Finite Automata Theory and Formal Languages
TMV027/DIT321 – LP4 2018

Formal Proofs, Alphabets and Words

Assignment 1 – Deadline: Thursday 29th of March 23:59

Assignments are part of the examination; they should be done and submitted individually!

To obtain full points, the answers should contain enough explanation/description so that they are easy to understand. So you need to justify all the steps in your proofs!

Please state clearly the property you are trying to prove, the method you will use, the base case(s) and the inductive hypothesis (hypotheses).

1. (1.75pts) Prove using induction that $\forall n \in \mathbb{N}.\ 9^n - 1$ is divisible by 8.

2. Consider the following definitions for $f, g, h : \mathbb{N} \to \mathbb{N}$:

$f(0) = 1$	$h(0) = 0$	$g(0) = 1$
$f(n+1) = h(n) + 2 \times g(n)$	$h(n+1) = n + g(n)$	$g(n+1) = n + g(n) - h(n)$

   (a) (0.3pts) Compute $f(1), g(1), h(1), f(2), g(2), h(2), f(3), g(3)$ and $h(3)$.

   (b) (2.2pts) Prove that $\forall\, n \in \mathbb{N}.\, f(n) = n + 1$.
   Hint: Try to prove by mutual induction that $\forall\, n \in \mathbb{N}.\, h(n) = n \wedge g(n) = 1$.

3. Consider the alphabet $\Sigma = \{a, b\}$. In this exercise you will define a set S containing certain words over the alphabet Σ. The simplest word in S is the empty word. If we have two words in S, we can form a new word by putting an a followed by one of the words, then another a followed by the other word and finally putting a b. If we have three words, we can also form a new word by putting a b followed by one of the words, then putting an a followed by another of the words, and finally another a followed by the third word.

   (a) (1.25pts) Give the inductive definition of the set S of words according to this description;

   (b) (0.75pts) Define the functions $\#_a : S \to \mathbb{N}$ and $\#_b : S \to \mathbb{N}$ that count the number of a's and the number of $b's$, respectively, in a word from S;

   (c) (2pts) Prove using induction that $\forall\, w \in S.\, \#_a(w) = 2 \times \#_b(w)$.

4. (1.75pts) Σ here denotes (finite) alphabets.
   Prove using induction that $\forall\, n \in \mathbb{N}.\, \forall \Sigma.\, |\Sigma^n| = |\Sigma|^n$.
   (See lecture 4, slide 16 for the definition of Σ^n.)