Exam
Data structures DAT036/DAT037/DIT960

Time	Thursday 20th August 2015, 14:00–18:00
Place	Maskinhuset
Course responsible	Nick Smallbone, tel. 0707 183062

The exam consists of **six questions**.

For a **3** (Chalmers) or **G** (GU), you need to answer **three** questions correctly.
You can ignore any parts labelled **"For a 4"** or **"For a 5/VG"**.

For a **4** (Chalmers only), you need to answer **four** questions correctly.
You must also answer all parts labelled **"For a 4"** in those questions but can ignore any parts labelled **"For a 5/VG"**.

For a **5** (Chalmers) or **VG** (GU), you need to answer **five** questions correctly.
You must also answer all parts labelled **"For a 4/5/VG"** in those questions.

For an answer to be considered correct, it must contain no major mistakes.
Minor mistakes might be accepted, but this is at the discretion of the marker.

When a question asks for **pseudocode**, you can use a mixture of English and programming notation to describe your solution, and should give enough detail that a competent programmer could easily implement your solution.

Allowed aids	One A4 piece of paper of hand-written notes, which should be handed in after the exam. You may write on both sides. You may also bring a dictionary.
Note	Begin each question on a new page. Write your anonymous code (*not* your name) on every page.

Good luck!

1. Consider the following algorithm for sorting an array a with the help of a priority queue.

```
q = new priority queue
for every element x in a
  q.insert(x)
i = 0
while q is not empty
  a[i] = q.findMinimum()
  q.deleteMinimum()
  i = i+1
```

Assuming that n is the length of the input array, what is the big-O complexity of this algorithm, if the priority queue is implemented using:

a) an unsorted array,

b) a binary heap,

c) **For 4/5/VG only:** a sorted array?

2. Consider the following hash table implemented using *linear probing*, where the hash function is the identity, *h(x)* = *x* (modulo 10).

0	1	2	3	4	5	6	7	8	9
10	1	11		14	5	XXX	7	18	

a) The value that was previously at index 6 has been deleted, which is represented by the **XXX** in the hash table.

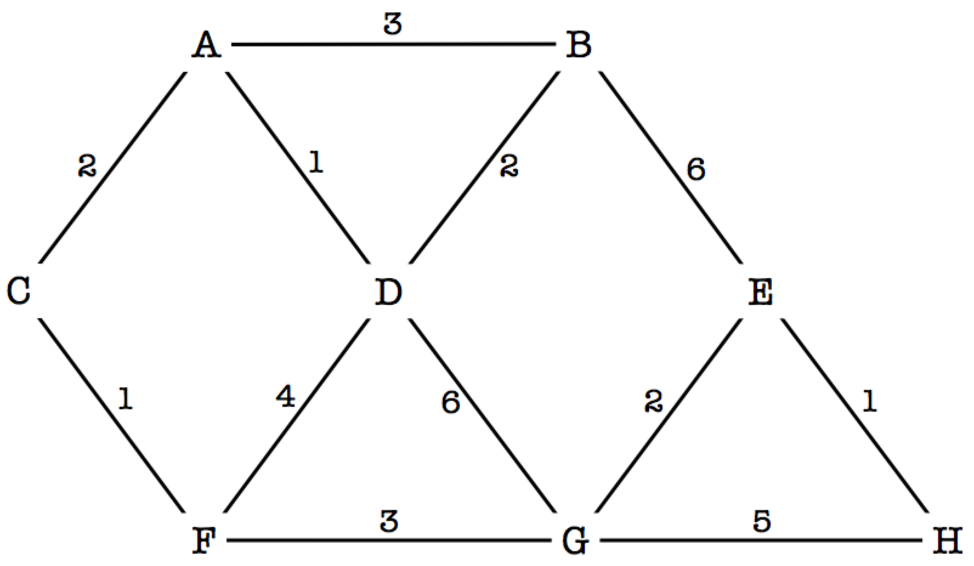
Which value might have been stored there, before it was deleted? There may be several correct answers, and you should write down **all** of them.

A) 26 **B**) 2 **C**) 17 **D**) 24 **E**) 6 **F**) 13

b) **For 4/5/VG only:**
Hash tables typically have better performance than balanced binary search trees. Even so, both are widely used in practice. One reason is that a hash table does not support all the operations that a BST does.

Give an example of an operation which can be efficiently implemented for a binary search tree but not for a hash table.

3. You are given the following weighted graph:



a) Suppose we perform Dijkstra's algorithm starting from node **D**. In which order does the algorithm visit the nodes, and what is the computed distance to each of them?

b) In its basic form, Dijkstra's algorithm computes the *distance* from one node to all other nodes, but it doesn't tell you the shortest path.

   Suppose you have run Dijkstra's algorithm on a graph, in other words you know the shortest distance from a node *x* to all other nodes in the graph. How can you use this information to find the shortest path from *x* to another node *y*?

   Answer either in pseudocode or English. If you want you can use finding the shortest path from **D** to **H** as an example – but your answer should make it clear how to do this for any graph.

4. In this question you should design a data structure that is almost a priority queue, except that you can find and remove the *second-smallest* element. It should support the following operations:

   - `new()`: create a new, empty priority queue
   - `insert(x)`: add an integer `x` to the priority queue
   - `findSecondSmallest()`: return the second-smallest element
   - `deleteSecondSmallest()`: remove the second-smallest element

   You may freely use standard data structures and algorithms from the course in your solution, without explaining how they are implemented.

   You should say what design or existing data structure you have chosen, and give **pseudocode** for each of the operations – you don't need to write fully detailed Java code.

   The operations must have the following time complexities:

   - **For a 3/G**:
     O(1) for `new`,
     O(log n) for `insert`,
     O(log n) for `findSecondSmallest`
     O(log n) for `deleteSecondSmallest`
     (where n is the number of elements in the priority queue)
   - **For a 4:**
     as for G but the complexity of `findSecondSmallest` must be O(1).

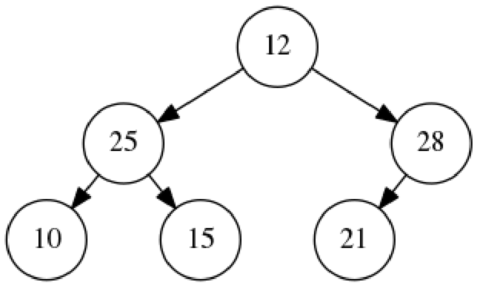
   For a 5/VG (you may wish to answer the earlier part first):
   Generalise your data structure so that it supports finding and removing the *k*th-smallest element, where *k* is a constant chosen by the user when the data structure is created. The time complexity should be:

     O(1) for `new` and `findKthSmallest`;
     O(log n) for `insert` and `deleteKthSmallest`.

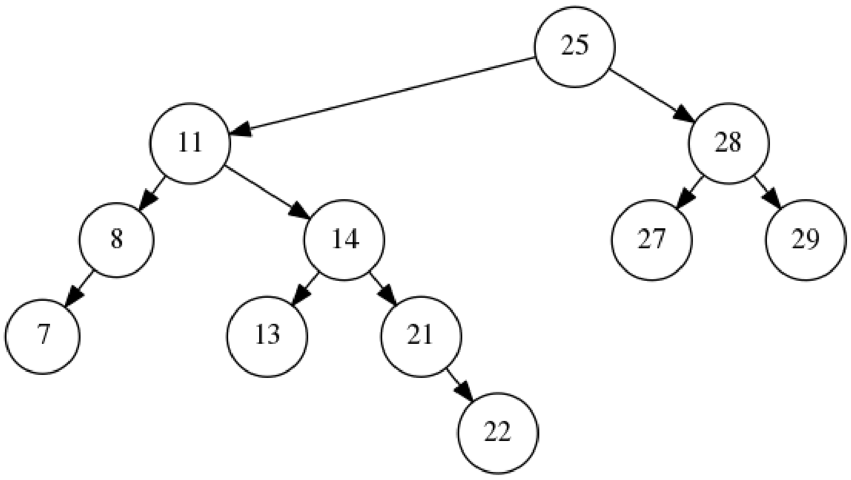
   Hint for a 5/VG: it might help to store the *k* smallest elements separately from the rest.

5. Have a look at the following three binary trees.

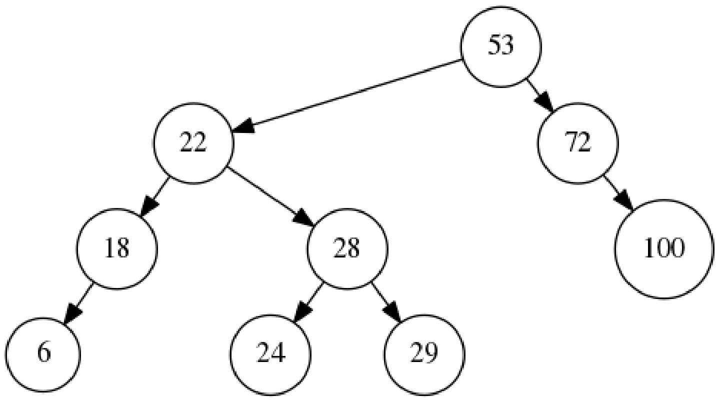
Tree A:



Tree B:



Tree C:



a) One of these trees is an AVL tree. Which one?

b) Insert 30 into the tree using the AVL insertion algorithm. Write down the final tree.

6. Suppose we are given the following type of binary search trees in Haskell:

```
data Tree a = Nil | Node a (Tree a) (Tree a)
```

a) Implement a Haskell function

```
deleteGreater :: Ord a => a -> Tree a -> Tree a
```

which takes an element `x` and a tree `t`, and returns `t` with all elements greater than or equal to `x` removed. For example on a tree `t` containing 1, 2, 3, 4, 5, `deleteGreater 3 t` should return a tree containing 1 and 2.

The complexity of your function should be O(height of tree), i.e., O(log n) for balanced trees, O(n) for unbalanced trees.

Hint: the correct solution is very small – mine is four lines of code. Try drawing some example trees and look for the general pattern of what is deleted. Think of what happens with a single node – which parts should be kept and which parts should be deleted.

b) **For 5/VG only:**
Does your function work on AVL trees, i.e., does it preserve the AVL invariant? If yes, explain why; if no, give an example that shows why not.