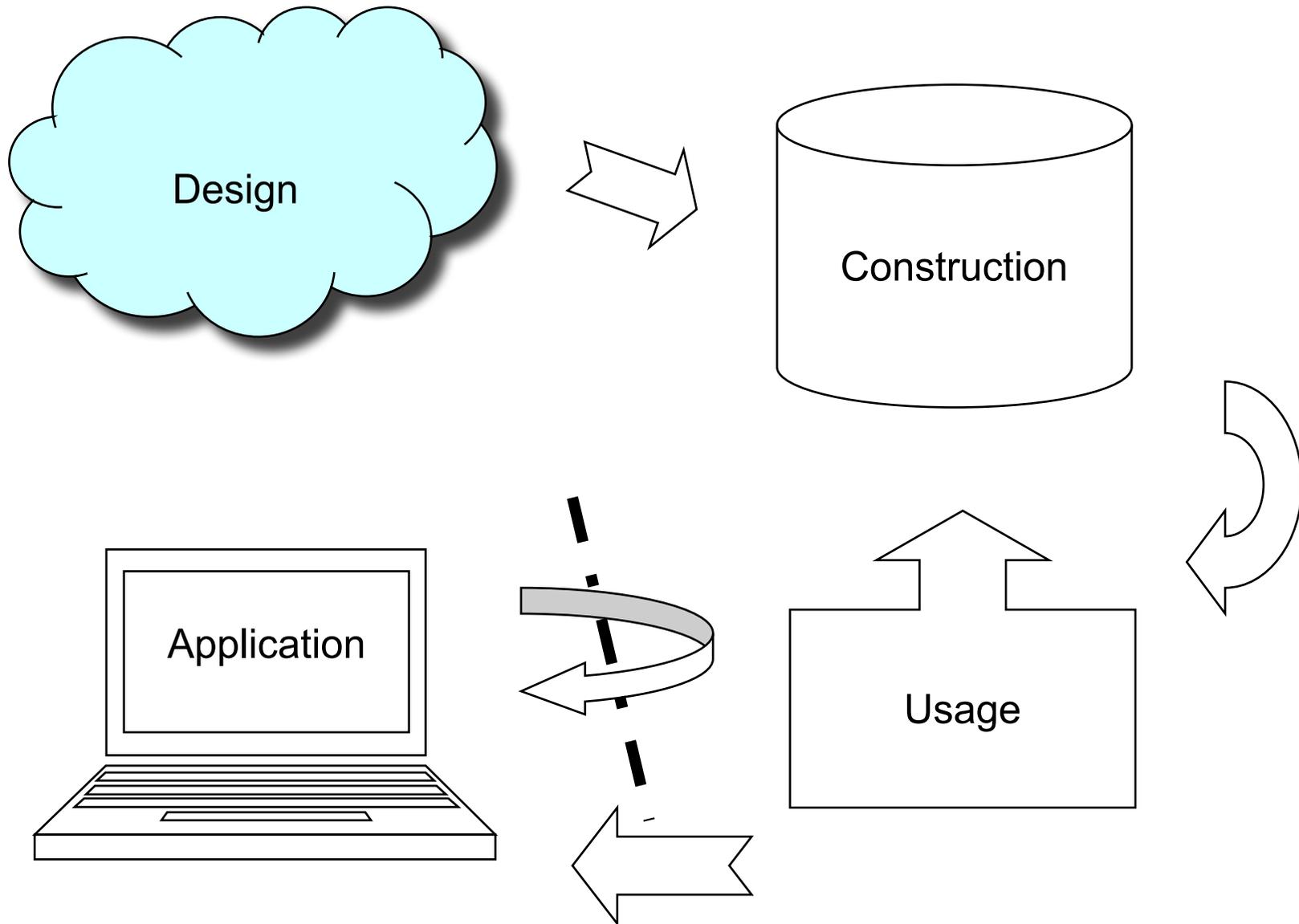


# Database design II

## Functional Dependencies

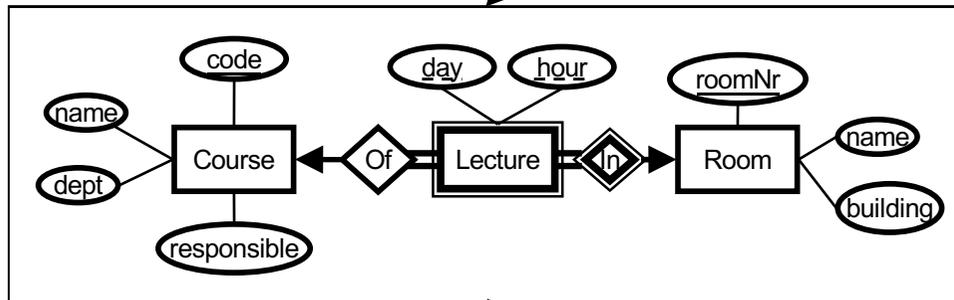
# Course Objectives



Problem description

"We want a database that we can use for scheduling courses and lectures. This is how it's supposed to work: ..."

TODAY



ER diagram

Functional dependencies  
Decomposition using normal forms to remove anomalies

Fix errors

Add constraints

Relational database schema

```
Courses (code, name, dept, examiner)
Rooms (roomNr, name, building)
Lectures (roomNr, day, hour, course)
roomNr -> Rooms.roomNr
course -> Courses.code
```

# Functional dependencies (FDs)

- $X \rightarrow A$ 
  - "X determines A", "X gives A"
  - "A depends on X"
- X and A are sets of attributes
- Examples:
  - **code**  $\rightarrow$  **name**
  - **code, period**  $\rightarrow$  **teacher**

# Assertions on a schema

- $X \rightarrow A$  is an assertion about a schema  $R$ 
  - If two tuples in  $R$  agree on the values of the attributes in  $X$ , then they must also agree on the value of  $A$ .
- Example: **code, period  $\rightarrow$  teacher**
  - If two tuples in the GivenCourses relation have the same course code and period, then they must also have the same teacher.

# Assertions on a domain

- $X \rightarrow A$  is really an assertion about a *domain* D
  - Let D be the relation that is the join (along references) of all relations in the database of the domain.
    - E.g. The Scheduler domain
  - If two tuples in D agree on the values of the attributes in X, then they must also agree on the value of A.
- Example: **code**, **period**  $\rightarrow$  **teacher**
  - If two tuples in the D relation (i.e. the domain) have the same course code and period, then they must also have the same teacher.

# What are FDs really?

- Functional dependencies represent a special kind of constraints of a domain – *dependency constraints*.
- The database we design should properly capture all constraints of the domain.
- We can use FDs to verify that our design indeed captures the constraints we expect, and add more constraints to the design when needed.

# What's so functional?

- $X \rightarrow A$  is a (deterministic) function from  $X$  to  $A$ . Given values for the attributes in the set  $X$ , we get the value of  $A$ .
- Example:
  - **code**  $\rightarrow$  **name**
  - imagine a deterministic function  $\mathbf{f}(\mathbf{code})$  which returns the name associated with a given code.

# A note on syntax

- A **functional dependency** exists between attributes in the same relation, e.g. in relation Courses we have FD:  
**code** → **name**
- A **reference** exists between attributes in two different relations, e.g. for relation GivenCourses we have reference:  
**course** -> **Courses.code**
- Two completely different things, but with similar syntax. Clear from context which is intended.

# Quiz!

*What are reasonable FDs for the scheduler domain?*

- Course codes and names
- The period a course is given
- The number of students taking a course
- The name of the course responsible
- The names of all lecture rooms
- The number of seats in a lecture room
- Weekdays and hours of lectures

# Quiz: (an) answer

*What are reasonable FDs for the scheduler domain?*

`code → name`

`code, period → #students`

`code, period → teacher`

`room → #seats`

`code, period, weekday → hour`

`code, period, weekday → room`

`room, period, weekday, hour → code`

# Multiple attributes on R/LHS

- $X \rightarrow A, B$ 
  - Short for  $X \rightarrow A$  and  $X \rightarrow B$
  - If we have both  $X \rightarrow A$  and  $X \rightarrow B$ , we can combine them to  $X \rightarrow A, B$ .
  - **`code, period → teacher, #students`**
- Multiple attributes on LHS can be crucial!
  - **`code, period → teacher`**
    - **`code ↗ teacher`**
    - **`period ↗ teacher`**

# Quiz!

- What's the difference between the LHS of a FD, and a key?
  - both uniquely determine the values of other attributes.
  - ...but a key must determine *all* other attributes in a relation!
  - We *use* FDs when determining keys of relations (will see how shortly).

# Example

Schedules (code, name, period, numStudents, teacher, room, numSeats, weekday, hour)

<i>code</i>	<i>name</i>	<i>per.</i>	<i>#st</i>	<i>teacher</i>	<i>room</i>	<i>#seats</i>	<i>day</i>	<i>hour</i>
TDA357	Databases	2	200	Steven Van Acker	HB2	186	Tuesday	10:00
TDA357	Databases	2	200	Steven Van Acker	HB2	186	Wednesday	8:00
TDA357	Databases	3	93	Graham Kemp	HC4	216	Tuesday	10:00
TDA357	Databases	3	93	Graham Kemp	VR	228	Friday	10:00
TIN090	Algorithms	1	64	Devdatt Dubhashi	HC1	126	Wednesday	08:00
TIN090	Algorithms	1	64	Devdatt Dubhashi	HC1	126	Thursday	13:15

`code, period` → `teacher` ? Yes! This is a FD

...but `{code, period}` is not a key...

# Example (decomposed)

```
Courses(code, name)
GivenCourses(course, period, #students, teacher)
  course -> Courses.code
Lectures(course, period, room, weekday, hour)
  (course, period) -> GivenCourses.(course, period)
  room             -> Rooms.name
Rooms(name, #seats)
```

`code, period` → `teacher` ?

Quiz: Given values for a code and a period, starting from any relation where they appear, is it possible to reach more than one teacher value by following keys and references?

Answer: No, so the FD constraint is properly captured. No need to fix schema

# Trivial FDs

- A FD is *trivial* if all attributes on the RHS are also on the LHS.
  - Example: `course, period → course`

Quiz: Is this a trivial FD?

`course, period → course, name`

Shorthand for

`course, period → course` (trivial)

`course, period → name` (not trivial)

# Inferring FDs

- In general we can find more FDs
  - course, period, weekday  $\rightarrow$  room
  - room  $\rightarrow$  #seats

$\Rightarrow$  course, period, weekday  $\rightarrow$  #seats
- We will need *all* FDs for doing a proper design.

# Closure of attribute set $X$

- Computing the *closure* of  $X$  means finding all FDs that have  $X$  as the LHS.
- If  $A$  is in the closure of  $X$ , then  $X \rightarrow A$ .  
E.g. If `teacher` is in the closure of `code, period`  
Then `code, period`  $\rightarrow$  `teacher`
- The closure of  $X$  is written  $X^+$ .
  - $X^+$  = all attributes that "follow" from  $X$

# Computing the closure

- Given a set of FDs,  $F$ , and a set of attributes,  $X$ :
  1. Start with  $X^+ = X$ .
  2. For all FDs  $Y \rightarrow B$  in  $F$  where  $Y$  is a subset of  $X^+$ , add  $B$  to  $X^+$ .
  3. Repeat step 2 until there are no more FDs that apply.

# Quiz!

*What is the closure of  
{code, period, weekday}?*

`code → name`

`code, period → #students`

`code, period → teacher`

`room → #seats`

`code, period, weekday → hour`

`code, period, weekday → room`

`room, period, weekday, hour → code`

`{code, period, weekday}+ =`

`{code, period, weekday, name, #students,  
teacher, hour, room, #seats}`

# Finding all implied FDs: $F^+$

- $F^+$  is also called the closure of  $F$
- Simple, exponential algorithm
  - For each set of attributes  $X$  in a relation  $R$ :
    1. compute  $X^+$ .
    2. Add  $X \rightarrow A$  to  $F^+$  for all  $A$  in  $X^+ - X$ .
    3. However, drop  $XY \rightarrow A$  whenever we discover  $X \rightarrow A$ .
      - Because  $XY \rightarrow A$  follows from  $X \rightarrow A$ .

# Example: Finding $F^+$

code, period, weekday → name (1)  
code, period, weekday → #students (2)  
code, period, weekday → teacher (3)  
code, period, weekday → hour (5)  
code, period, weekday → room (6)  
code, period, weekday → #seats **Implied: (6)+(4)**

(1) code → name  
(2) code, period → #students  
(3) code, period → teacher  
(4) room → #seats  
(5) code, period, weekday → hour  
(6) code, period, weekday → room  
(7) room, period, weekday, hour → code

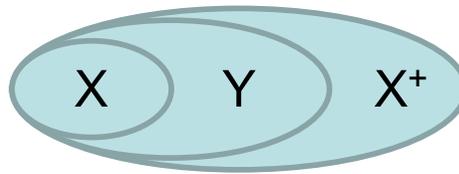
$X = \{\text{code, period, weekday}\}$  (remember: must repeat for all other  $X$  too)

$X^+ = \{\text{code, period, weekday, name, #students, teacher, hour, room, #seats}\}$

$X^+ - X = \{\text{name, #students, teacher, hour, room, #seats}\}$

# Finding $F^+$ : a simplifying trick

If  $X \subseteq Y \subseteq X^+$



then  $Y^+ == X^+$  and no new FDs will be found

e.g.

$X = \{\text{code, period, weekday}\}$

$Y = \{\text{code, period, weekday, name, room}\}$

$X^+ = \{\text{code, period, weekday, name, \#students, teacher, hour, room, \#seats}\}$

In particular, if  $X^+$  is the set of all attributes, then the closure of all supersets of  $X$  will also be the set of all attributes.

# Finding keys

- For a relation  $R$ , any subset  $X$  of attributes of  $R$  such that  $X^+$  contains all attributes of  $R$  is a *superkey* of  $R$ .
  - Intuitively, a superkey is any set of attributes that determine all other attributes.
  - The set of all attributes is a superkey.
- A *key* for  $R$  is a *minimal* superkey.
  - A superkey  $X$  is minimal if no proper subset of  $X$  is also a superkey.
    - Minimal – no subset is a key
    - Minimum – the smallest, i.e. the one with the fewest number of attributes

**Schedules (code, name, period, #students,  
teacher, room, #seats, weekday, hour)**

Example:

**$X = \{\text{code, period, weekday, hour}\}$**

is a superkey of the relation Schedules since  $X^+$  is the set of all attributes of Schedules.

However,

**$Y = \{\text{code, period, weekday}\}$**

is also a superkey, and is a subset of  $X$ , so  $X$  is not a key of Schedules. No subset of  $Y$  is a superkey, so  $Y$  is also a key.

# Quiz!

*What is the key of Schedules?*

`code → name`

`code, period → #students`

`code, period → teacher`

`room → #seats`

`code, period, weekday → hour`

`code, period, weekday → room`

`room, period, weekday, hour → code`

Two keys exist:

`{code, period, weekday}`

`{room, period, weekday, hour}`

# Primary keys

- There can be more than one key for the same relation.
- We choose one of them to be the *primary key*, which is the key that we actually use for the relation.
- Other keys could be asserted through uniqueness constraints.
  - E.g. for the self-referencing relation

## Example:

For NextTo we have both

- **left** → **right**
- **right** → **left**

```
Rooms (name, #seats)
NextTo (right, left)
    right -> Rooms.name
    left  -> Rooms.name
    left  unique
```

Both **left** and **right** are keys, but we have chosen **right** to be the primary key for **NextTo**. We can add a constraint stating that **left** should be unique.

Note: The syntax for constraints is not well specified. Both the reference syntax, as well as the uniqueness assertion, are my suggestions only (but they're rather good).

# Where do FDs come from?

- "Keys" of entities (from ER diagram)
  - If code is the key for the entity Course, then all other attributes of Course are functionally determined by code, e.g. **code** → **name**
- Relationships (from ER diagram)
  - If all courses hold lectures in just one room, then the key for the Course entity also determines all attributes of the Room entity, e.g.  
**code** → **room**
- Physical reality (domain description)
  - No two courses can have lectures in the same room at the same time, e.g.  
**room, period, weekday, hour** → **code**

# Make reality match theory

- In some cases reality is not suitably deterministic. We may need to invent key attributes in order to have a key at all.

Quiz: Give examples of this phenomenon from reality!

Social security numbers, course codes, product numbers, user names etc.

# How NOT to find FDs

- Do an E-R diagram, look at the entities and many-to-one relationships, pick the proper FDs.

Quiz: Why not?

- FDs should be used to find *more* constraints, and also to check that your diagram is correct. If the FDs are taken from the diagram, no more constraints will be added, and it will contain the same errors!

# Example: Scheduler domain

```
Courses(code, name)
GivenCourses(course, period, #students, teacher)
  course -> Courses.code
Lectures(course, period, room, weekday, hour)
  (course, period) -> GivenCourses.(course, period)
  room              -> Rooms.name
Rooms(name, #seats)
```

```
code -> name
code, period -> #students
code, period -> teacher
room -> #seats
code, period, weekday -> hour
code, period, weekday -> room
room, period, weekday, hour -> code
```

Quiz: Fix the  
schema!

# Scheduler domain (fixed)

```
Courses(code, name)
GivenCourses(course, period, #students, teacher)
  course -> Courses.code
Lectures(course, period, room, weekday, hour)
  (course, period) -> GivenCourses.(course, period)
  room             -> Rooms.name
  (room, period, weekday, hour) unique
Rooms(name, #seats)
```

```
code → name
code, period → #students
code, period → teacher
room → #seats
code, period, weekday → hour
code, period, weekday → room
room, period, weekday, hour → code
```

Add a key to  
Lectures!

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  - <https://goo.gl/TfFTxA>



Break! In part 2:

BCNF decomposition

3NF