Binary search

Searching

Suppose I give you an array, and ask you to find if a particular value is in it, say 4.

5	3	9	2	8	7	3	2	1	4

The only way is to look at each element in turn.

This is called *linear search*.

You might have to look at every element before you find the right one.

Searching

But what if the array is sorted?

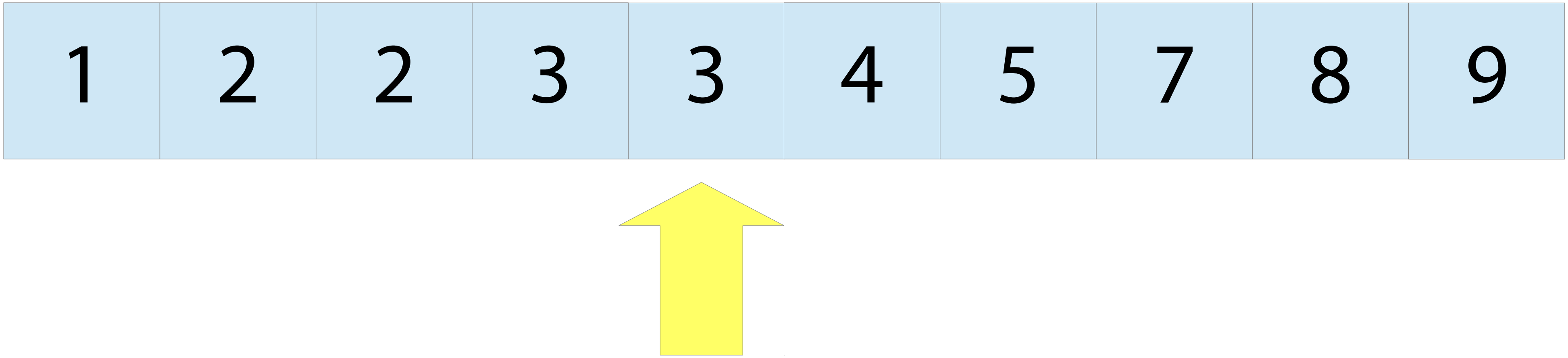
1	2	2	3	3	4	5	7	8	9

Then we can use *binary search*.

Binary search

Suppose we want to look for 4.

We start by looking at the element half way along the array, which happens to be 3.



1	2	2	3	3	4	5	7	8	9

Binary search

3 is less than 4.

Since the array is sorted, we know that 4 must come after 3.

We can ignore everything before 3.

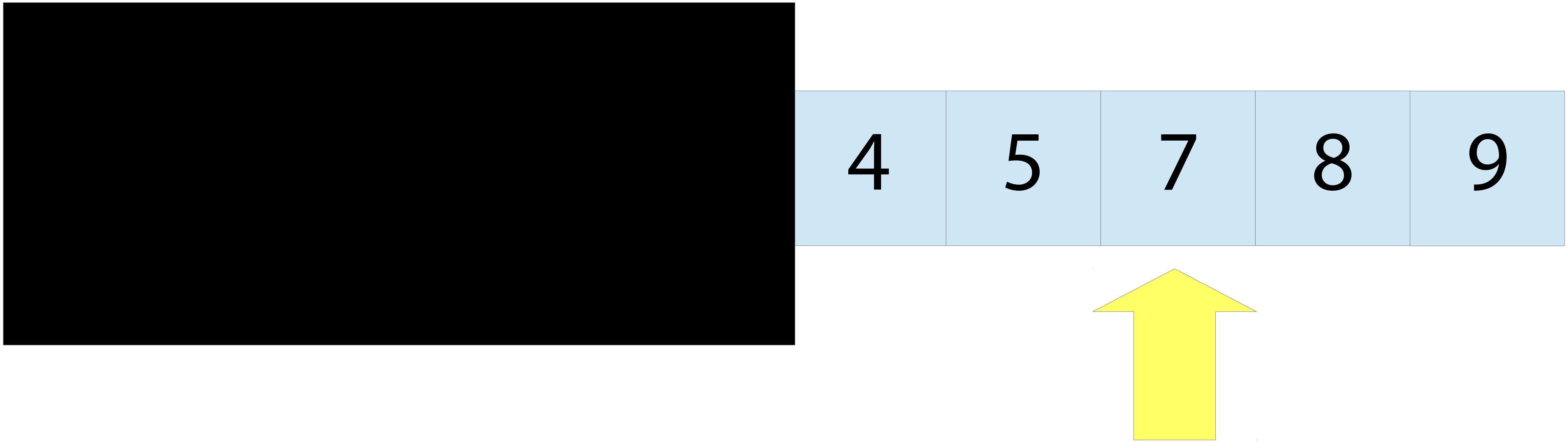
[redacted]

4	5	7	8	9

Binary search

Now we repeat the process.

We look at the element half way along what's left of the array. This happens to be 7.



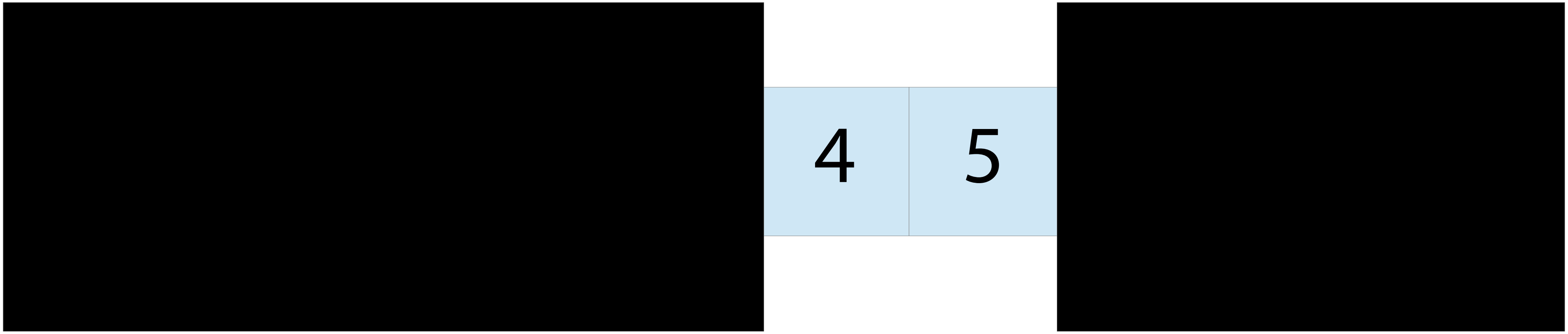
4	5	7	8	9

Binary search

7 is greater than 4.

Since the array is sorted, we know that 4 must come before 7.

We can ignore everything after 7.

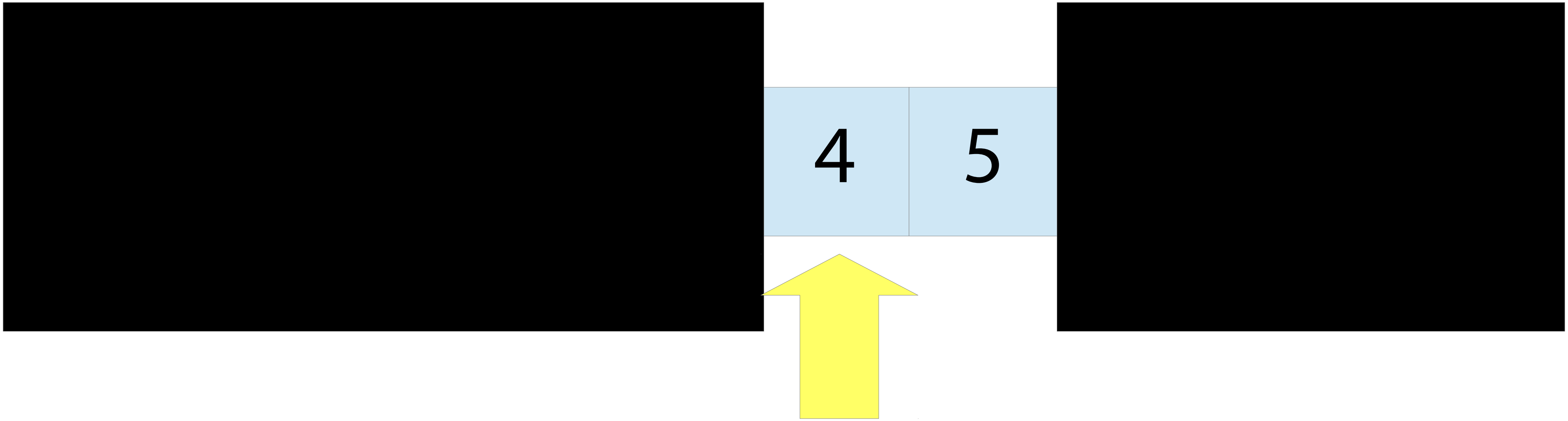


Binary search

We repeat the process.

We look half way along the array again.

We find 4!



Performance of binary search

Binary search repeatedly *chops the array in half*

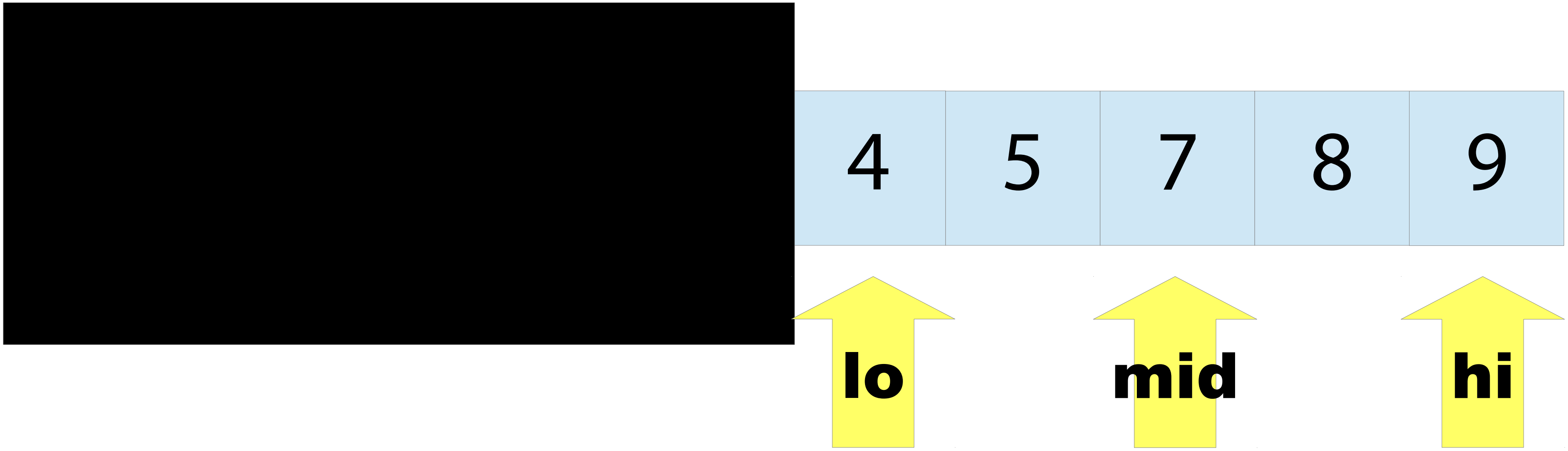
- If we double the size of the array, we need to look at one more array element
- With an array of size 2^n, after n tries, we are down to 1 element
- On an array of size n takes **O(log n)** time!

On an array of a billion elements, need to search **30** elements

(compared to a billion tries for linear search!)

Implementing binary search

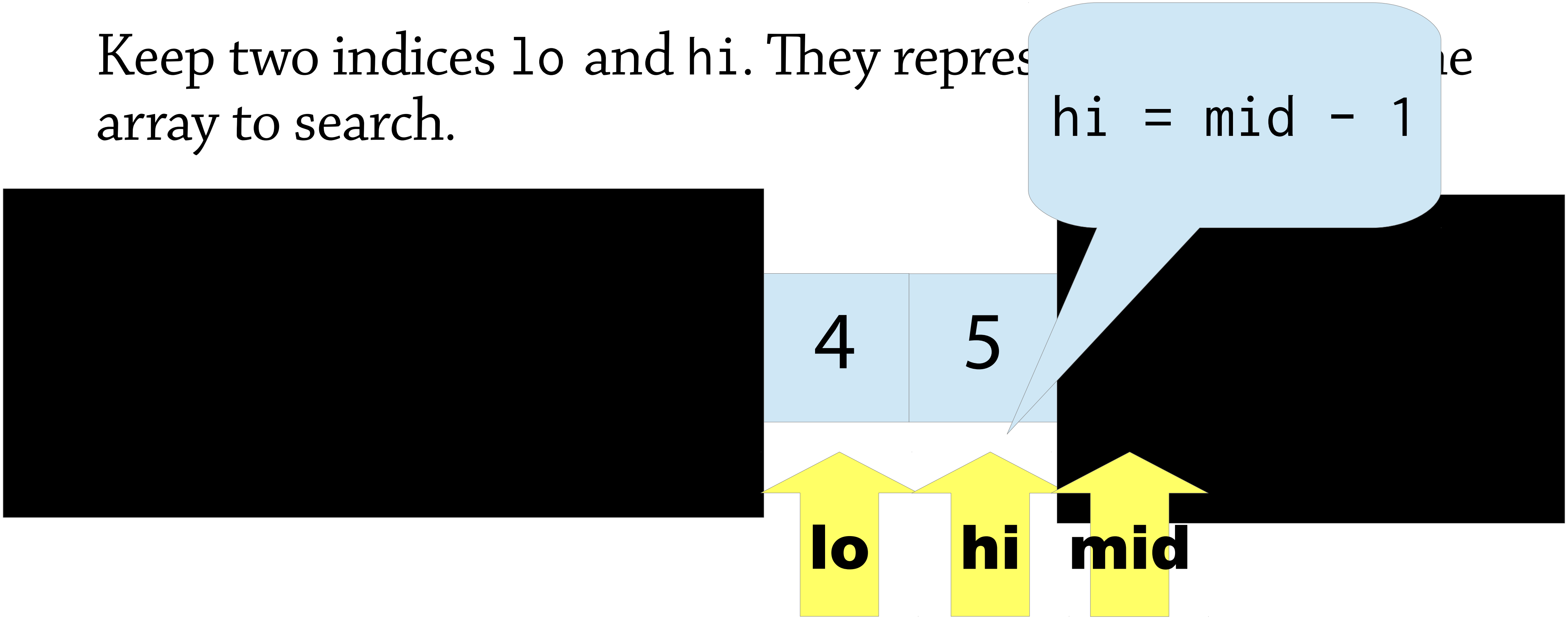
Keep two indices `lo` and `hi`. They represent the part of the array to search.



Let `mid = (lo + hi) / 2` and look at `a[mid]` – then either set `lo = mid+1`, or `hi = mid-1`, depending on the value of `a[mid]`

Implementing binary search

Keep two indices `lo` and `hi`. They repres e
array to search.



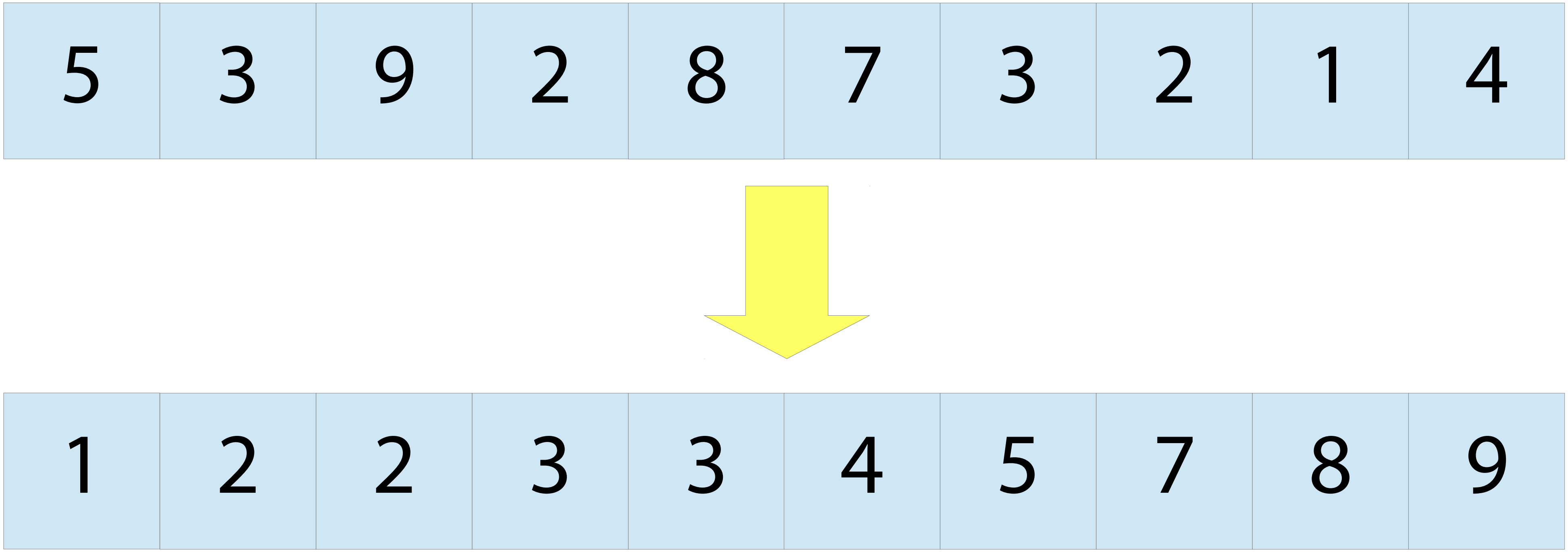
[redacted]

[redacted]

Let `mid = (lo + hi) / 2` and look at `a[mid]` – then either set `lo = mid+1`, or `hi = mid-1`, depending on the value of `a[mid]`

Sorting

Sorting



5	3	9	2	8	7	3	2	1	4

1	2	2	3	3	4	5	7	8	9

Zillions of sorting algorithms (bubblesort, insertion sort, selection sort, quicksort, heapsort, mergesort, shell sort, counting sort, radix sort, ...)

Insertion sort

Imagine someone is dealing you cards. Whenever you get a new card you put it into the right place in your hand:

This is the idea of *insertion sort*.

Insertion sort

Sorting

5	3	9	2	8

Start by “picking up” the 5:

5				

Insertion sort

Sorting

5	3	9	2	8

Then insert the 3 into the right place:

3	5			

Insertion sort

Sorting

5	3	9	2	8

Then the 9:

3	5	9		

Insertion sort

Sorting

5	3	9	2	8

Then the 2:

2	3	5	9	

Insertion sort

Sorting

5	3	9	2	8

Finally the 8:

2	3	5	8	9

Complexity of insertion sort

Insertion sort does n insertions for an array of size n

Does this mean it is O(n)? *No!* An insertion is not constant time.

To insert into a sorted array, you must move all the elements up one, which is O(n).

Thus total is $O(n^2)$.

In-place insertion sort

This version of insertion sort needs to make a new array to hold the result

An *in-place* sorting algorithm is one that doesn't need to make temporary arrays

- Has the potential to be more efficient

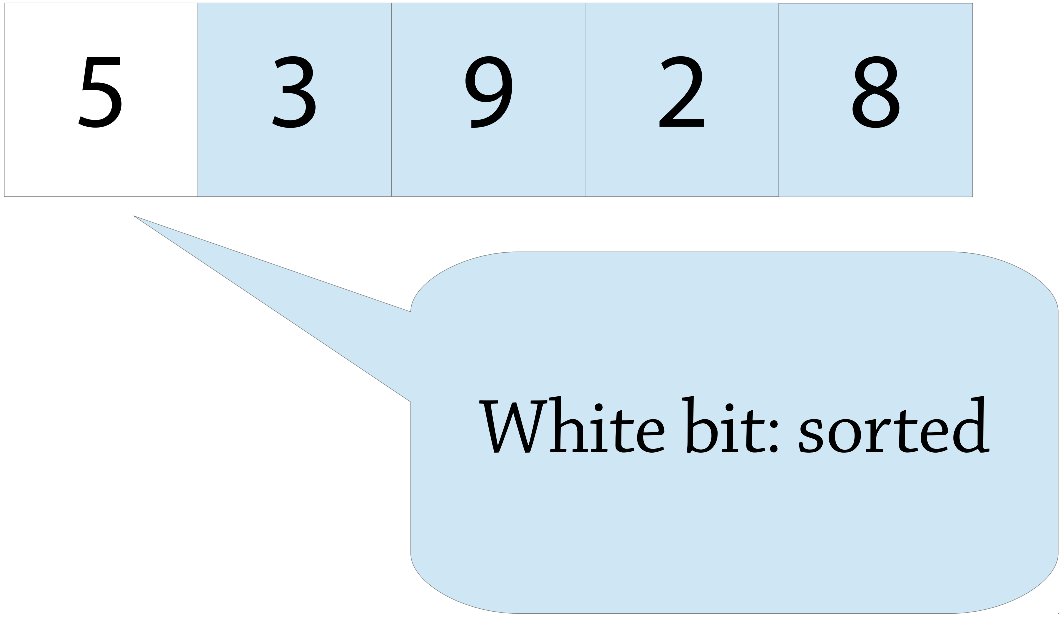
Let's make an in-place insertion sort!

Basic idea: loop through the array, and insert each element into the part which is already sorted

In-place insertion sort

5	3	9	2	8

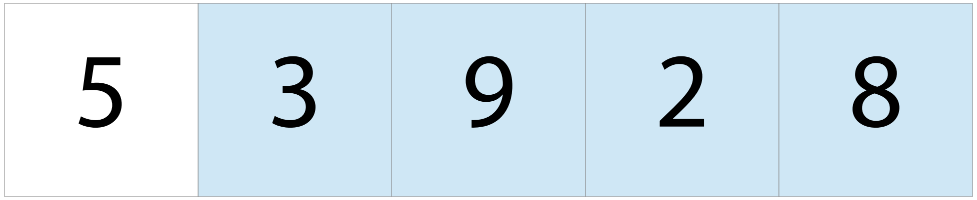
The first element of the array is sorted:



5	3	9	2	8

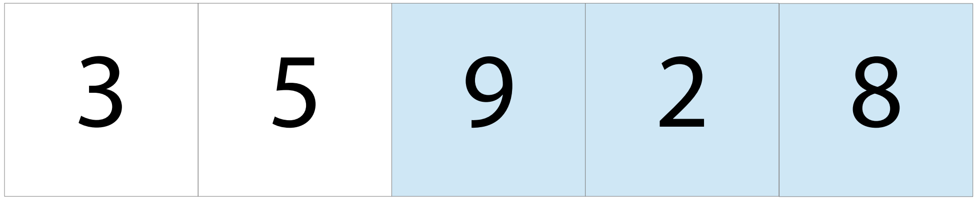
White bit: sorted

In-place insertion sort



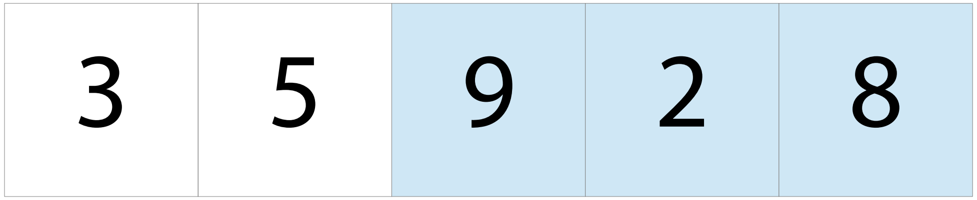
5	3	9	2	8

Insert the 3 into the correct place:



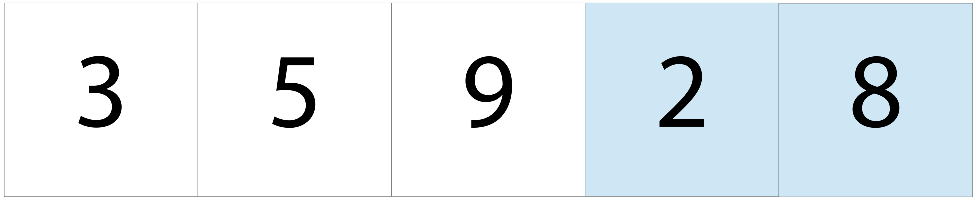
3	5	9	2	8

In-place insertion sort



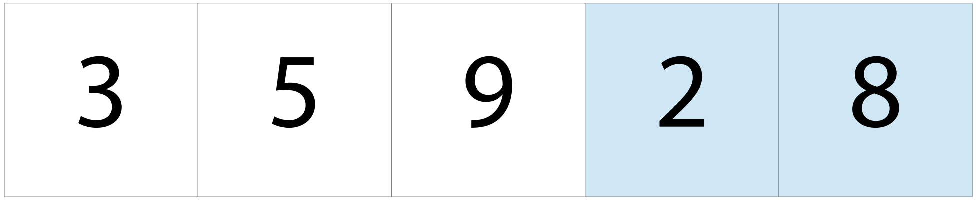
3	5	9	2	8

Insert the 9 into the correct place:



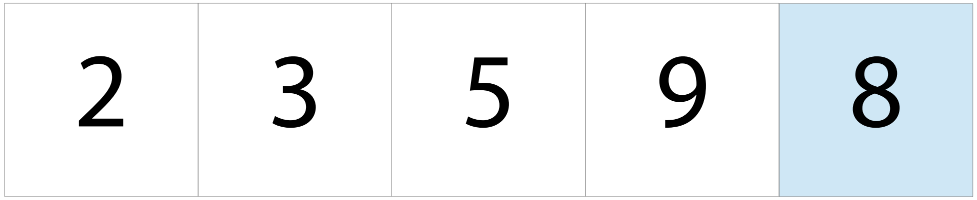
3	5	9	2	8

In-place insertion sort



3	5	9	2	8

Insert the 2 into the correct place:



2	3	5	9	8

In-place insertion sort

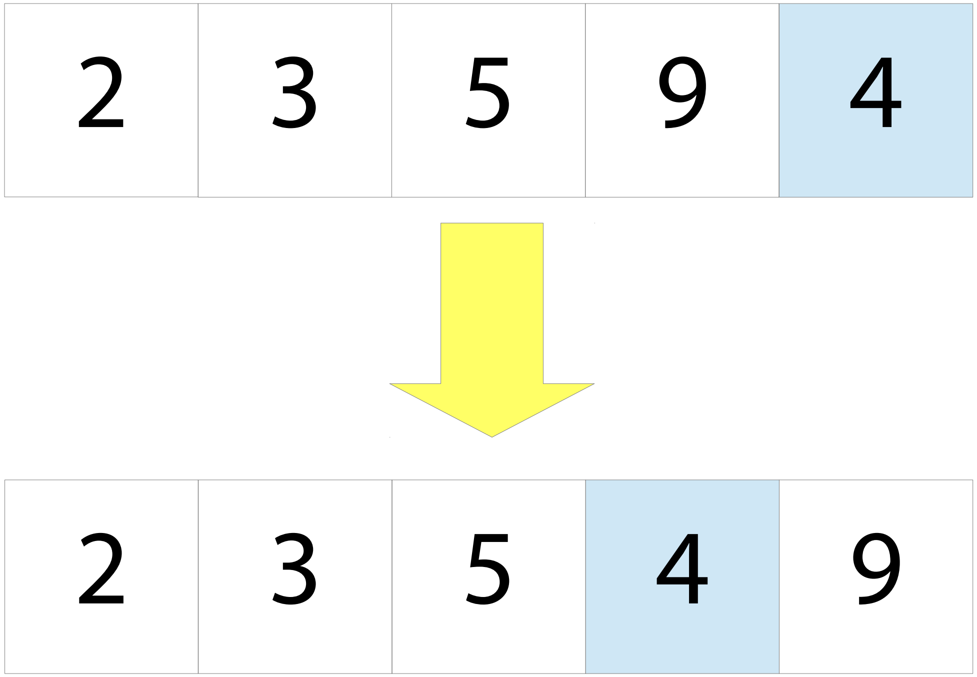
2	3	5	9	8

Insert the 8 into the correct place:

2	3	5	8	9

In-place insertion

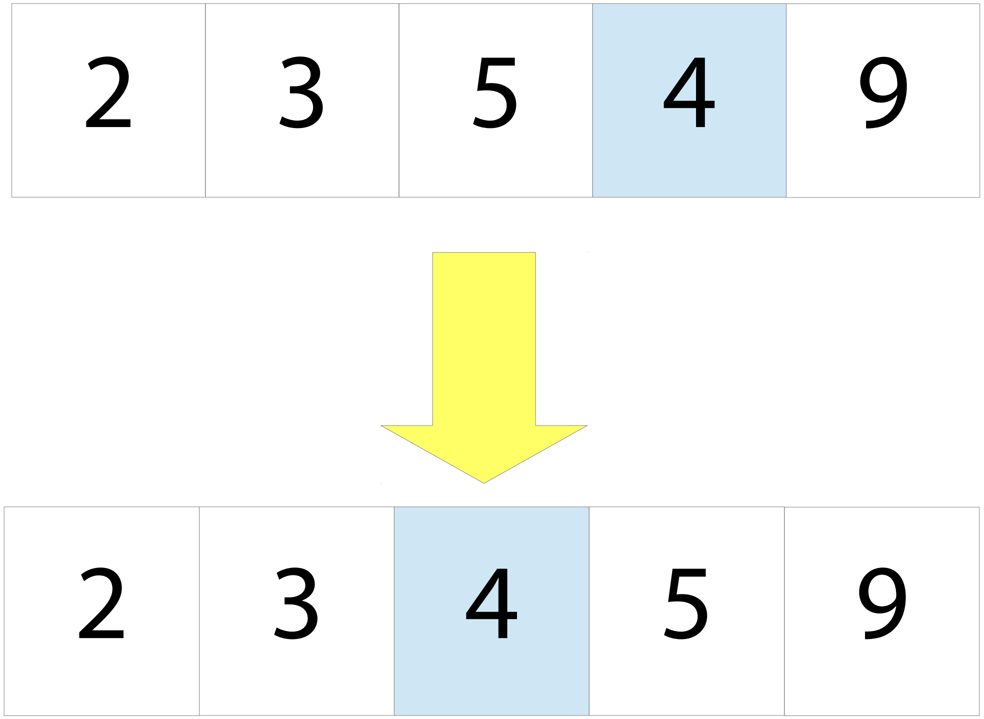
One way to do it: repeatedly swap the element with its neighbour on the left, until it's in the right position



2	3	5	9	4

2	3	5	4	9

In-place insertion



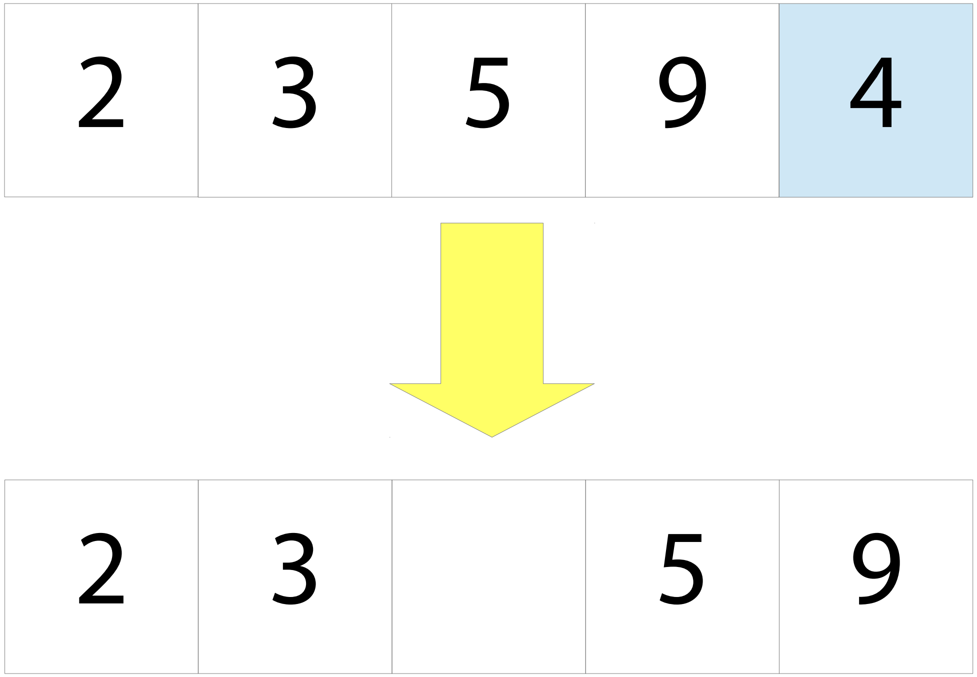
2	3	5	4	9

2	3	4	5	9

```
while n > 0 and array[n] < array[n-1]
  swap array[n] and array[n-1]
  n = n-1
```

In-place insertion

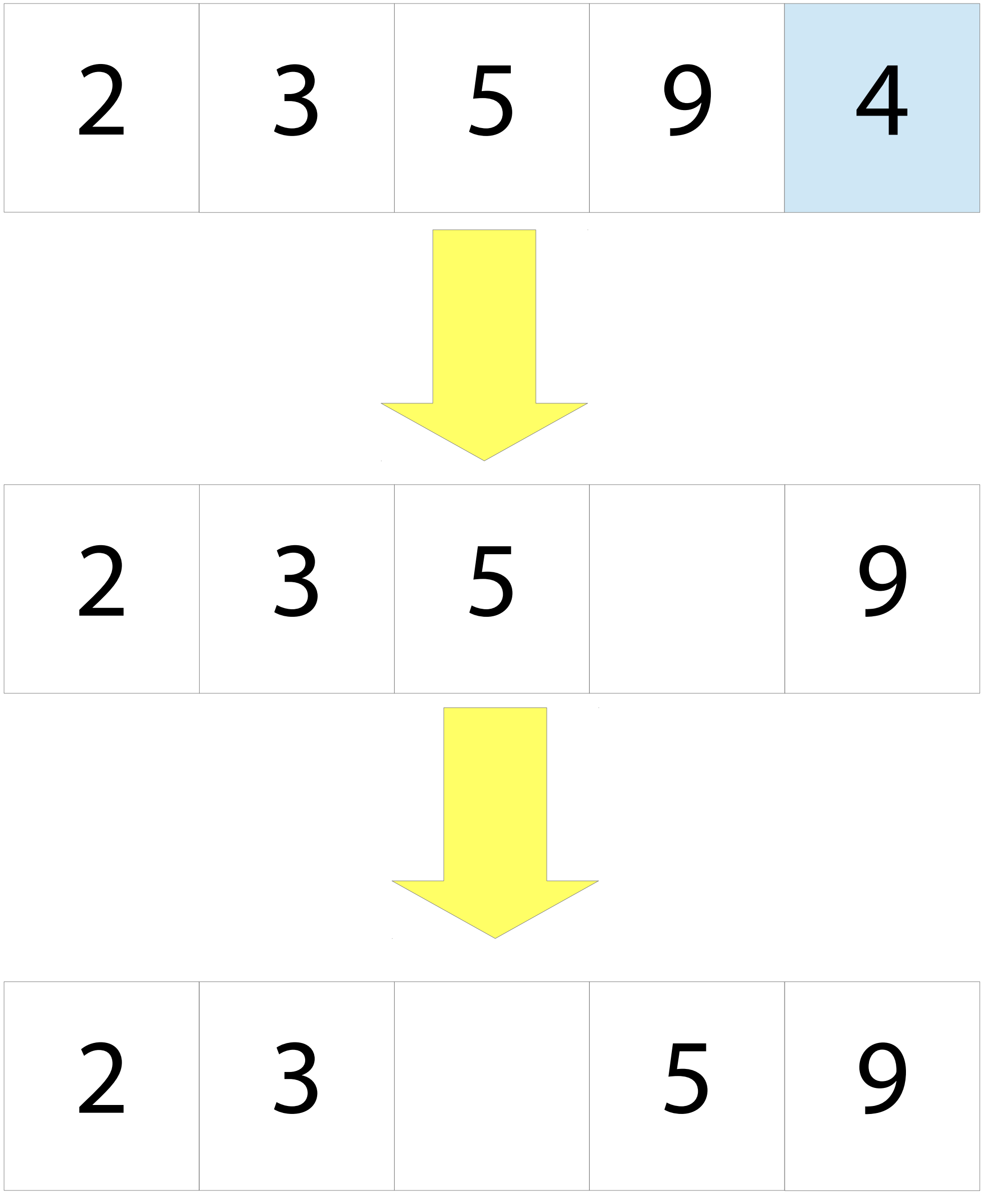
An improvement: instead of swapping, move elements upwards to make a "hole" where we put the new value



2	3	5	9	4

2	3		5	9

In-place insertion



2	3	5	9	4

2	3	5		9

2	3		5	9

In-place insertion sort

```
for i = 1 to n
  insert array[i] into arr
```

This notation means 0, 1, ..., i-1

An aside: we have the *invariant* that `array[0..i)` is sorted

- An invariant is something that holds whenever the loop body starts to run
- Initially, i = 1 and `array[0..1)` is sorted
- As the loop runs, more and more of the array becomes sorted
- When the loop finishes, `i = n`, so `array[0..n)` is sorted – the whole array!

Insertion sort

$O(n^2)$ in the worst case

$O(n)$ in the best case (a sorted array)

Actually the fastest sorting algorithm in general for small lists – it has low constant factors

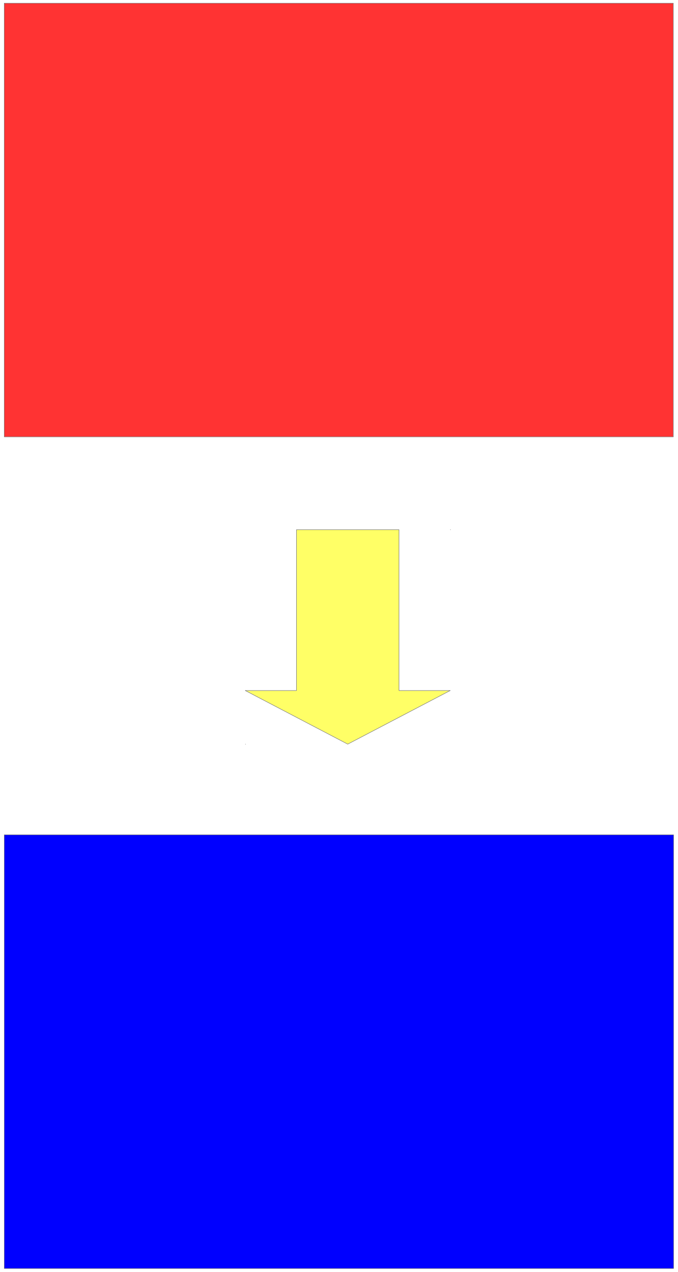
Divide and conquer

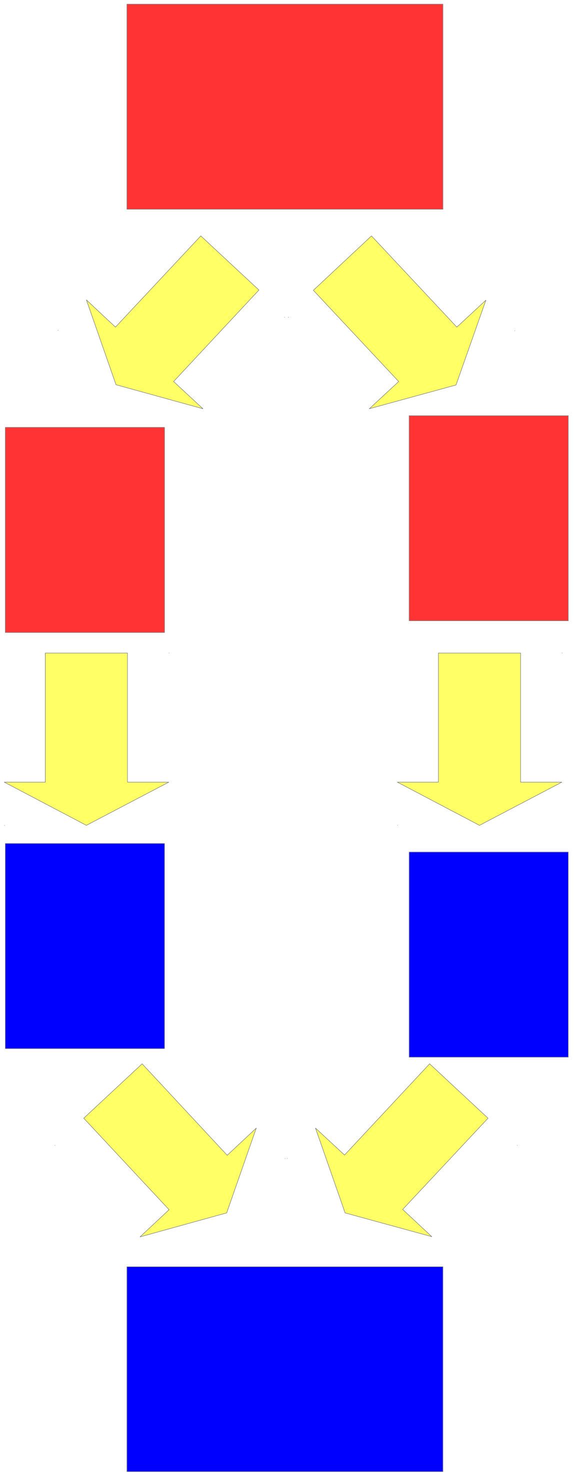
Very general name for a type of recursive algorithm

You have a problem to solve.

- *Split* that problem into smaller subproblems
- *Recursively* solve those subproblems
- *Combine* the solutions for the subproblems to solve the whole problem

To solve this...





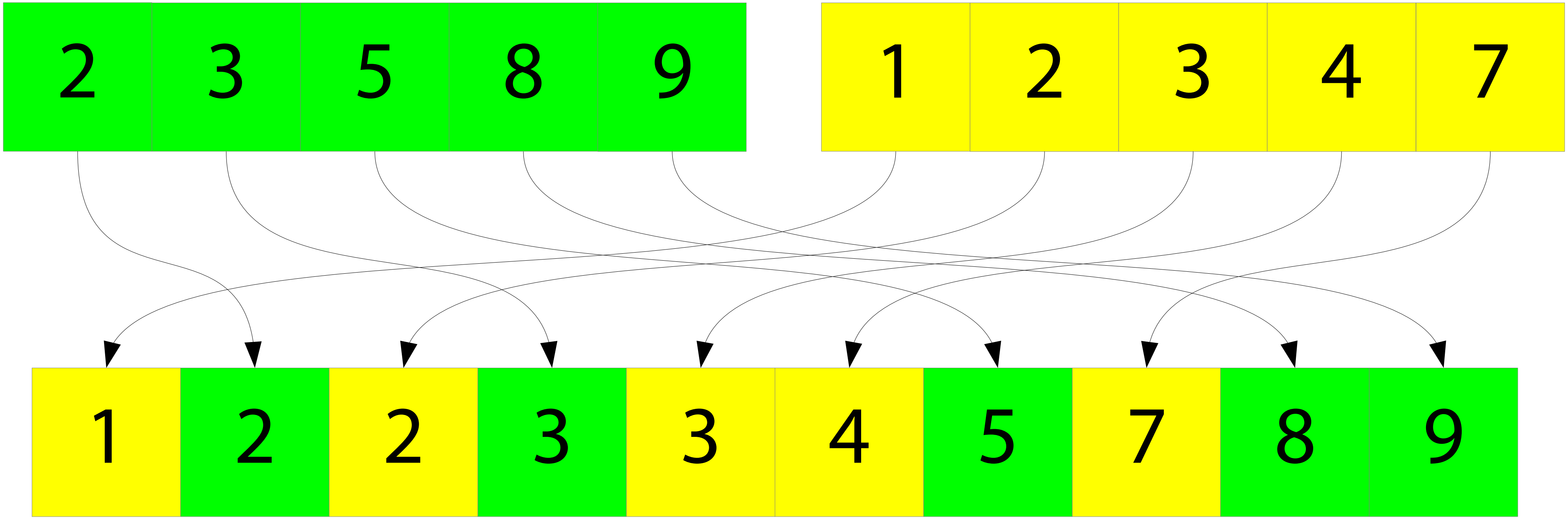
1. *Split* the problem into subproblems

2. *Recursively* solve the subproblems

3. *Combine* the solutions

Mergesort

We can *merge* two sorted lists into one in linear time:



2 3 5 8 9

1 2 3 4 7

1 2 2 3 3 4 5 7 8 9

Mergesort

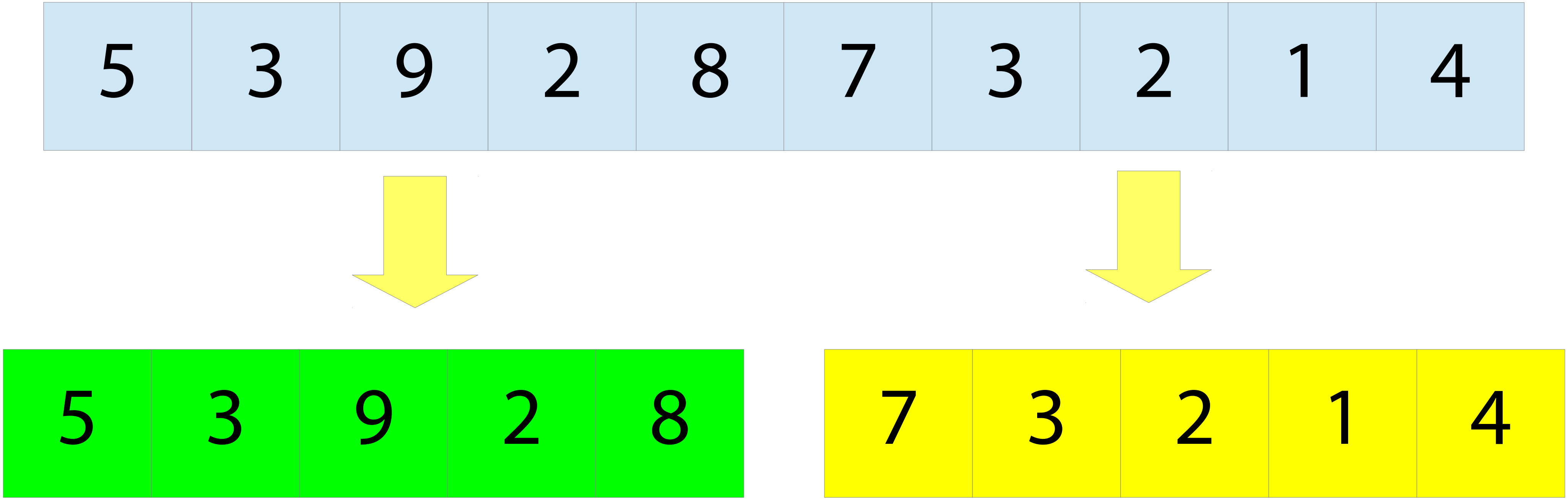
A divide-and-conquer algorithm

To mergesort a list:

- *Split* the list into first and second halves
- *Recursively* mergesort the two halves
- *Merge* the two sorted lists together

Mergesort

1. *Split* the list into two equal parts



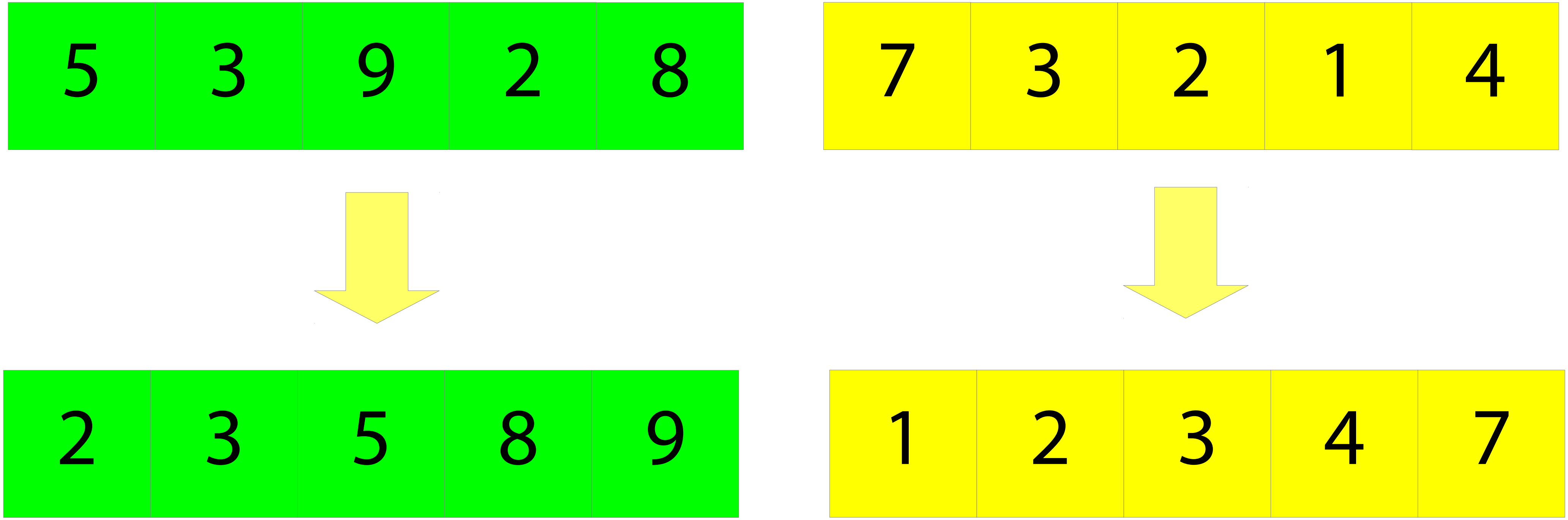
5	3	9	2	8	7	3	2	1	4

5	3	9	2	8

7	3	2	1	4

Mergesort

2. *Recursively* mergesort the two parts



5	3	9	2	8

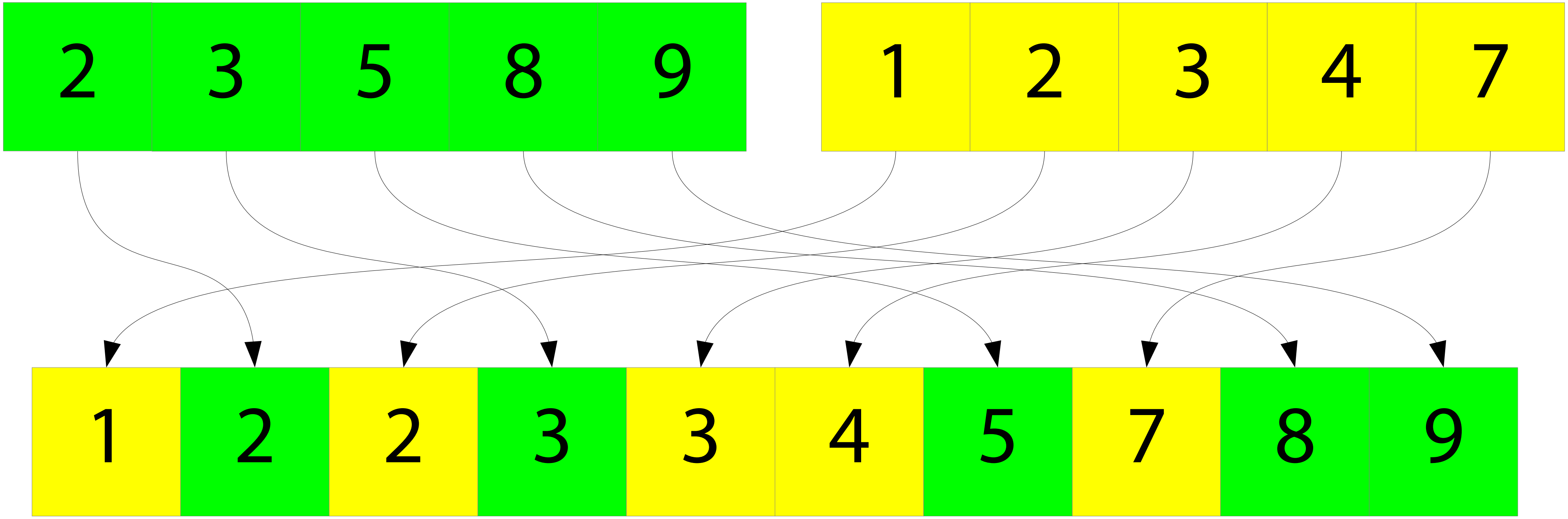
7	3	2	1	4

2	3	5	8	9

1	2	3	4	7

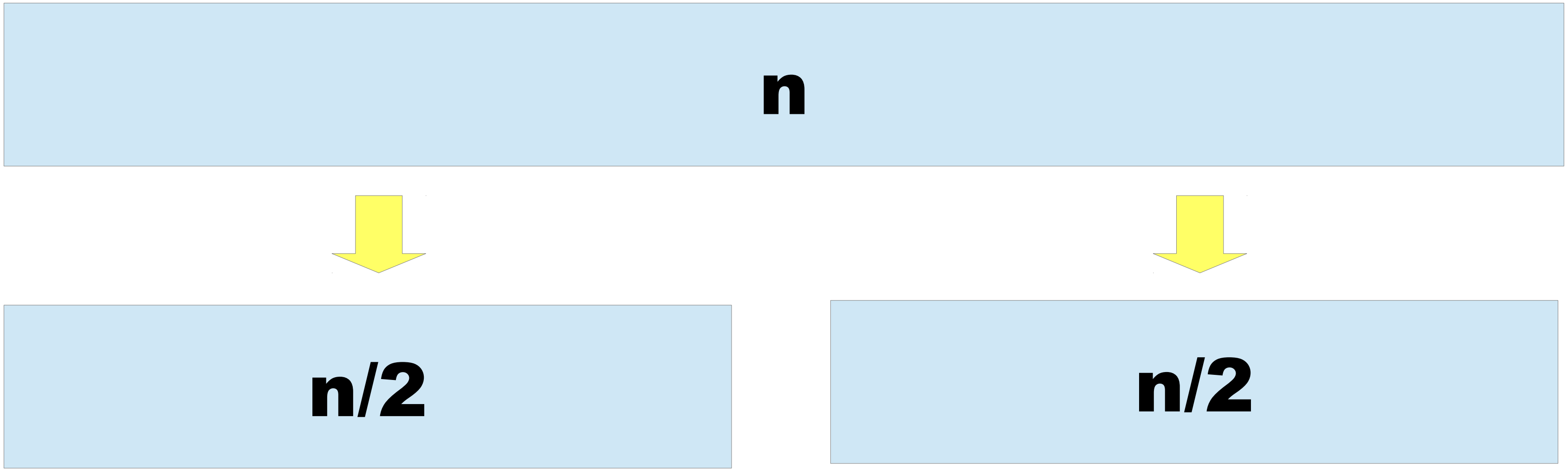
Mergesort

3. *Merge* the two sorted lists together



Complexity of mergesort

An array of size n gets split into two arrays of size n/2:



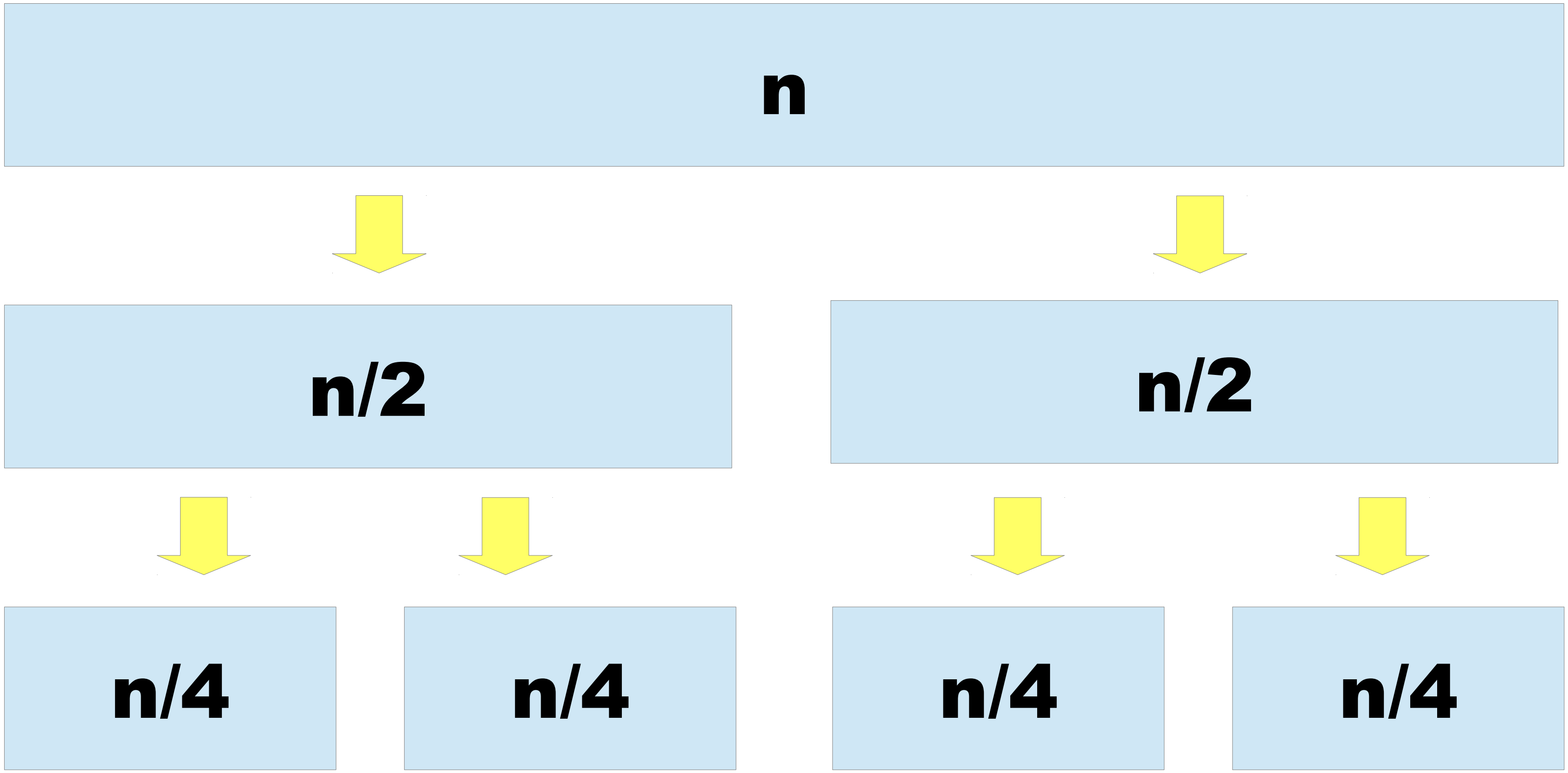
n

n/2

n/2

Complexity of mergesort

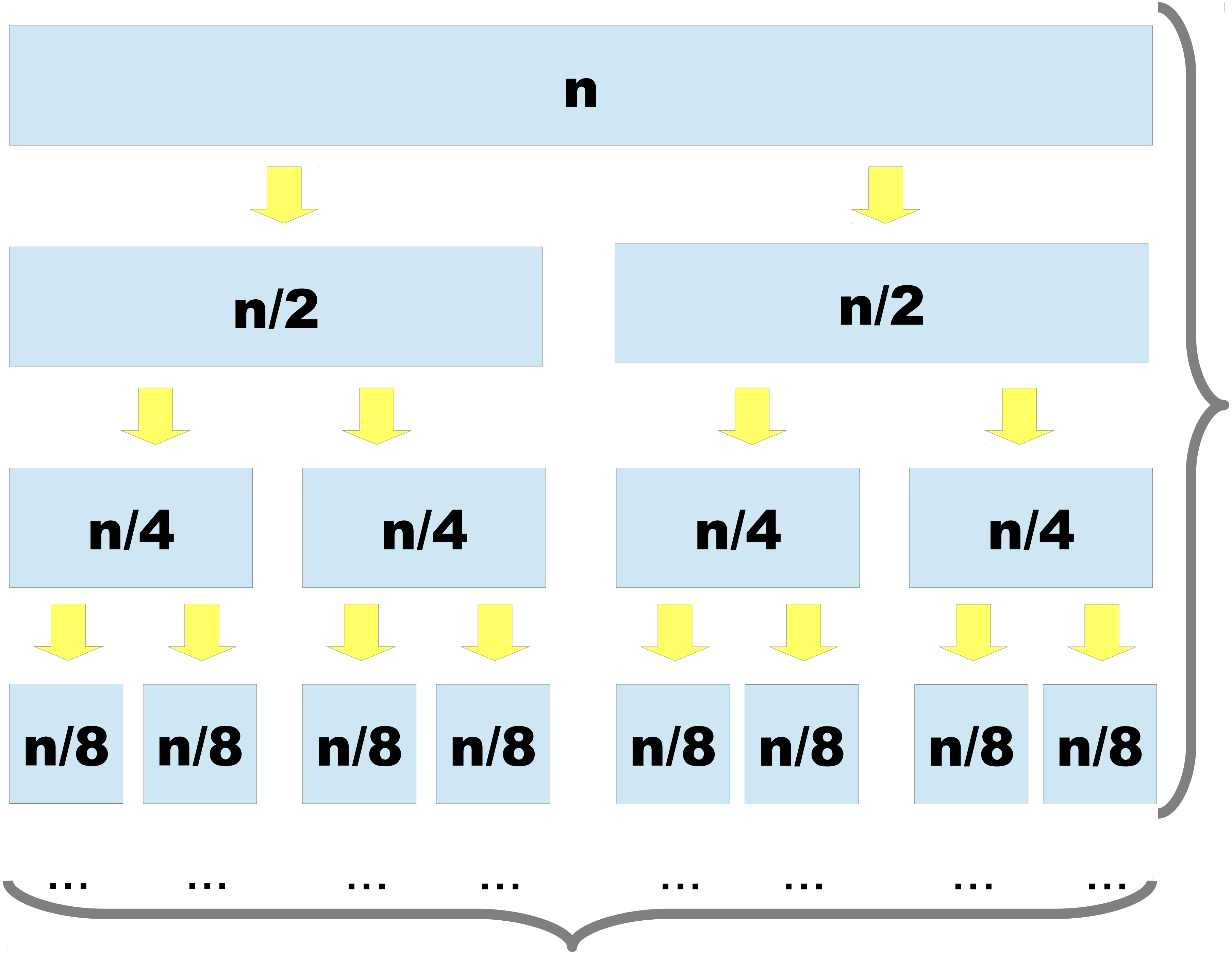
The recursive calls will split these arrays into four arrays of size n/4:



n

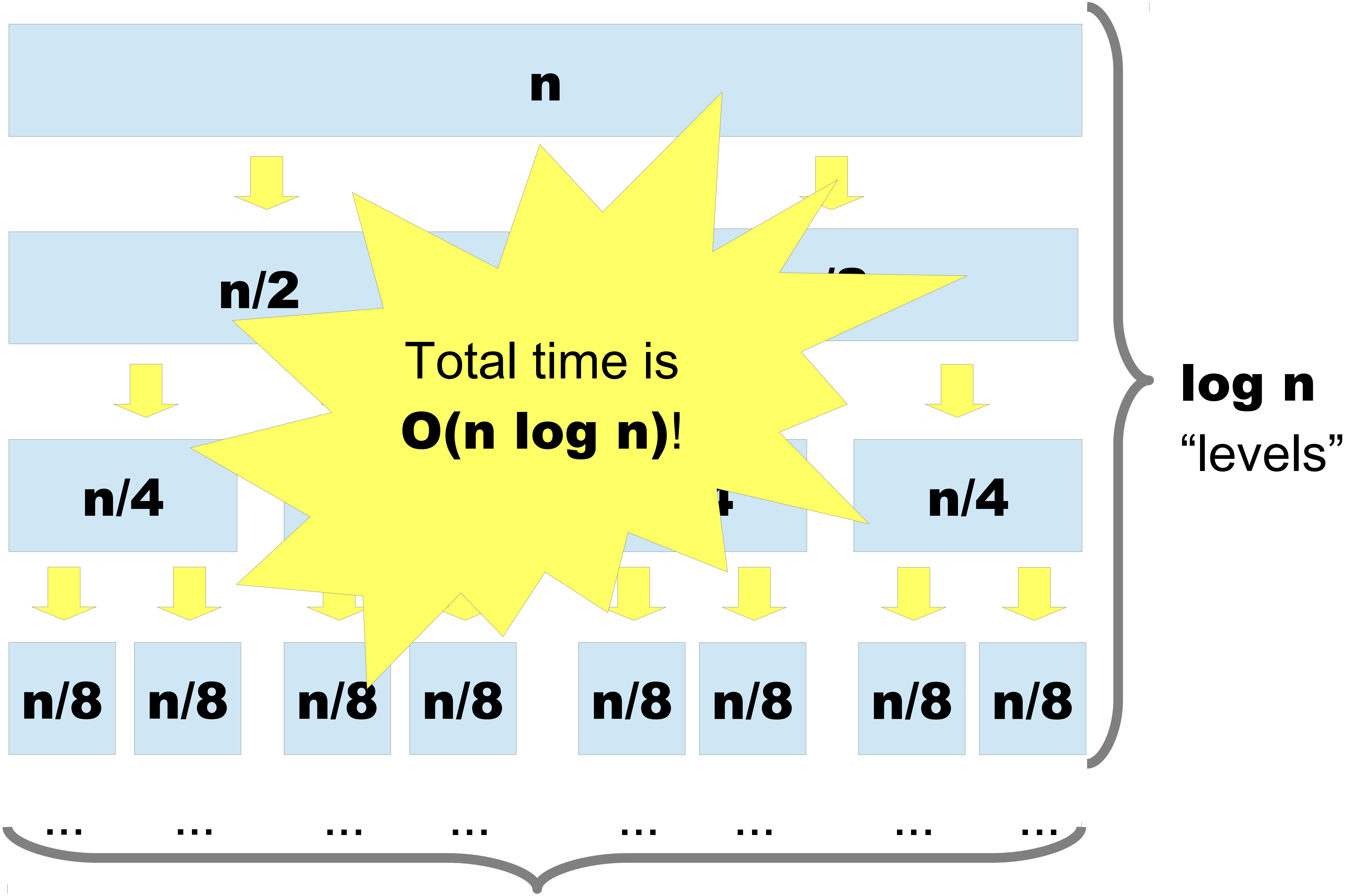
n/2 n/2

n/4 n/4 n/4 n/4



log n
“levels”

O(n) time per level



n

n/2

n/4 **n/4**

n/8 **n/8** **n/8** **n/8** **n/8** **n/8** **n/8** **n/8**

… … … … … … … …

Total time is **O(n log n)**!

log n “levels”

O(n) time per level

Merge sort is not in-place

- The merge operation is tricky to do in-place (i.e. without using a second array). There is no obvious way to do it, although several attempts have been suggested.
- Therefore merge sort is not in-place. This is a drawback of the algorithm.

Complexity analysis

Mergesort's complexity is O(n log n)

- Recursion goes log n "levels" deep
- At each level there is a total of O(n) work

General "divide and conquer" theorem:

- If an algorithm does O(n) work to split the input into two pieces of size n/2 (or k pieces of size n/k)...
- ...then recursively processes those pieces...
- ...then does O(n) work to recombine the results...
- ...then the complexity is O(n log n)

Sorting so far

There are a *huge* number of sorting algorithms

- No single best one, each has advantages (hopefully) and disadvantages

Insertion sort:

- O(n^2) so not good overall
- Good on small arrays though – low *constant factors*

Merge sort:

- O(n log n), hooray!
- But not in-place and high constant factors