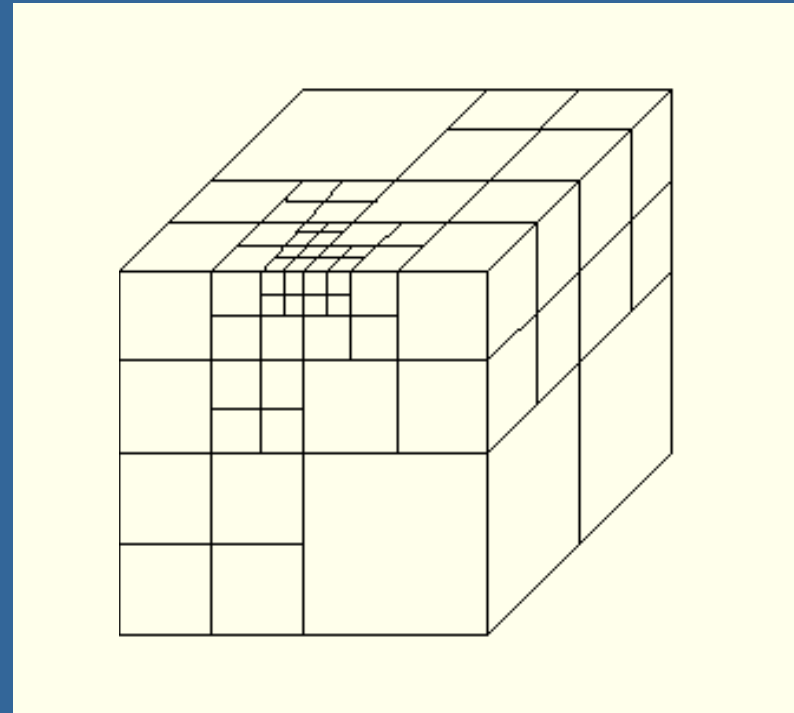


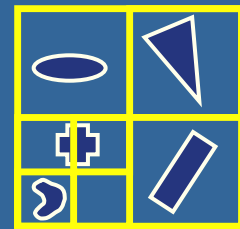
- In 3D, each square (or rectangle) becomes a box, and 8 children

# Example of Octree



Recursively split space  
in eight parts – equally  
along x,y,z dimension  
simultaneously for each  
level

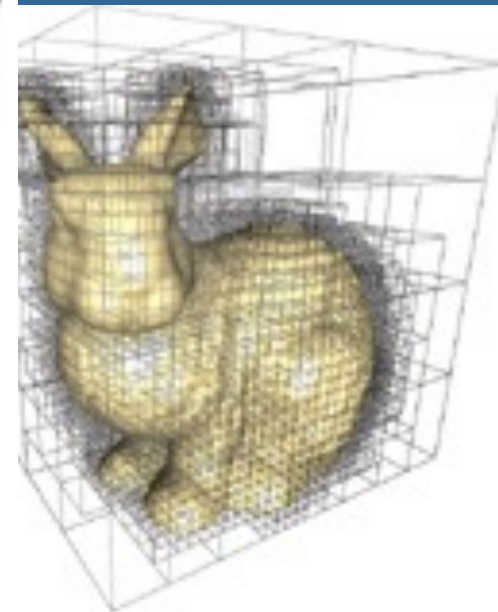
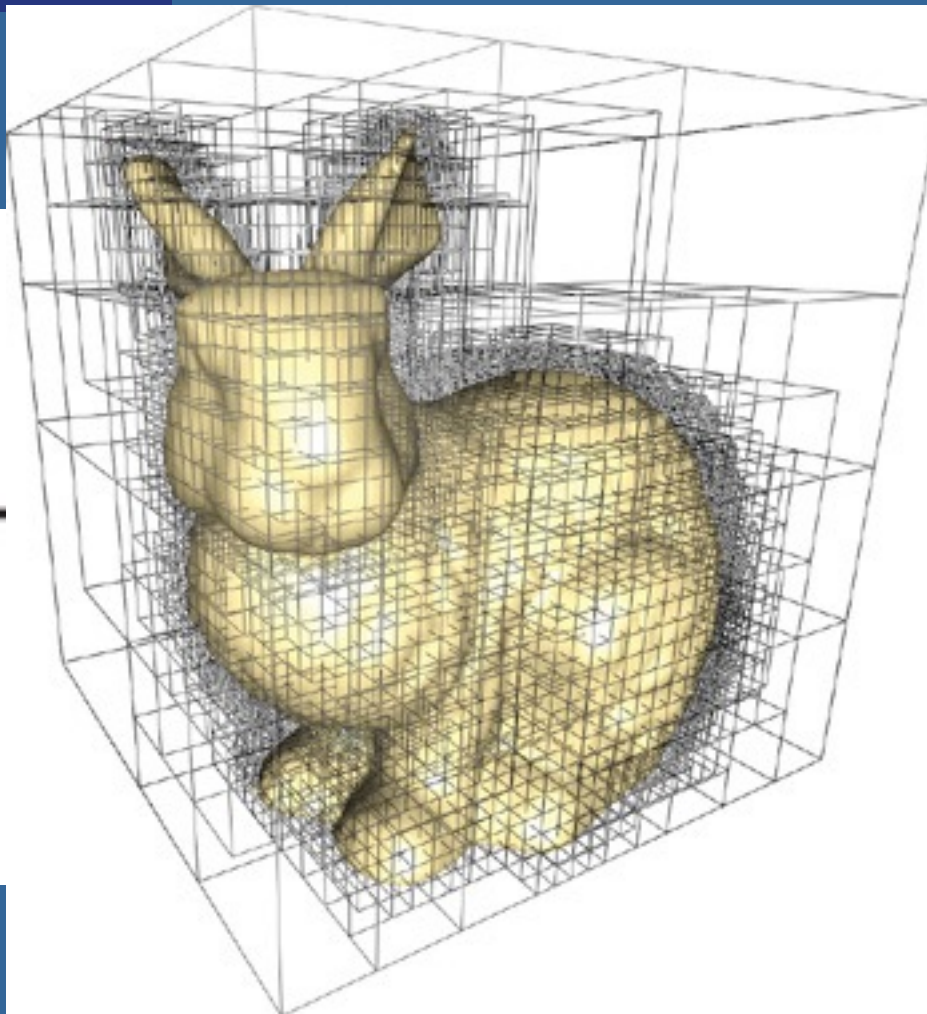




# Example of octree

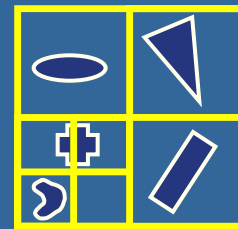


(a)

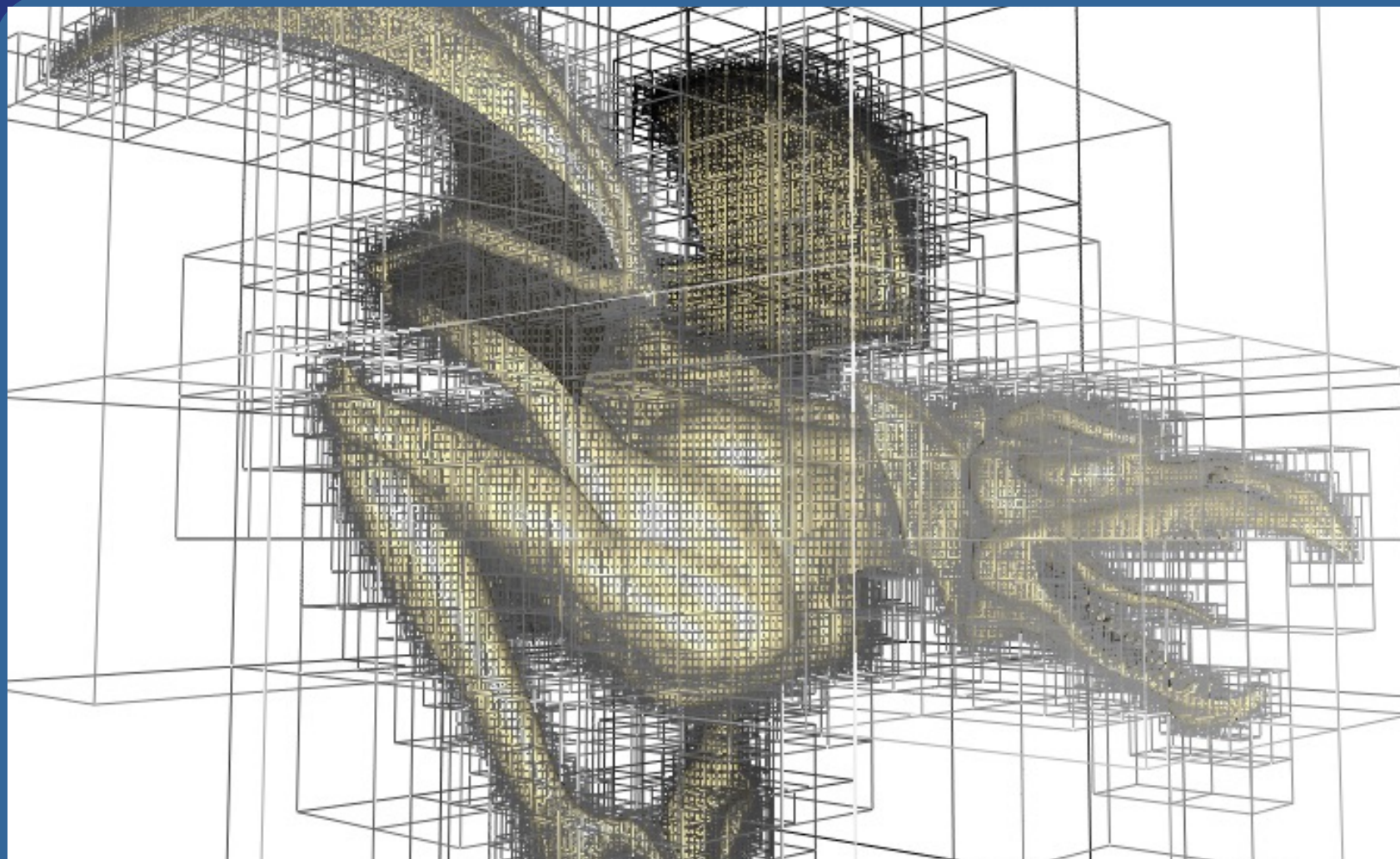


(c)





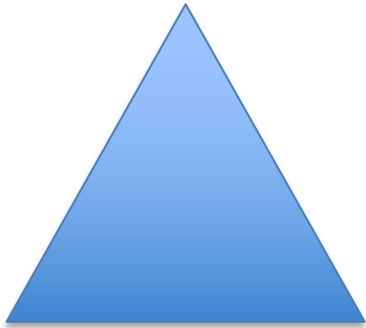
# Example of octree



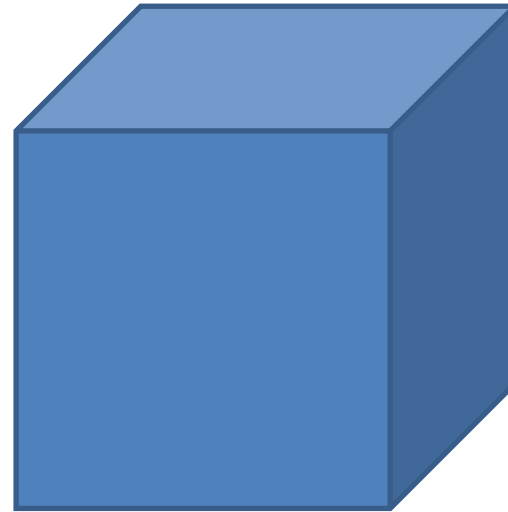
## Octrees (2)

- Expensive to rebuild (BSPs are too)
- Octrees can be used to
  - Speed up ray tracing
  - Faster picking
  - Culling techniques...
  - But are not used that often these days, except for **Sparse Voxel Octrees (SVO:s)**

# Voxels



Triangle  
36 bytes



Voxel  
Volume – element  
1 bit



# Voxels

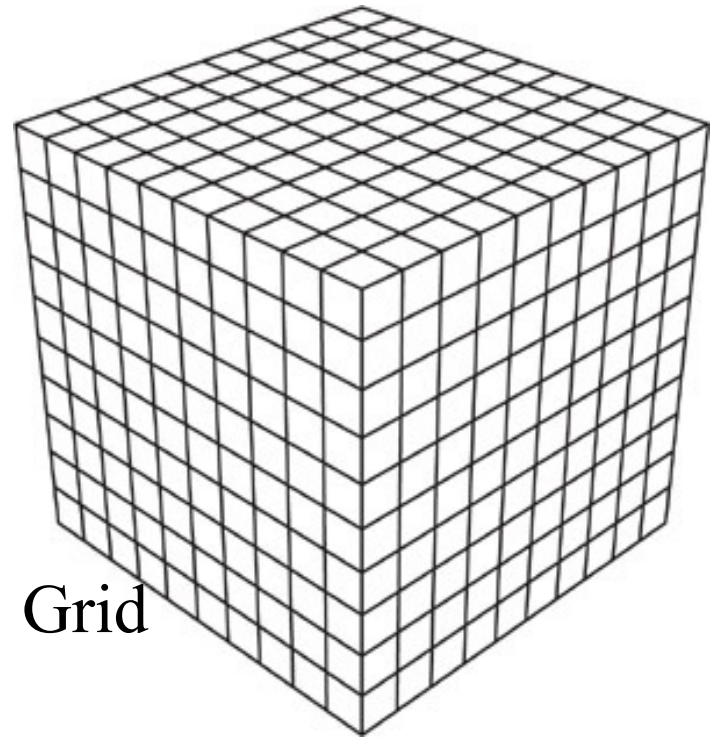
- Desirable to be able to use very high resolutions



# Voxels

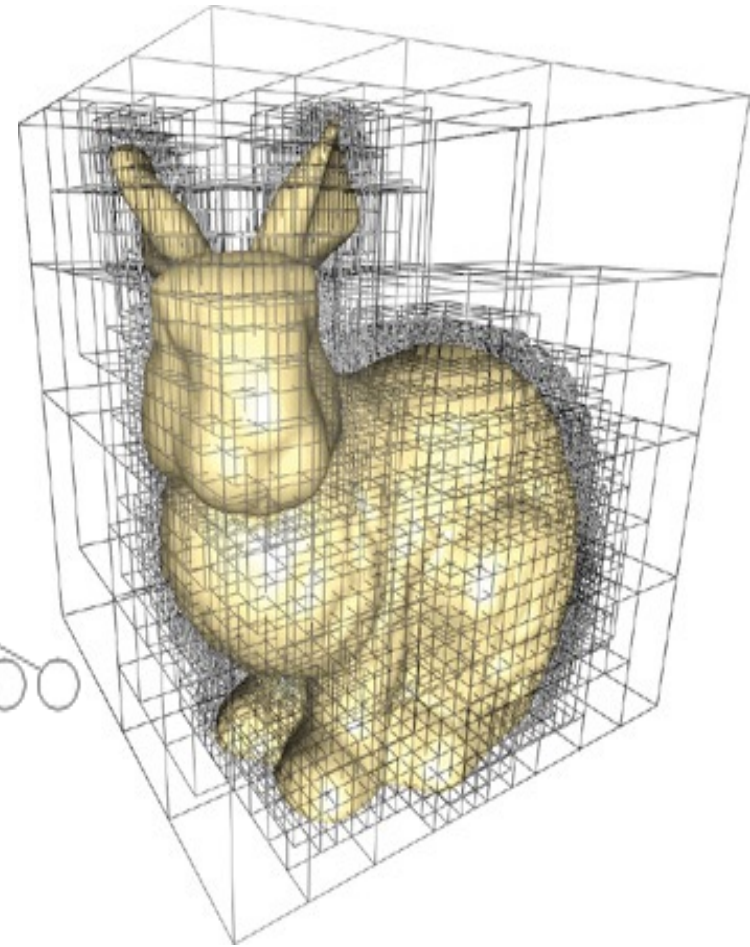
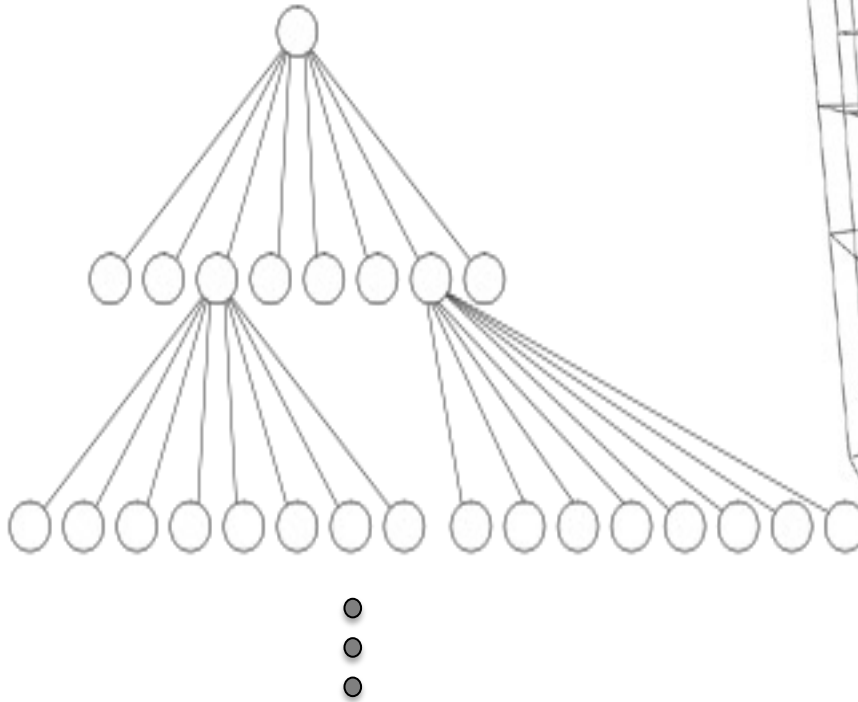
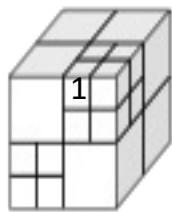
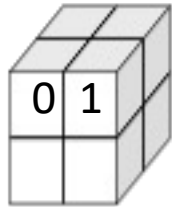
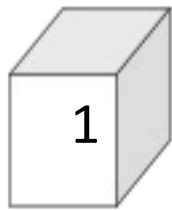
One possible data structure:

- Voxel Grids – 3D array of 0:s and 1:s



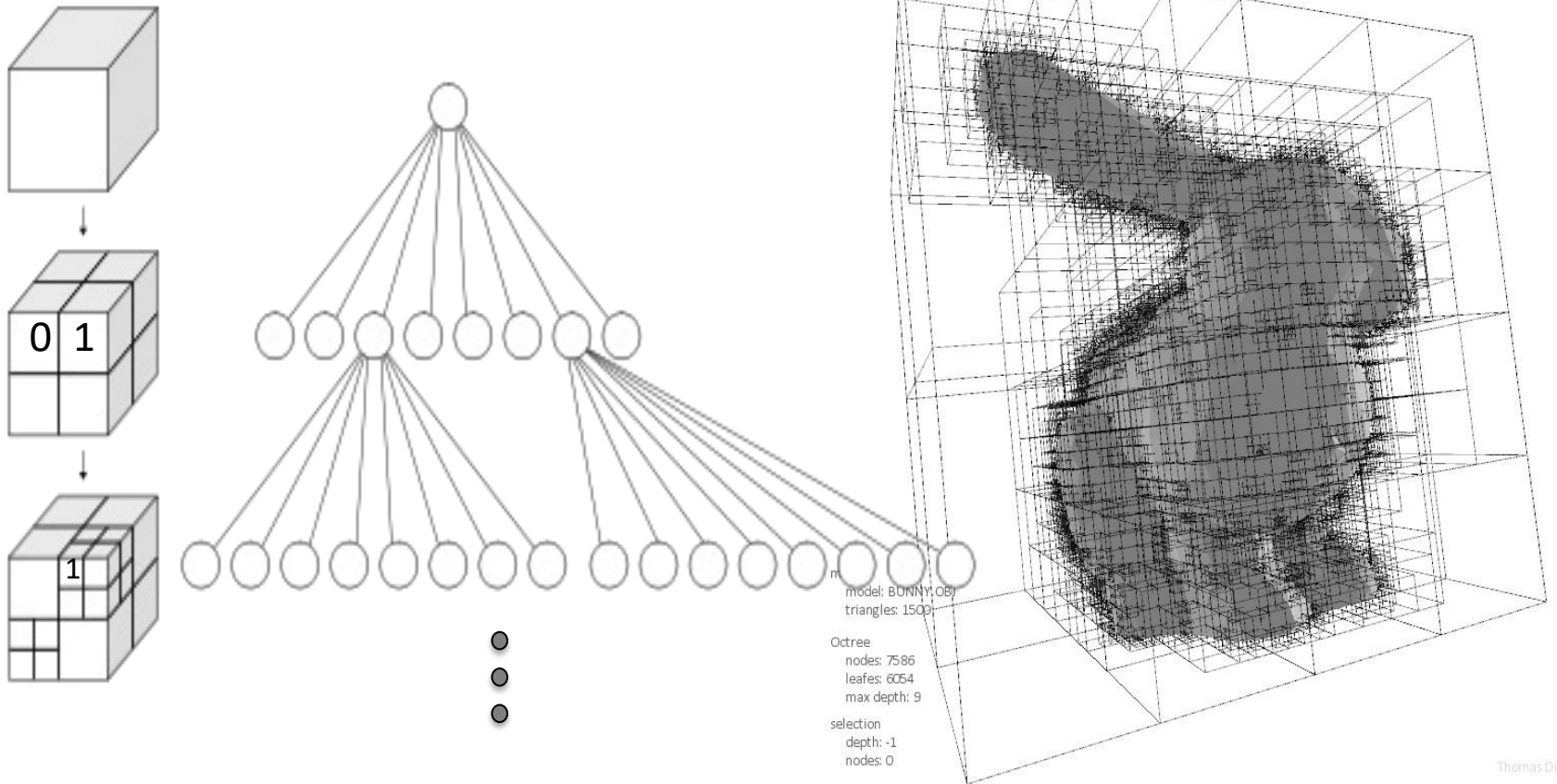
# Sparse Voxel Octree

Each node has eight children, representing an octant of the parent node's volume.



# Sparse Voxel Octree

Each node has eight children, representing an octant of the parent node's volume.





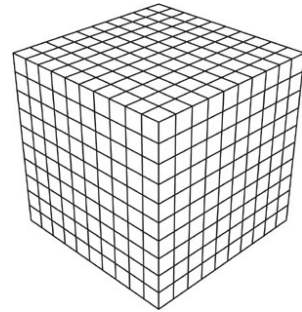
# Sparse Voxel Octree

- SVO: Id Software, rage 6
- 1.15 bits/ non-empty voxel
- DAGs: e.g., down to 0.08 bit/non-empty voxel



# Sparse Voxel DAGs

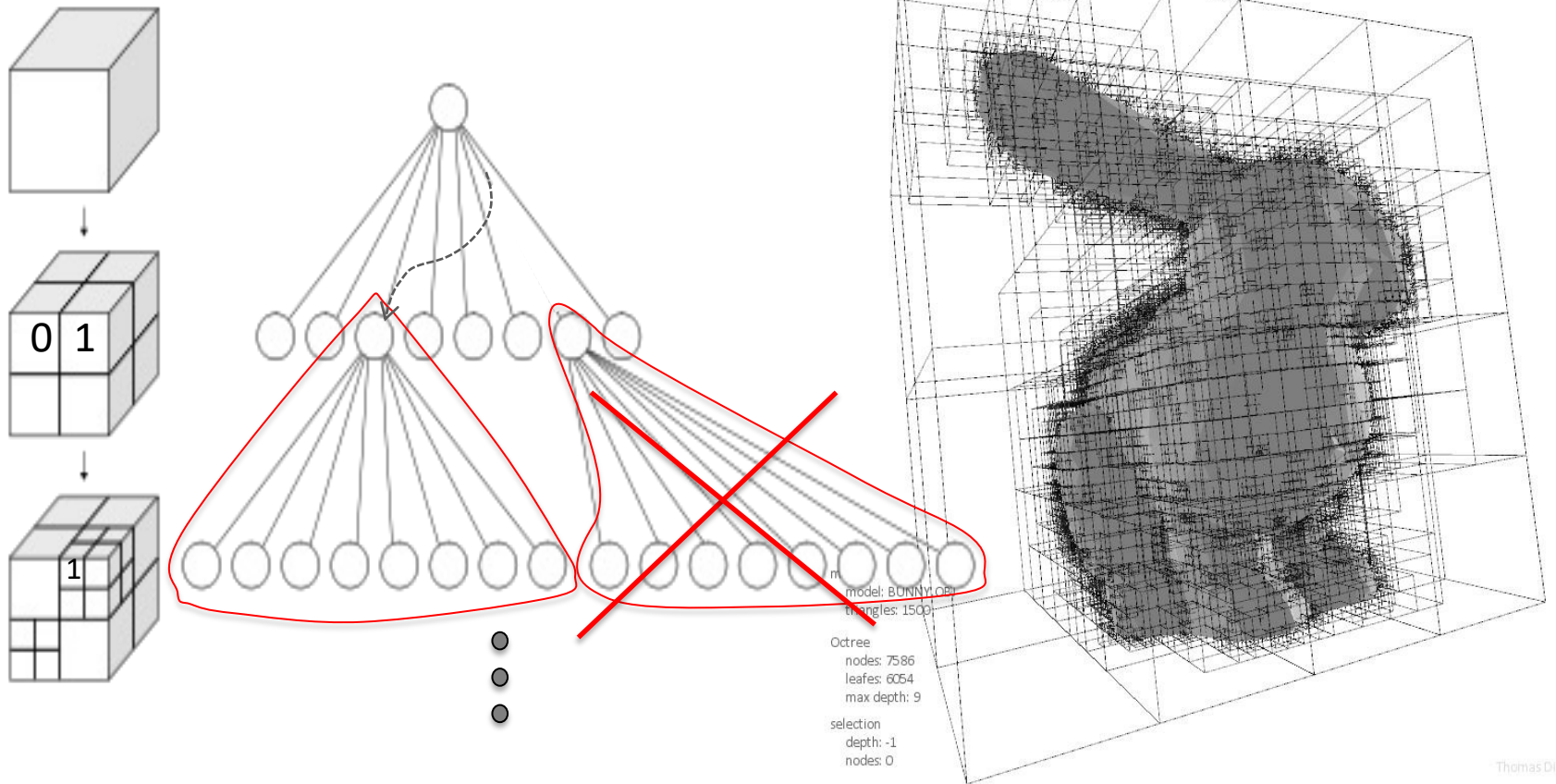
- Voxel = 1 bit.
- SVDags can currently handle scene of res =  $128.000^3$ 
  - Naively with bit grid: 262 TB
  - SVDAGs => < 1GB can be possible





# Sparse Voxel DAGs

For identical subgraphs, only store one instance, and point to that instance.



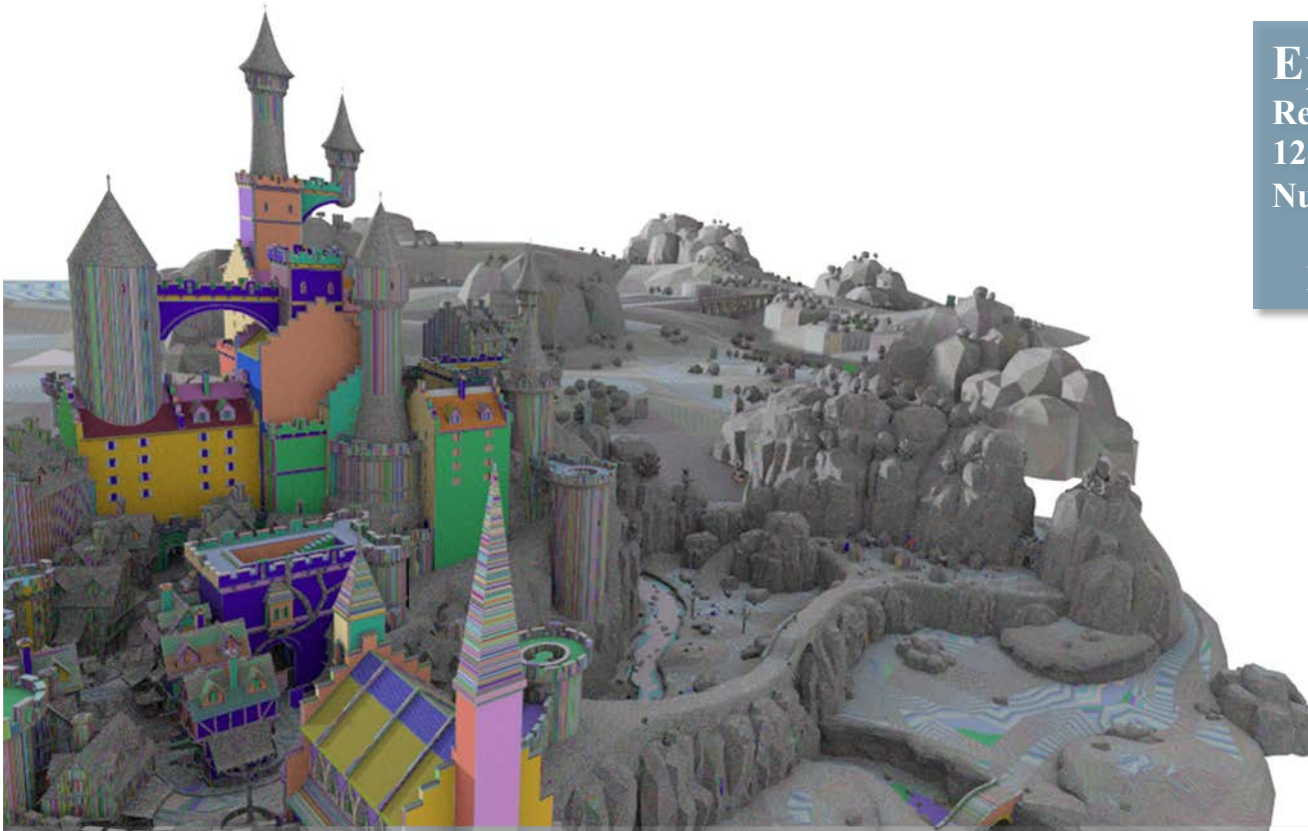
# Sparse Voxel DAGs



<https://youtu.be/6zpbV6hZPWU>



# Visualizing Identical Subtrees



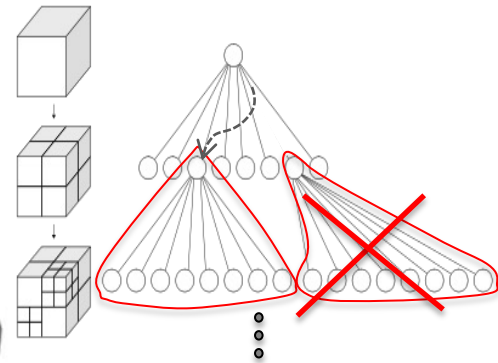
## Epic Citadel

Resolution:  $128K \times 128K \times 128K$

Number of nodes

SVO: 5.5 billion

DAG: 45 million (0.8%)



# Visualizing Identical Subtrees

## Hairball

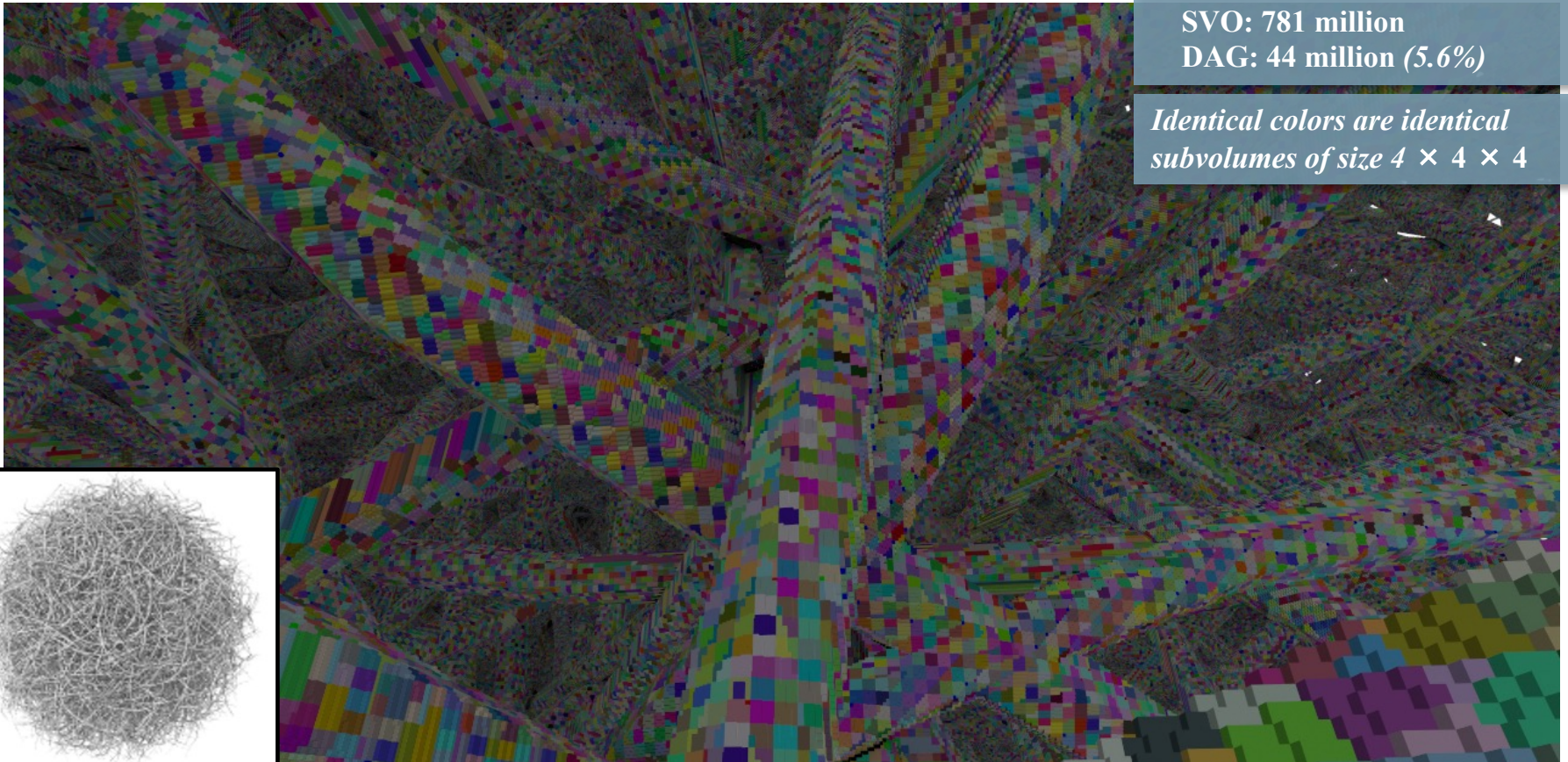
Resolution: 8K × 8K × 8K

Number of nodes

SVO: 781 million

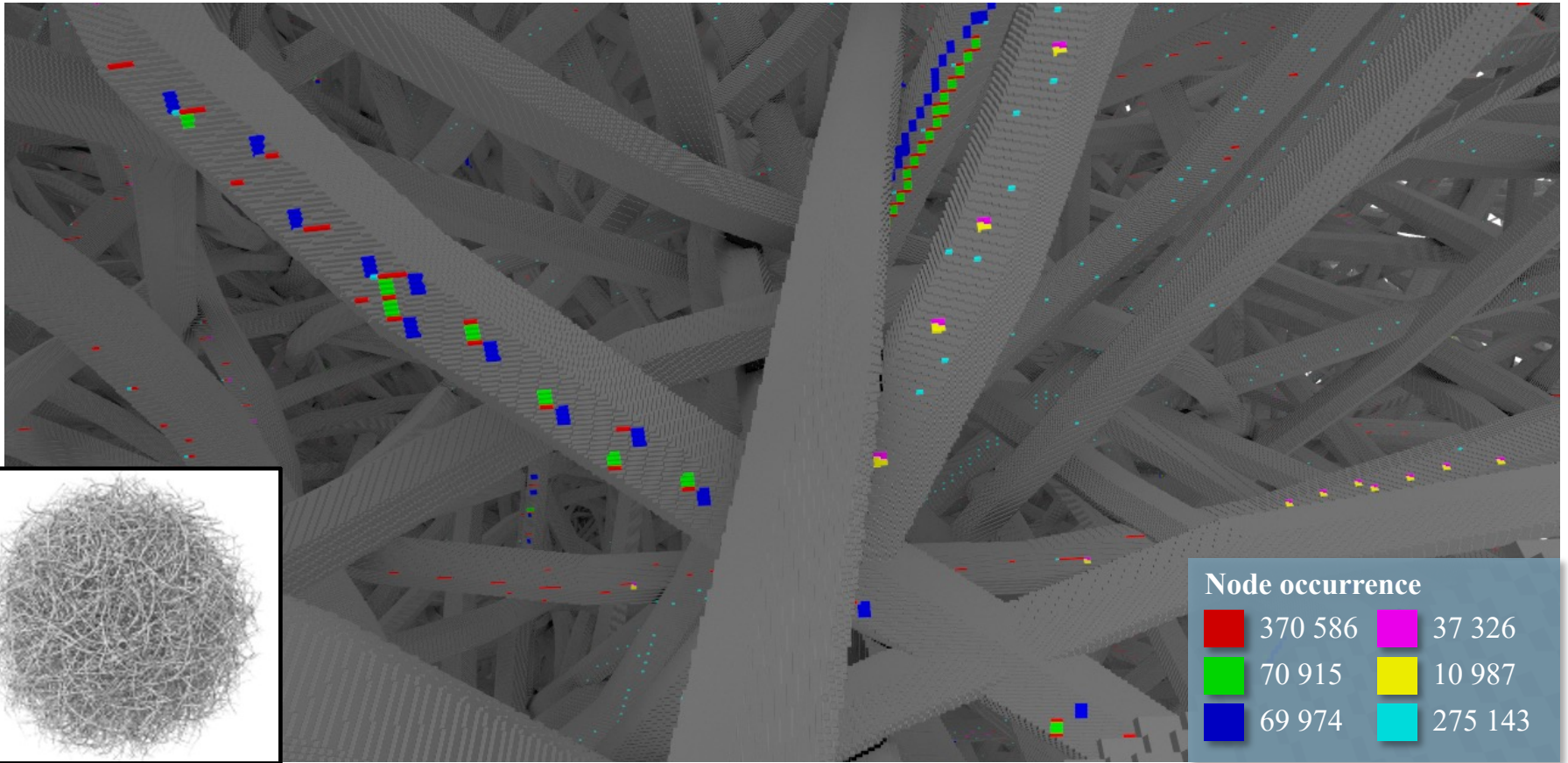
DAG: 44 million (5.6%)

*Identical colors are identical  
subvolumes of size 4 × 4 × 4*





# Visualizing Identical Subtrees

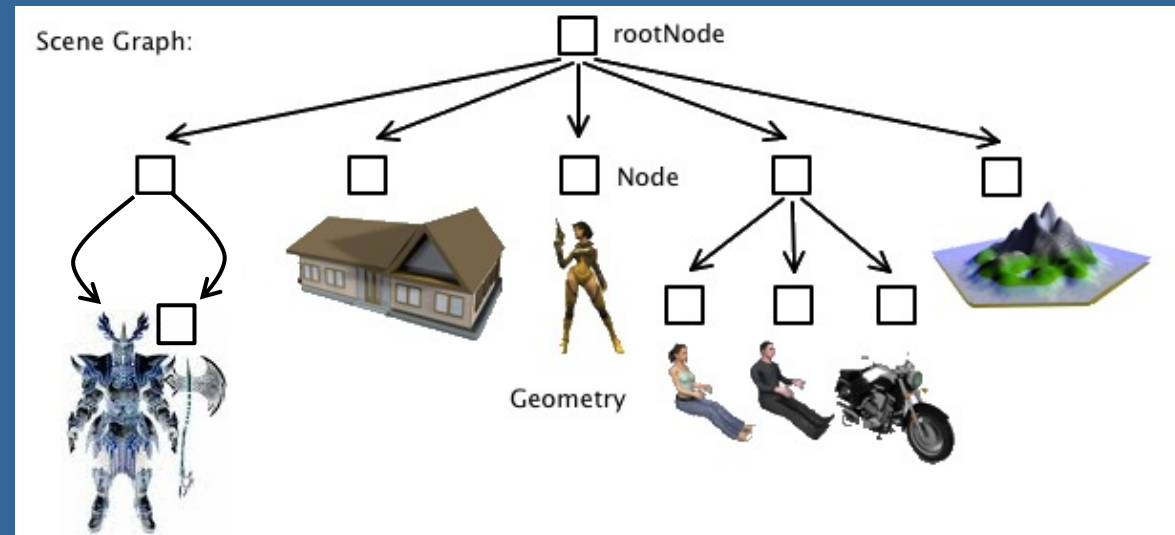


A Scene Graph is a hierarchical scene description – more typically a **logical** hierarchy (than e.g. **spatial**)

# Scene graphs

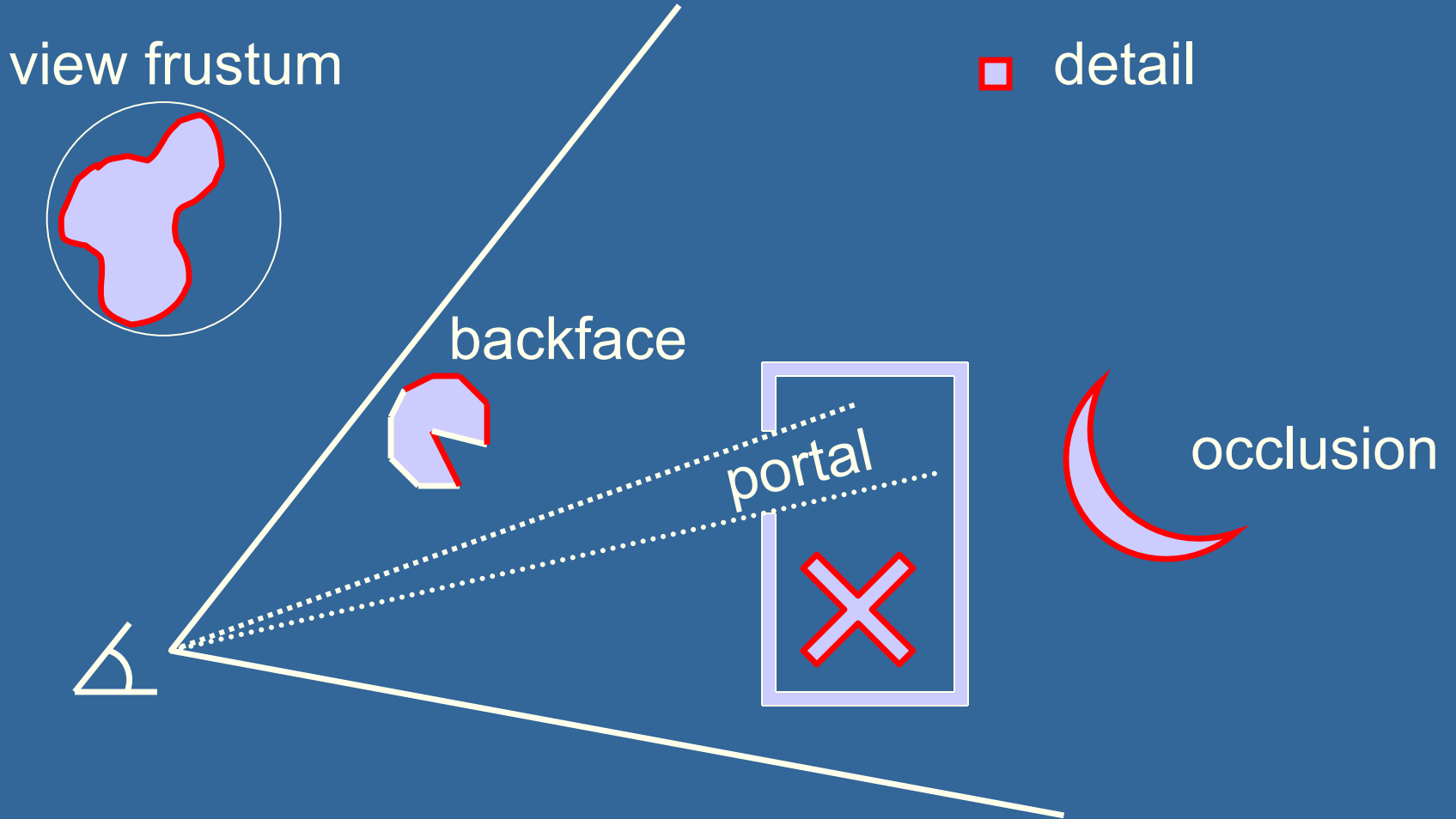
– a node hierarchy

- A scene graph is a node hierarchy, which often reflects a logical hierarchical scene description
  - often in combination with a BVH such that each node has a BV.
- Common hierarchical features include:
  - Lights
  - Materials
  - Transforms
  - Transparency
  - Selection



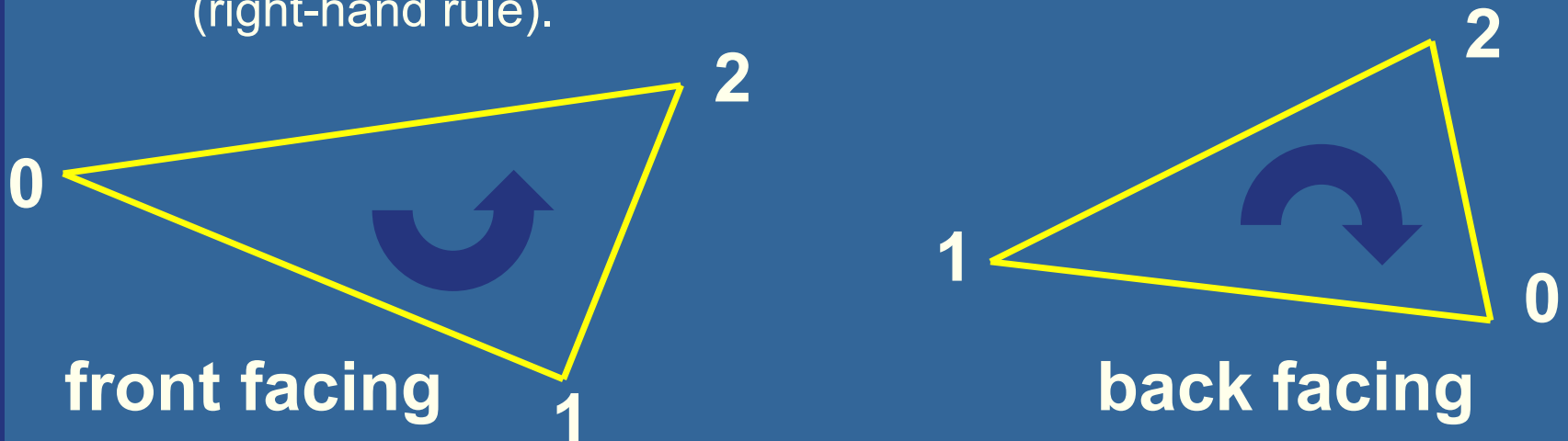
# Different culling techniques

(red objects are skipped)



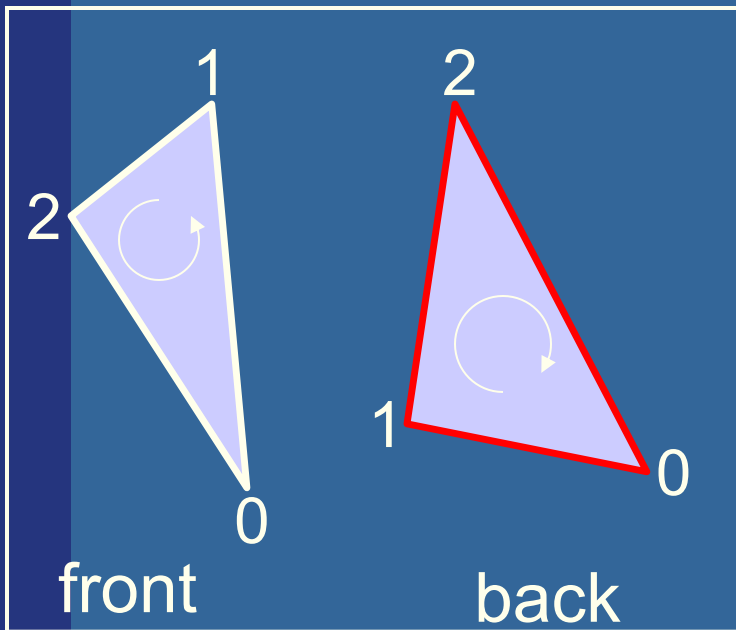
# Backface culling

- Can be used when back-faces are never seen (closed objects)
- OpenGL:
  - `glCullFace(GL_BACK)` ;
  - `glEnable(GL_CULL_FACE)` ;
- First, define front/back-faces
  - Let counterclockwise vertex-winding order define front face (right-hand rule).

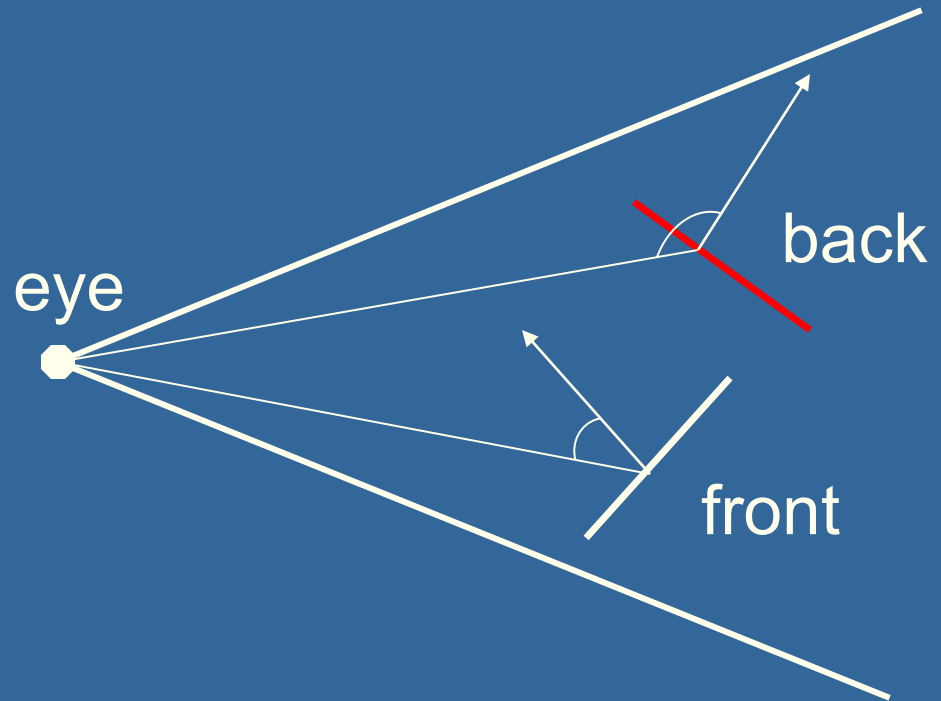


# How to cull backfaces

- Two ways in different spaces:



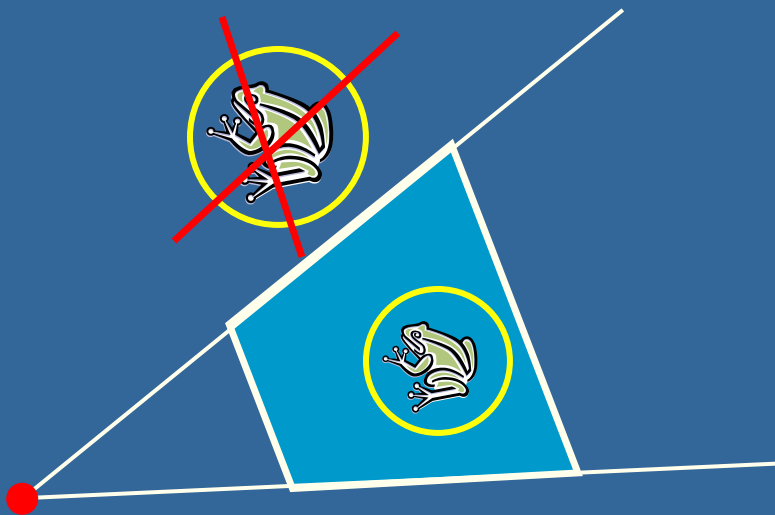
screen space



eye space

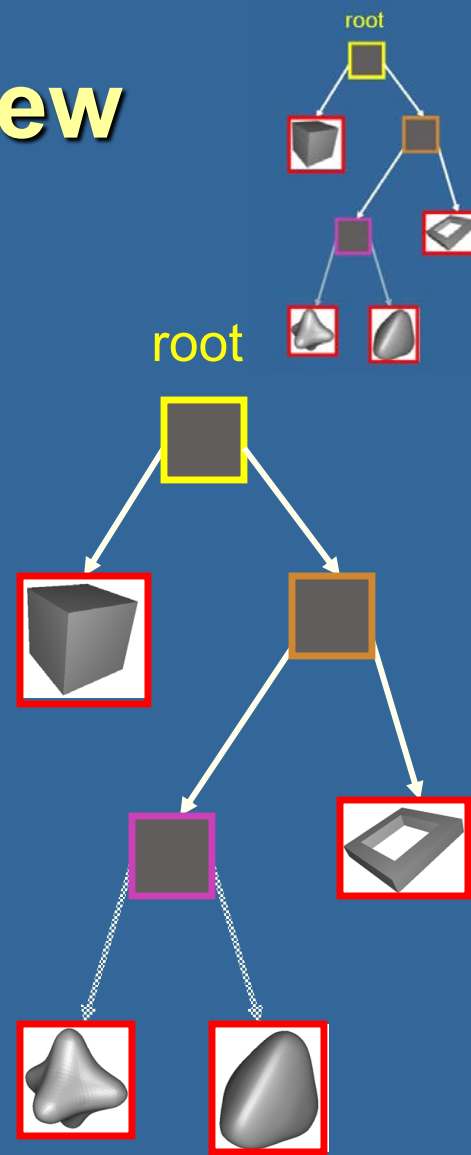
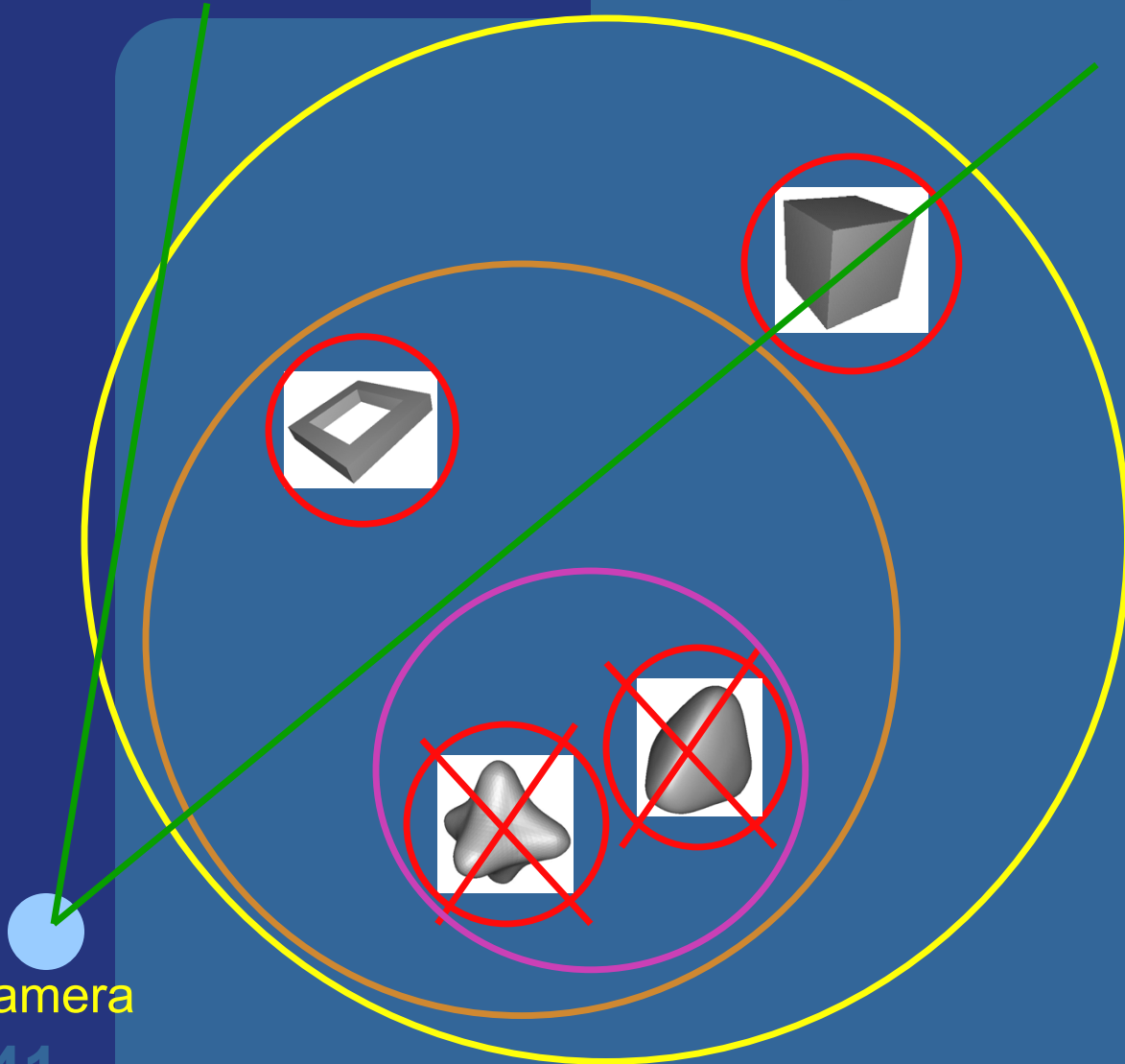
# View-Frustum Culling

- Bound every “natural” group of primitives by a simple volume (e.g., sphere, box)
- If a bounding volume (BV) is outside the view frustum, then the entire contents of that BV is also outside (not visible)





# Example of Hierarchical View Frustum Culling



camera

Refined view frustum culling:  
frustum gets smaller for each door

# Portal Culling

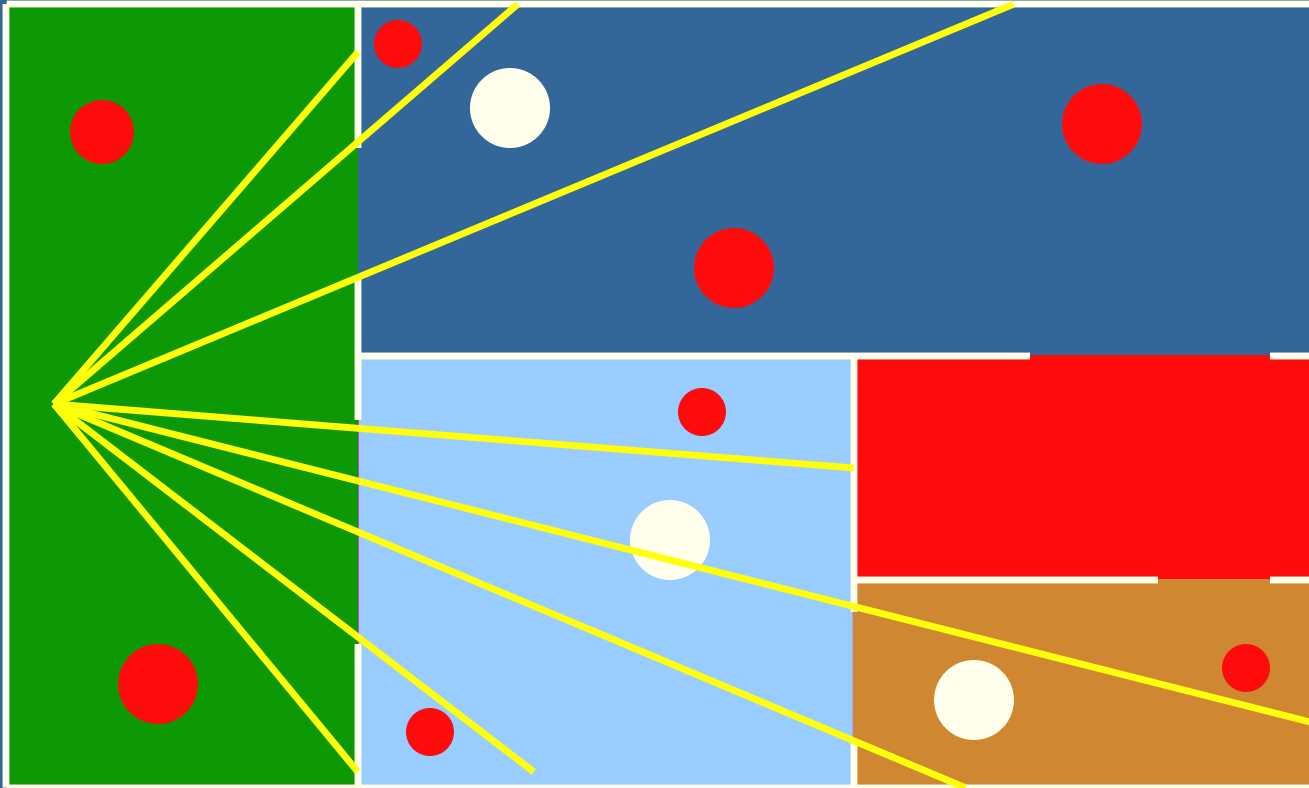
Images courtesy of David P. Luebke and Chris Georges



- Average: culled 20-50% of the polys in view
- Speedup: from slightly better to 10 times

# Portal culling example

- In a building from above
- Circles are objects to be rendered



# Portal Culling Algorithm (1)

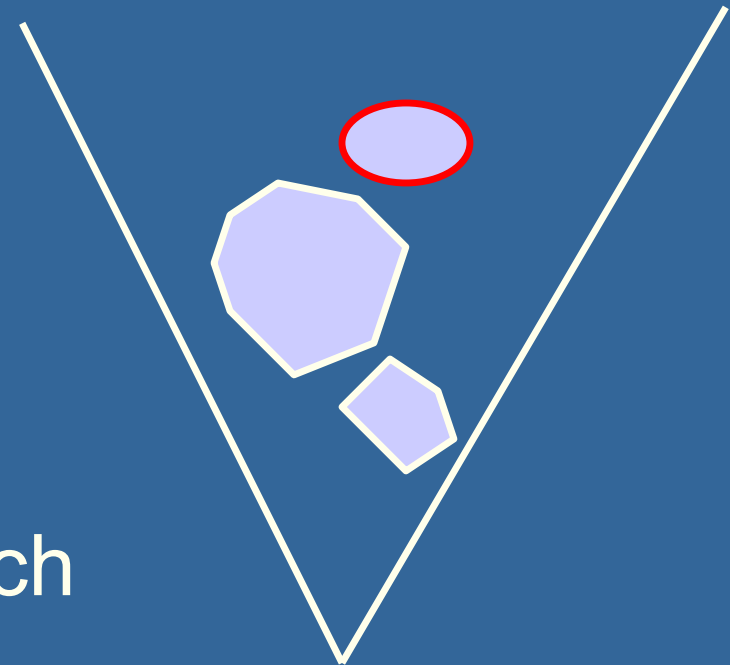
- "Recursively do VFC through visible portals (i.e., doors & mirrors)"

## Algorithm:

- Build a graph of the scene with cells (rooms) and portals (doors/mirrors)
- For each frame:
  - Locate cell of viewer and init 2D AABB to whole screen
  - \* Render current cell with View Frustum culling w.r.t. AABB
  - Traverse to closest cells (through portals)
  - Intersection of AABB & AABB of traversed portal
  - Goto \*

# Occlusion Culling

- Main idea: Objects that lies completely “behind” another set of objects can be culled
- Hard problem to solve efficiently
- Has been lots of research in this area
  - OpenGL: “Occlusion Queries”



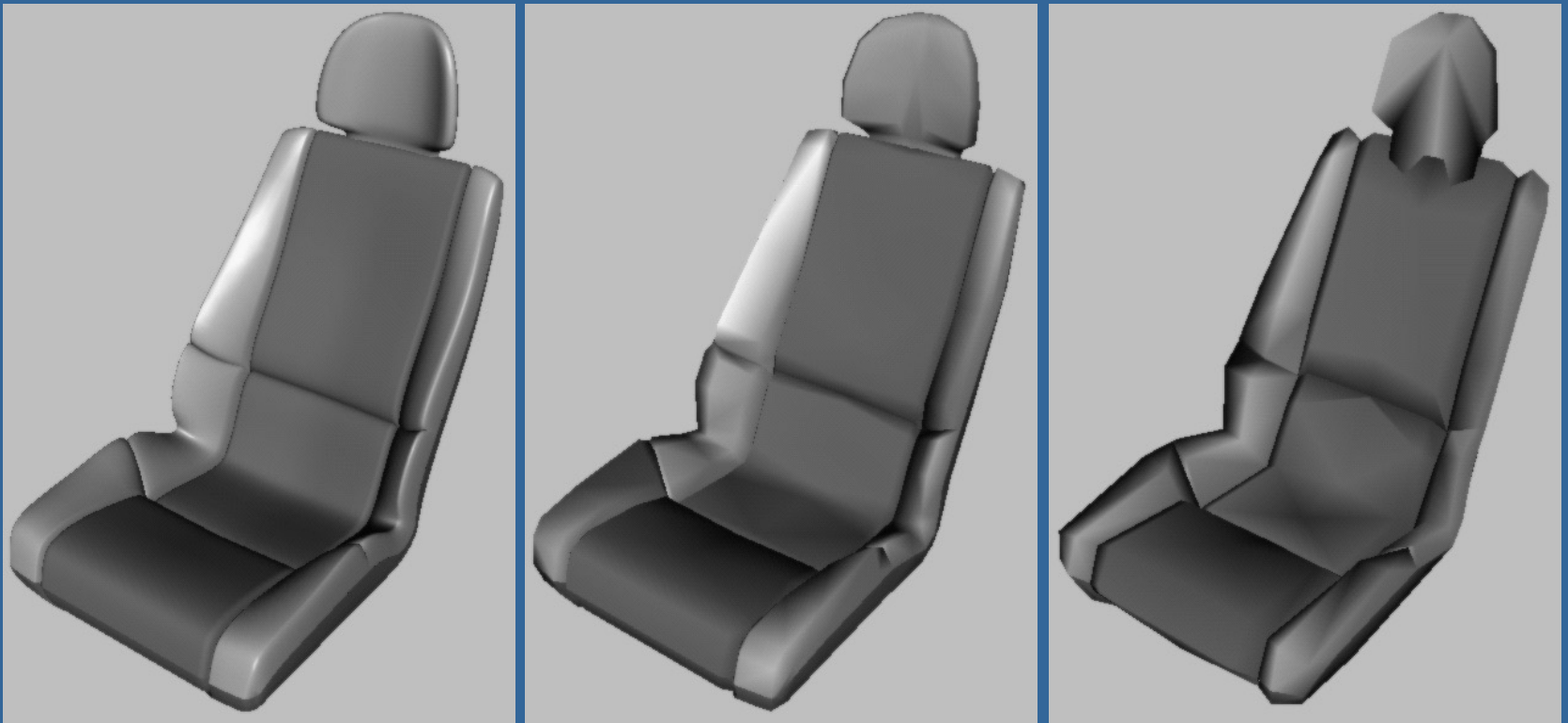
# Occlusion culling algorithm

Use some kind of occlusion representation  $O_R$

```
for each object  $g$  do:  
    if( not Occluded( $O_R, g$ ))  
        render( $g$ );  
        update( $O_R, g$ );  
    end;  
end;
```

# Level-of-Detail Rendering

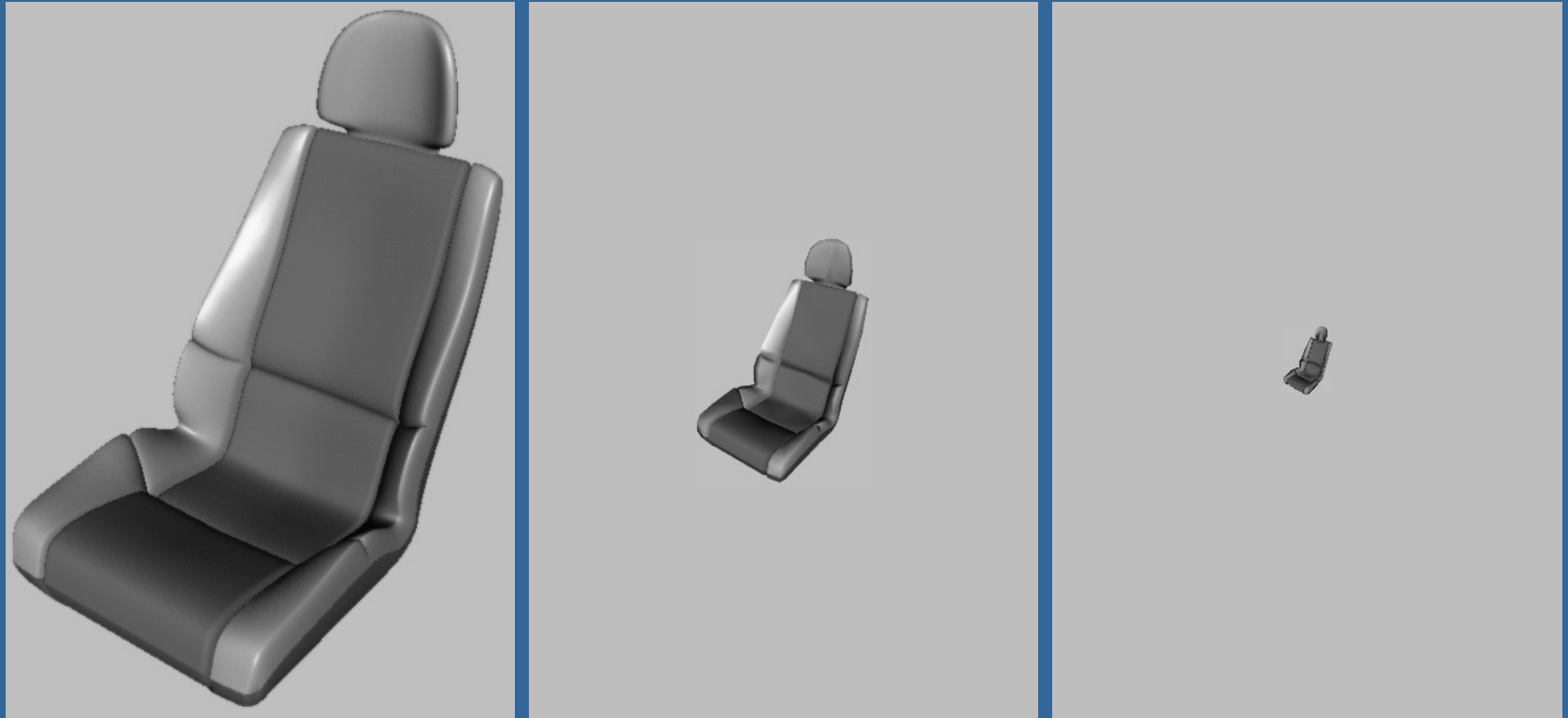
- Use different levels of detail at different distances from the viewer
- More triangles closer to the viewer





# LOD rendering

- Not much visual difference, but a lot faster



- Use area of projection of BV to select appropriate LOD

# Far LOD rendering

- When the object is far away, replace with a quad of some color
- When the object is *really far away*, do not render it (called: detail culling)!
- Use projected area of BV to determine when to skip

# Exercise

- Create a function (by writing code on paper) that performs hierarchical view frustum culling
  - void hierarchicalVFC(BVHnode\* node)

# What you need to know

- Describe how use BVHs.
- Top-down construction of BVH, AABSP-tree,
- Construction + sorting with AABSP and Polygon-Aligned BSP
- Octree/quadtree
- Culling – VFC, Portal, Detail, Backface, Occlusion
  - Backface culling – screenspace is robust, eyespace non-robust.
- What is LODs

**THE END**