# Written examination
# TIN172/DIT410, Artificial Intelligence

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(I will visit some time around 15:30 to answer questions)

19 August 2014, 14:00–18:00

This examination consists of eight basic questions (numbered 1–8) and three advanced (numbered A–C). There are no points awarded for the questions, but you can either give a correct answer, or fail.

**Grading**

The number of questions (basic plus advanced) that you need to answer correctly in order to get a certain grade is shown in the following table:

| Basic questions | Advanced questions | Final grade |
|---|---|---|
| $\geq 5$ | — | 3/G |
| $\geq 6$ | $\geq 1$ | 4/VG |
| $\geq 7$ | $\geq 2$ | 5 |

**Accessories**

- Paper and pencil.
- Crayons, paper glue, scissors.
- One A4 cheat sheet with any information you want on it.
- No books or calculators.

**Notes**

- Answer directly on the question page,
  but you can also use empty papers if you run out of space.
- ***Write readable, and explain your answers!***

# 1 Complexity dimensions of poker

Assume that you want to write a program that can play the card game of poker against other players. Classify this program according to the complexity dimensions below. (Note that you could select more than one value). Also explain why you chose that/those value(s).

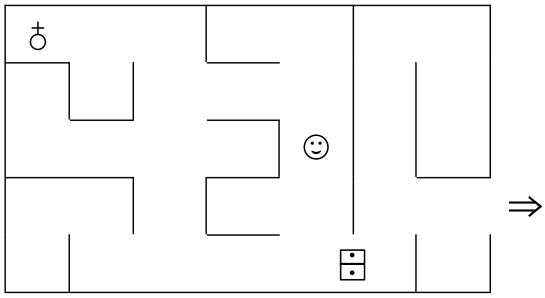
| Dimension | Possible values | Value(s) | Explanation |
|---|---|---|---|
| Modularity | (F)lat<br>(M)odular<br>(H)ierarchical | | Anything goes (with a good explanation). |
| Representation | (S)tates<br>(F)eatures<br>(I)ndividuals +relations | | Anything goes (with a good explanation). |
| Planning horizon | (S)tatic<br>(F)inite<br>(I)ndefinite<br>($\infty$)Infinite | | It's not static, but the other ones are fine for some implementations |
| Observability | (F)ull<br>(P)artial | P | The agent cannot see the other players' cards, or the deck. |
| Dynamics | (D)eterministic<br>(S)tochastic | | Depends on how you view dynamicity: it's deterministic in the sense that you know the results of your own actions, but it's stochastic in the way the other agents react. |
| Number of agents | (S)ingle<br>(M)ultiple | M | Since the computer plays against several opponents, it has to be able to reason about them. |

**Answer:** Most answers are fine, if you have a good explanation. For specific comments, see the table.

## 2 Escaping a maze

You (☺) are trapped in a maze and needs to escape from it. Unfortunately there is a locked door (⌸) between you and the exit (⇒), and you have to take the key (♁) before you can unlock the door.

You are not facing any special direction and can move one step in any direction at any time step, as long as there is no wall in the way. Alternatively you can pick up the key (if you are in the right location), or you can unlock the door (if you have the key and are in the right location). Neither the key nor the door can move around on their own. Your goal is to find a plan for escaping the maze using as few moves as possible. Assume that the grid has size $M \times N$.



**Give a suitable representation of the states in this searching problem.**

**Answer:** $\langle m, n, k, d \rangle$ or $\langle m, n, k \rangle$, where

- $1 \leq m \leq M$, $1 \leq n \leq N$, representing the position of the agent,
- $k \in \{\mathbf{true}, \mathbf{false}\}$ tells whether the key has been collected, and
- $d \in \{\mathbf{true}, \mathbf{false}\}$ tells whether the door is unlocked.

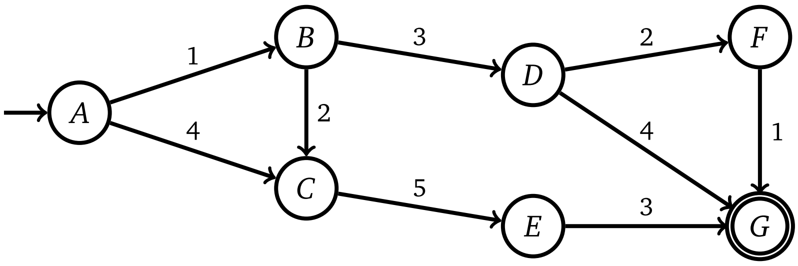
Note that $d$ is not strictly necessary, since you can never drop the key or lock the door2 anyway.

**What is the size of the state space?**

**Answer:**
$M \times N \times 2 \times 2$, alternatively $M \times N \times 2$.
One can also reason that it's impossible that the door is unlocked but the agent is not carrying the key, which means that one of the $2 \times 2$ boolean alternatives disappear, so we get $M \times N \times 3$.
Finally one can argue that to the right of the door, the agent has to carry the key and the door must be unlocked, so we can shrink the state space even further.

## 3 Cost-based search

The following is a representation of a search problem, where $A$ is the start node and $G$ is the goal. There is also a heuristics $h$ which is defined in the table. Note that $h(B)$ is unknown.




| $n$ | $h(n)$ |
|---|---|
| $A$ | 5 |
| $B$ | ? |
| $C$ | 4 |
| $D$ | 3 |
| $E$ | 3 |
| $F$ | 1 |
| $G$ | 0 |

**What values of $h(B)$ make $h$ admissible?**

**Answer:** $0 \leq h(B) \leq 6$, since the minimal cost from $B$ to $G$ is 6

**For each of the following search strategies, mark which of the listed paths it could return.**

Assume that the heuristics $h$ is admissible. Note that for some search strategies, the returned path might depend on tie-breaking. In these cases, make sure to mark *all* paths that could be returned.

| **Search algorithm** | $A-C-E-G$ | $A-B-C-E-G$ | $A-B-D-G$ | $A-B-D-F-G$ |
|---|---|---|---|---|
| Depth-first | X | X | X | X |
| Breadth-first | X | | X | |
| $A*$ with heuristic $h$ | | | | X |

**Answer:** See the table

*Write your answers directly on the exam, but you can also use extra papers if you run out of space.*
***Write readable, and don't forget to explain your answers!***

## 4 Cryptoarithmetic puzzle

A cryptarithmetic puzzle is a mathematical game where the digits of some numbers are represented by letters (or symbols). Each letter represents a unique digit. The goal is to find the digits such that a given mathematical equation is verified.

**Formulate the following puzzle as a CSP:**

$$\begin{array}{ccccc} & & O & D & D \\ + & & O & D & D \\ \hline & E & V & E & N \end{array}$$

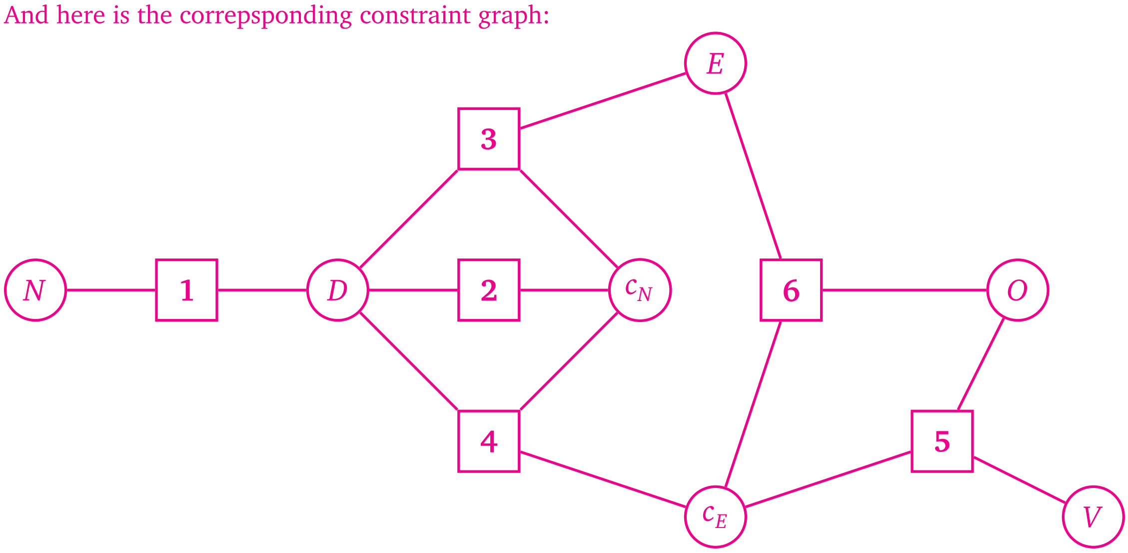
**Answer:**

One possibility is to introduce two additional carry variables, $c_N, c_E$:

$$\begin{aligned}
(2D)\,\%\,10 &= N && [1] \\
(2D) \div 10 &= c_N && [2] \\
(2D + c_N)\,\%\,10 &= E && [3] \\
(2D + c_N) \div 10 &= c_E && [4] \\
(2O + c_E)\,\%\,10 &= V && [5] \\
(2O + c_E) \div 10 &= E && [6] \\
D, N, V &\in \{0, \ldots, 9\} \\
O, E &\in \{1, \ldots, 9\} \\
c_N, c_E &\in \{0, 1\}
\end{aligned}$$

(Where $\div$ and % correspond to integer division and remainder).

And here is the correpsponding constraint graph:




Note that it can be solved without extra carry variables, but then the constraints become much larger.

# 5 Knowledge base

| | | | | |
|---|---|---|---|---|
| $a \leftarrow b \wedge c.$ | $a \leftarrow g.$ | $a \leftarrow c \wedge k.$ | $b \leftarrow g \wedge j.$ | $b \leftarrow k.$ |
| $b \leftarrow j \wedge a.$ | $c \leftarrow f.$ | $d \leftarrow m \wedge j.$ | $d \leftarrow p \wedge a.$ | $f \leftarrow m.$ |
| $g \leftarrow f \wedge p.$ | $k \leftarrow j.$ | $k \leftarrow b \wedge p.$ | $m.$ | $p.$ |

**Give a *nontrivial* model of the knowledge base above.**

(A *trivial* interpretation or model is where all atoms are true or all atoms are false).

**Answer:**
There are only two nontrivial models:

- the minimal model: $M = \{a, c, d, f, g, m, p\}$
- the minimal model plus $b$ and $k$: $M \cup \{b, k\}$

**Give a *nontrivial* interpretation that is not a model of the knowledge base.**

**Answer:**
Any interpretation that assigns at least one of the following atoms to **false**: $a, c, d, f, g, m, p$.
Alternatively, any interpretation that assigns $k$ to **true** but $b$ and $j$ to **false**.
(And a lot of other interpretations)

**Give all atoms that are logical consequences of the knowledge base.**

**Answer:** $a, c, d, f, g, m, p$

**Give all atoms that are *not* logical consequences of the knowledge base.**

**Answer:** $b, j, k$

# 6 Rolling the dice

Assume that you roll two fair 6-sided dice.

**What is the probability that one of them turns up six (⚅), given that their sum is 8?**

**Answer:**
Let $S$ = "one of the dice turns up six" = $\{16, \ldots, 56, 66, 65, \ldots, 61\}$,
and $T$ = "their sum is 8" = $\{26, 35, 44, 53, 62\}$.

$S \cap T = \{26, 62\}$, so $|S \cap T| = 2$ and $|T| = 5$.

$P(S|T) = P(S, T)/P(T) = |S \cap T|/|T| = 2/5$

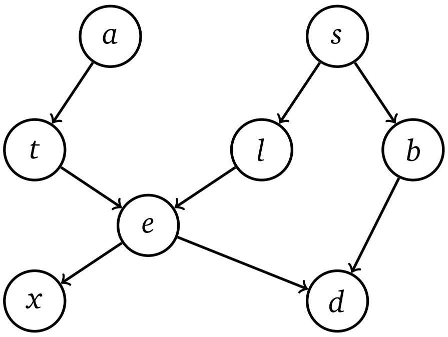
**What is the probability that one of them turns up six (⚅), given that they show different values?**

**Answer:**
Let $S$ = "one of the dice turns up six" = $\{16, \ldots, 56, 66, 65, \ldots, 61\}$,
and $T$ = "they show different numbers" = $\Omega - \{11, \ldots, 66\}$.

$S \cap T = S - \{66\}$, so $|S \cap T| = |S| - 1 = 10$ and $|T| = 6^2 - 6 = 30$.

$P(S|T) = P(S, T)/P(T) = |S \cap T|/|T| = 1/3$

# 7 Belief network




**Mark which of the following independence assumptions that are true in the belief network above.**

| **Assumption** | **true / false** |
|---|---|
| $a$ and $t$ are independent | false |
| $a$ and $s$ are independent | true |
| $l$ and $b$ are conditionally independent given $s$ | true |
| $s$ and $d$ are conditionally independent given $e$ | false |
| $s$ and $x$ are conditionally independent given $e$ | true |

**Answer:** See the table

# 8 A medical conundrum

(*Note:* the figures below are not really accurate, as they are liberally adapted from different studies.)

Many patients arriving at an emergency room, suffer from chest pain. This may indicate acute coronary syndrome (ACS). Patients suffering from ACS that go untreated may die with probability 2% in the next few days. Successful diagnosis results lowers the short-term mortality rate to 0.2%. Consequently, a prompt diagnosis is essential.

Approximately 50% of patients presenting with chest pain turn out to suffer from ACS (either acute myocardial infraction or unstable angina pectoris). Approximately 10% suffer from lung cancer.

Of ACS sufferers in general, 2⁄3 are smokers and 1⁄3 non-smokers. Only 1⁄4 of non-ACS sufferers are smokers. In addition, 90% of lung cancer patients are smokers. Only 1⁄4 of non-cancer patients are smokers.

**Assumption 1** A patient may suffer from none, either or both conditions!

**Assumption 2** When the smoking history of the patient is known, the development of cancer or ACS are independent.

One can perform an ECG to test for ACS. An ECG test has *sensitivity* of 66.67% (i.e., it correctly detects 2⁄3 of all patients that suffer from ACS), and a *specificity* of 75% (i.e., 1⁄4 of patients that do not have ACS, still test positive).

An X-ray can diagnose lung cancer with a sensitivity of 90% and a specificity of 90%.

**Assumption 3** Repeated applications of a test produce the same result for the same patient, i.e. that randomness is only due to patient variability.

**Assumption 4** The existence of lung cancer does not affect the probability that the ECG will be positive. Conversely, the existence of ACS does not affect the probability that the X-ray will be positive.

*For the problem questions, please see next page.*

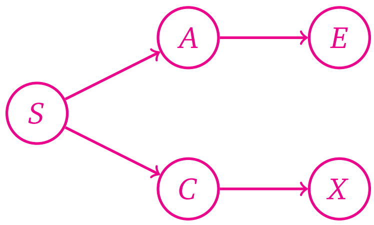
## A medical conundrum, the questions

What does the description on the previous page imply about the dependencies between the patient condition, smoking and test results? Assume the following events (i.e., variables that can be either true or false):

$A$: ACS  $C$: Lung cancer  $S$: Smoking  $E$: Positive ECG result  $X$: Positive X-ray result

**Draw a belief network for the problem.**

**Answer:**
According to our information, only the presence of ACS affects the results of ECG, and only the presence of lung cancer affects the results of the X-ray. Consequently $P(E|A,C) = P(E|A)$ and $P(X|A,C) = P(X|C)$. At the same time, smoking is linked with both ACS and cancer.




**What is the probability that the patient suffers from ACS if the patient is a smoker?**

**Answer:**
We need to find out $P(A|S)$, and Bayes theorem says that $P(A|S) = P(S|A)P(A)/P(S)$. From the description we know that $P(S|A) = 2/3$ ("of ACS sufferers in general, 2⁄3 are smokers") and $P(A) = 1/2$ ("approximately 50% of patients presenting with chest pain turn out to suffer from ACS"). $P(S)$ can be calculated like this:

$$P(S) = P(S|\neg A)P(\neg A) + P(S|A)P(A) = 1/4 \times 1/2 + 2/3 \times 1/2 = \frac{11}{24}$$

From the text we know that $P(S|\neg A) = 1/4$ ("only 1⁄4 of non-ACS sufferers are smokers"), and we also know that $P(\neg A) = 1 - P(A) = 1/2$.

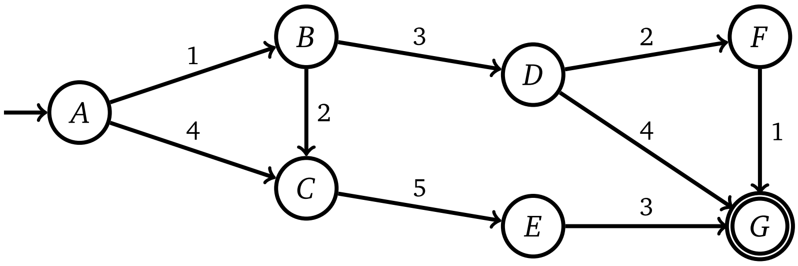
Then we plug this in Bayes formula to obtain:

$$P(A|S) = \frac{P(S|A)P(A)}{P(S)} = \frac{2/3 \times 1/2}{11/24} = \frac{24}{33} = \frac{8}{11} \approx 72.7\%$$

So smoking is a strong indicator for heart attack.

# A [Advanced] Cost-based search, continued

*The search graph and heuristics from question 3 is repeated here.*




| $n$ | $h(n)$ |
|---|---|
| $A$ | 5 |
| $B$ | ? |
| $C$ | 4 |
| $D$ | 3 |
| $E$ | 3 |
| $F$ | 1 |
| $G$ | 0 |

Recall that a heuristics is *monotone*, or *consistent*, if the difference in the heuristic values for any two nodes is always less than or equal to the actual cost of the lowest-cost path between the nodes. Or in mathematical notation: for any two nodes $n$ and $n'$, $|h(n) - h(n')| \leq \textit{lowest-cost}(n, n')$.

**What values of $h(B)$ make $h$ monotone/consistent?**

**Answer:** It's enough to check these constraints:

$$\begin{aligned} h(A) &\leq cost(AB) + h(B) \\ h(B) &\leq cost(BC) + h(C) \\ h(B) &\leq cost(BD) + h(D) \end{aligned}$$

This gives us $4 \leq h(B) \leq 6$. (Note that this inequality is *inclusive*.)

**What values of $h(B)$ will cause $A*$ search to expand node $A$, then node $C$, then node $B$ in order?**

**Answer:**

$$\begin{aligned} f(AB) &= cost(AB) + h(B) = 1 + x \\ f(AC) &= cost(AC) + h(C) = 4 + 4 = 8 \\ f(ACE) &= cost(ACE) + h(E) = 9 + 3 = 12 \end{aligned}$$

$AC$ should be expanded before $AB$, which should be expanded before $ACE$. I.e., $f(AC) < f(AB) < f(ACE)$, i.e., $8 < 1 + x < 12$. We don't have to reason about the $D$ node, since it cannot be expanded before $B$ anyway.

So, $7 < h(B) < 11$. (Note that this inequality is *strict*.)

**What values of $h(B)$ will cause $A*$ to return a sub-optimal path?**

**Answer:** We need to make it expand both $C$ and $E$ before $B$:

$$f(ACEG) < f(AB) = 1 + h(B)$$

Since $f(ACEG) = 12$, we get $h(B) > 11$. (Note that this inequality is *strict*.)

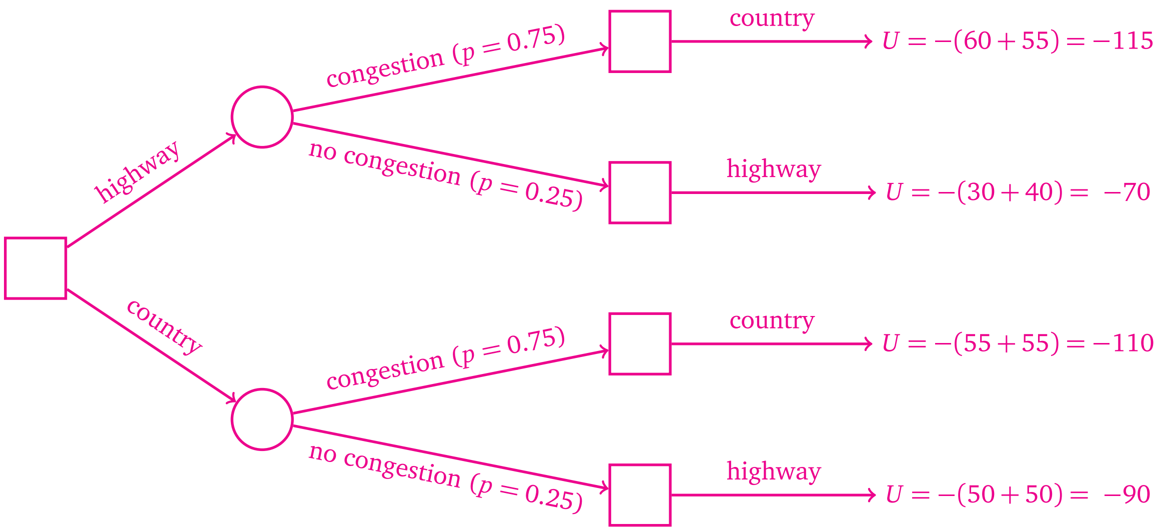
# B [Advanced] Optimal policy for driving

Imagine that you are driving a car from town $A$ to town $C$, passing through town $B$. For each part of the trip, you have the choice of either taking the highway, or the country road. The travel time depends on whether the roads are congested or not. They are shown in the table below. For simplicity, assume that whenever that the state of congestion is global, i.e. that the roads are congested everywhere or nowhere and that your utility is the negative travel time, i.e. that you want to minimise expected travel time. Initially, you know that the probability of congestion is $p_c$. However, as soon as you have completed the first part of the trip ($A \to B$) you will know whether or not there is congestion.

| Route | Highway | | Country | |
|---|---|---|---|---|
| | No congestion | Congestion | No congestion | Congestion |
| $A \to B$ | 30 mins | 60 mins | 50 mins | 55 mins |
| $B \to C$ | 40 mins | 60 mins | 45 mins | 55 mins |

**What is the complete optimal policy for the case when $p_c = 0.75$?**
**Make a diagram to illustrate your calculations.**

**Answer:**
The decision for the second part ($B \to C$) is not a problem, since you know by then if there is a congestion or not. So you don't have to calculate any policy for that part, only for the first ($A \to B$). Here is a decision tree for the problem:




Now, the expected utilities are like follows:

$$\begin{aligned}
\mathscr{E}(U \mid \text{highway}_{AB}) &= U(\text{highway}_{AB} \wedge \text{congestion}) \times P_c + U(\text{highway}_{AB} \wedge \neg\text{congestion}) \times (1 - P_c) \\
&= -115 \times 0.75 + -70 \times 0.25 \\
&= -103.75 \\
\mathscr{E}(U \mid \text{country}_{AB}) &= U(\text{country}_{AB} \wedge \text{congestion}) \times P_c + U(\text{country}_{AB} \wedge \neg\text{congestion}) \times (1 - P_c) \\
&= -110 \times 0.75 + -90 \times 0.25 \\
&= -105
\end{aligned}$$

So, since $-103.75 > -105$, the optimal policy is to take the highway for the first part ($A \to B$), and then select for the second part ($B \to C$) depending on if there is a congestion or not.

# C [Advanced] A medical conundrum, continued

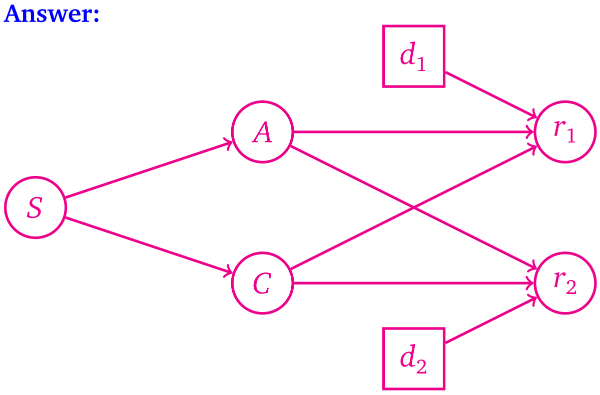
*This is a continuation of question 8*

Now consider the case where you have the choice between tests to perform. First, you observe $S$, whether or not the patient is a smoker. Then, you select a test to make: $d_1 \in \{\textit{X-ray, ECG}\}$. Finally, you decide whether or not to treat for ACS: $d_2 \in \{\textit{treat, not treat}\}$. An untreated ACS patient may die with probability 2%, while a treated one with probability 0.2%. Treating a non-ACS patient results in death with probability 0.1%.

**Draw a decision diagram, where**

- $S$ is an observed random boolean variable, taking values in $\{\textit{true, false}\}$.
- $A$ and $C$ are hidden boolean variables, taking values in $\{\textit{true, false}\}$.
- $d_1$ and $d_2$ are choice variables, taking values in $\{\textit{X-ray, ECG}\}$ and $\{\textit{treat, not treat}\}$, respectively.
- $r_1$ is a boolean result variable, corresponding to a positive (*true*) or a negative (*false*) test result.
- $r_2$ is a boolean result variable, corresponding to the patient living (*true*) or dying (*false*).

**Answer:**




**Let $d_1 = \textit{X-ray}$, and assume the patient suffers from ACS, i.e. $A = \textit{true}$. How is the posterior distributed?**

**Answer:**
This is sort of a trick question, since the result doesn't depend on the value of $A$ at all.

The only possible variable that $d_1$ can influence is $r_1$, so we calculate the posterior of $r_1$. But as stated in the premises, the result of an X-ray is independent from the patient having ACS or not. So, the posterior of $r_1$ is the same as the probability that an X-ray gives a positive result (without knowing anything). I.e.:

$$\begin{aligned} P(r_1 = \textbf{true}) &= P(C)P(\textit{X-ray} = \textbf{positive} \mid C) + P(\neg C)P(\textit{X-ray} = \textbf{negative} \mid \neg C) \\ &= 0.1 \times 0.9 + (1 - 0.1) \times (1 - 0.9) \\ &= 0.18 \end{aligned}$$