ON IRRELEVANCE AND ALGORITHMIC EQUALITY IN PREDICATIVE TYPE THEORY — ERRATA —

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1. Definition of algorithmic equality

p.13 Algorithmic type equality: The correct rule for function types is:

$$\frac{\Delta \vdash U \stackrel{\frown}{\Longleftrightarrow} U' \qquad \Delta.\, x{\star}U \vdash T \stackrel{\frown}{\Longleftrightarrow} T'}{\Delta \vdash (x{\star}U) \stackrel{s,s'}{\to} T \Longleftrightarrow (x{\star}U') \stackrel{s,s'}{\to} T'}$$

2. Soundness of algorithmic equality

p.26 **Theorem 28**: For the inductive proof, (2) needs to be strengthened to:

Let $\Delta \vdash n : T$ and $\Delta \vdash n' : T'$. If $\Delta \vdash n \longleftrightarrow n' : U$ then $\Delta \vdash n = n' : U$ and $\Delta \vdash U = T = T'$.

Case $n = n' = x$ and

$$\frac{(x\!:\!U) \in \Delta}{\Delta \vdash x \stackrel{\frown}{\longleftrightarrow} x : U}$$

We have $\Delta \vdash x = x : U$ and $\Delta \vdash U = T = T'$ by inversion on $\Delta \vdash x : T$ and $\Delta \vdash x : T'$.

Case $\Delta \vdash n\, u : T$ and $\Delta \vdash n'\, u' : T'$ and

$$\frac{\Delta \vdash n \longleftrightarrow n' : (x\!:\!U_1) \to U_2 \qquad \Delta \vdash u \stackrel{\frown}{\Longleftrightarrow} u' : U_1}{\Delta \vdash n\, u \stackrel{\frown}{\longleftrightarrow} n'\, u' : U_2[u/x]}$$

By inversion on typing, we have $\Delta \vdash n : (x : T_1) \to T_2$ and $\Delta \vdash u : T_1$ and $\Delta \vdash T_2[u/x] = T$, and likewise $\Delta \vdash n' : (x : T_1') \to T_2'$ and $\Delta \vdash u' : T_1'$ and $\Delta \vdash T_2'[u'/x] = T'$. By the first induction hypothesis, $\Delta \vdash n = n' : (x\!:\!U_1) \to U_2$ and $\Delta \vdash (x\!:\!U_1) \to U_2 = (x\!:\!T_1) \to T_2 = (x\!:\!T_1') \to T_2'$. By function type injectivity, $\Delta \vdash U_1 = T_1 = T_1'$ and $\Delta, x : U_1 \vdash U_2 = T_2 = T_2'$. Thus $\Delta \vdash u, u' : U_1$ by conversion. By the second induction hypothesis, $\Delta \vdash u = u' : U_1$. This implies first $\Delta \vdash n\, u = n'\, u' : U_2[u/x]$, and further $\Delta \vdash U_2[u/x] = T_2[u/x] = T = T_2'[u'/x] = T'$.

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Case $\Delta \vdash n \overset{\div}{} u : T$ and $\Delta \vdash n' \overset{\div}{} u' : T'$ and

$$\frac{\Delta \vdash n \longleftrightarrow n' : (x{\div}U_1) \to U_2}{\Delta \vdash n \overset{\div}{} u \mathrel{\widehat{\longleftrightarrow}} n' \overset{\div}{} u' : U_2[u/x]}$$

By inversion on typing, we have $\Delta \vdash n : (x{\div}T_1) \to T_2$ and $\Delta^{\div} \vdash u : T_1$ and $\Delta \vdash T_2[u/x] = T$, and likewise $\Delta \vdash n' : (x{\div}T_1') \to T_2'$ and $\Delta^{\div} \vdash u' : T_1'$ and $\Delta \vdash T_2'[u'/x] = T'$. By the first induction hypothesis, $\Delta \vdash n = n' : (x{\div}U_1) \to U_2$ and $\Delta \vdash (x{\div}U_1) \to U_2 = (x{\div}T_1) \to T_2 = (x{\div}T_1') \to T_2'$. By function type injectivity, $\Delta \vdash U_1 = T_1 = T_1'$ and $\Delta, x \div U_1 \vdash U_2 = T_2 = T_2'$. Thus $\Delta^{\div} \vdash u, u' : U_1$ by conversion. This implies $\Delta \vdash n \overset{\div}{} u = n' \overset{\div}{} u' : U_1[u/x]$, and further $\Delta \vdash U_2[u/x] = T_2[u/x] = T = T_2'[u'/x] = T'$.