Exercise set 4. Decision problems and estimation

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Time and scoring The numbers indicate expected time to complete the exercise. If you take more time than indicated, please note that. A "!" indicates that it may require some extra thought. A "?" indicates that this is an open question. The exercises count towards your grade.

1 An insurance problem (90-120min)

Consider the insurance example of chapter 3. Therein, an insurer is covering customers by asking for a premium $d > 0$. In the event of an accident, which happens with probability $\epsilon \in [0, 1]$, the insurer pays out $h > 0$. The problem of the insurer is, given ϵ, h, what to set the premium d to so as to maximise its expected utility. We assume that the insurer's utility is linear.

We now consider customers with some baseline income level $x \in \mathcal{S}$. For simplicity, we assume that the only possible income levels are $\mathcal{S} = \{15, 20, 25, \ldots, 60\}$. let $V : \mathbb{R} \to \mathbb{R}$ denote the utility function of a customer. Customers who are *interested* the insurance product, will buy it if and only if:

$$V(x - d) > \epsilon V(x - h) + (1 - \epsilon)V(x). \tag{1.1}$$

We make the simplifying assumption that the utility function is the same for all customers, and that it has the following form:

$$V(x) = \begin{cases} \ln x, & x \geq 1 \\ 1 - (x - 2)^2, & \text{otherwise.} \end{cases} \tag{1.2}$$

Customers who are *not* interested the insurance product, will not buy it no matter what the price.

There is some unknown probability distribution $P_\omega(x)$ over the income level, such that the probability of n people having incomes $X = \{x_1, \ldots, x_n\}$ is $P^n_\omega(X) = \prod_{i=1}^n P_\omega(x_i)$. We have two data sources for this. The first is a model of the general population ω_1 not working in high-tec industry, and the second is a model of employees in high-tec industry, ω_2. The models are summarised in the table below.

Income Levels	15	20	25	30	35	40	45	50	55	60
Models	Probability (%) of income level $P_\omega(x)$									
ω_1	5	10	12	13	11	10	8	10	11	10
ω_2	8	4	1	6	11	14	16	15	13	12

Table 1: Income level distribution for the two models

Our goal is to find a premium d that maximises our expected utility. We assume that our firm is liquid enough that our utility is linear. In this exercise, we consider 4 different cases. For simplicity, you can let $\mathcal{D} = \mathcal{S}$ throughout this exercise.

Exercise 1.1 (50)**.** *Show that the expected utility for a given ω is the expected gain from a buying customer, times the probability that an interested customer will have an income x such that they would buy our insurance.*

$$U(\omega, d) = (d - \epsilon h) \sum_{x \in \mathcal{S}} P_\omega(x) \, \mathbb{I} \left\{ V(x - d) > \epsilon V(x - h) + (1 - \epsilon) V(x) \right\}. \tag{1.3}$$

Let $h = 150$ and $\epsilon = 10^{-3}$. Plot the expected utility for varying d, for the two possible ω. What is the optimal price level if the incomes of all interested customers are distributed according to ω_1? What is the optimal price level if they are distributed according to ω_2?

Exercise 1.2 (20)**.** *According to our intuition, customers interested in our product are much more likely to come from the high-tec industry than from the general population. For that reason, we have a prior probability $\xi(\omega_1) = 1/4$ and $\xi(\omega_2) = 3/4$. Plot the expected utility under this prior as the premium d varies. What is the optimal expected utility and premium?*

Exercise 1.3 (20)**.** *Instead of fully relying on our prior, the company decides to perform a random survey of 1000 people. We asked whether they would be interested in the insurance product (as long as the price is low enough). If they were interested, we asked them what their income is. Only 126 people were interested, with income levels given in Table 2. Each row column of the table shows the stated income and the number of people reporting it. Let $X = \{x_1, x_2, \ldots\}$ be*

Income	15	20	25	30	35	40	45	50	55	60
Number	7	8	7	10	15	16	13	19	17	14

Table 2: Survey results

the set of data we have collected. Assuming that that the responses *are truthful, calculate the posterior probability* $\xi(\omega \mid X)$*, assuming that the only possible models of income distribution are the two models* ω_1, ω_2 *used in the previous exercises. PLot the expected utility under the posterior distribution as d varies. What is the maximum expected utility we can obtain?*

Exercise 1.4 (30 – Bonus exercise)**.** *Having only two possible models is somewhat limiting, especially since neither of them might correspond to the income distribution of people interested in our insurance product. How could this problem be rectified? Describe the idea and implement it. When would you expect this to work better?*

2 Feedback

Finally, some questions about this unit:

1. Did you find the material interesting?
2. Did you find it potentially useful?
3. How much did you already know?
4. How much had you already seen but did not remember in detail?
5. How much have you seen for the first time?
6. Which aspect did you like the most?
7. Which aspect did you like the least?
8. Did the exercises help you to understand the material?
9. Feel free to add any further comments.