Homework 3

Question 1

Let $\to$ be a relation on a set E. We have defined the reflexive transitive closure $\to^*$ of $\to$ by the rules

$$\frac{}{a \to^* a} \qquad \frac{a \to b}{a \to^* b} \qquad \frac{a \to^* b \qquad b \to^* c}{a \to^* c}$$

We define another relation $R\ a\ b$ by the rules

$$\frac{}{R\ a\ a} \qquad \frac{a \to b}{R\ a\ b} \qquad \frac{a \to b \qquad R\ b\ c}{R\ a\ c}$$

Show that $a \to^* b$ if, and only if, $R\ a\ b$.

Question 2

Show that if we have

$$(a \to b \wedge a \to^* c) \Rightarrow \exists d\ (b \to^* d \wedge c \to d)$$

then $\to$ is confluent i.e.

$$(a \to^* b \wedge a \to^* c) \Rightarrow \exists d\ (b \to^* d \wedge c \to^* d)$$

Question 3

We define

$$e\ ::=\ v \mid \mathsf{add}\ e\ e \qquad v\ ::=\ \mathsf{zero} \mid \mathsf{succ}\ e$$

and

$$\frac{}{\mathsf{add}\ e\ \mathsf{zero} \to e} \qquad \frac{}{\mathsf{add}\ e_0\ (\mathsf{succ}\ e_1) \to \mathsf{succ}\ (\mathsf{add}\ e_0\ e_1)} \qquad \frac{e \to e'}{\mathsf{add}\ e_0\ e \to \mathsf{add}\ e_0\ e'}$$

and

$$\frac{}{v \Downarrow v} \qquad \frac{e \Downarrow \mathsf{succ}\ e'}{\mathsf{add}\ e_0\ e \Downarrow \mathsf{succ}\ (\mathsf{add}\ e_0\ e')} \qquad \frac{e \Downarrow \mathsf{zero} \qquad e_0 \Downarrow v}{\mathsf{add}\ e_0\ e \Downarrow v}$$

Show that $e \to^* v$ if, and only if, $e \Downarrow v$ (cf. exercise 3.5.17)