**Flexible Dynamic Information Flow Control in the Presence of Exceptions**

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Flexible Dynamic Information Flow Control in the Presence of Exceptions

Sequential LIO

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Abstract

We describe a language-based, dynamic information flow control (IFC) system called LIO. Our system presents a new design point for IFC, influenced by the challenge of implementing IFC as a Haskell library, as opposed to the more typical approach of modifying the language runtime system. In particular, we take a coarse-grained, floating-label approach, previously used by IFC Operating Systems, and associate a single, mutable label—the current label—with all the data in a computation’s context. This label is always raised to reflect the reading of sensitive information and it is used to restrict the underlying computation’s effects. To preserve the flexibility of fine-grained systems, LIO also provides programmers with a means for associating an explicit label with a piece of data. Interestingly, these labeled values can be used to encapsulate the results of sensitive computations which would otherwise lead to the creeping of the current label. Unlike other language-based systems, LIO also bounds the current label with a current clearance, providing a form of discretionary access control that LIO programs can use to deal with covert channels. Moreover, LIO provides programmers with mutable references and exceptions, the latter of which can be used to encode and recover from monitor failures, all while preserving data confidentiality and integrity.

1 Introduction

Information flow control (IFC) tracks the flow of data through a system and prohibits code from operating on data in violation of a security policy. Significant research, development, and experimental effort has been devoted to static information flow mechanisms. Static analysis has a number of benefits, including reduced runtime overhead, fewer runtime failures, and robustness against implicit flows (Denning & Denning, 1977). However, static analysis does not work well in environments where new classes of users and new kinds of data are encountered at runtime. For such systems, dynamic enforcement techniques are more appropriate; dynamic IFC systems address many of the shortcomings of static IFC systems while retaining permissiveness.

Dynamic IFC systems fall into roughly two categories: fine-grained and coarse-grained enforcement. Fine-grained approaches, typically employed by language-based systems, e.g., (Austin & Flanagan, 2009; Austin & Flanagan, 2010; Hedin & Sabelfeld, 2012; Hrițcu et al., 2013), explicitly associate security policies—or labels—with every value.
Such systems have the benefit of giving programmers the ability to associate a particular security policy with a particular value. Unfortunately, this also places the burden of understanding and specifying labels on values that are not relevant for certain tasks. Moreover, fine-grained IFC systems are typically implemented as new (or changes to) languages or runtimes, imposing a large start-up cost on programmers.

Coarse-grained approaches, typically employed by IFC Operating Systems (VanDeBogart et al., 2007; Zeldovich et al., 2006; Krohn et al., 2007), associate a single label with every value in the context of a computation, usually a process. The advantage of such systems is simplicity: programmers do not need to clutter code with labels and can easily understand the security policy of any value—it is simply the label of the context. However, this is also a downside; programmers cannot associate a particular, and heterogeneous, security policy with a particular value. Moreover, incorporating sensitive data into a context usually amounts to “tainting” the whole context, which can lead to the label creep problem. Label creep occurs when the context label is tainted to a point where the computation cannot perform any useful side-effects.

In this work, we present LIO, a language-based dynamic IFC system, implemented as a Haskell library, that borrows ideas from both fine- and coarse-grained IFC systems. Like coarse-grained systems, LIO associates a label—the current label—with the current context. In particular, we define a monad, LIO, that restricts computations to a safe, IFC sublanguage (of Haskell). This monad keeps track of the current label, which is, in turn, used to permit restricted access to IO functionality by executing actions in the underlying IO monad. Like many Operating Systems (OSes), LIO is a floating-label system; the current label is raised to allow reading of sensitive data, thereby “floating above” the labels of all data observed by the current computation. Of course, raising a computation’s label comes at the cost of restricting where the computation may subsequently write.

Like fine-grained systems, LIO allows code to associate explicit labels with particular values, thus allowing applications to handle differently-labeled data in the same context. Specifically, LIO provides a Labeled type and a value constructor label that wraps explicit labels around values. Typically, labels are created at run time and incorporate dynamic information such as usernames or email addresses. The label on a Labeled value is public and can always be inspected. The wrapped value, on the other hand, can be inspected only using unlabel, a monadic function that appropriately raises the current label before returning the value.

Explicit unlabeling trivially addresses the problem of implicit flows: code cannot branch on a labeled boolean value without first calling unlabel on the value. However, label creep could still occur if code keeps unlabeling differently labeled data. To address this problem, we introduce a function called toLabeled. This primitive executes a computation (that may raise the current label) and restores the current label upon its termination. Importantly, however, the result of the computation is encapsulated as a Labeled value—only when the (outer) computation wishes to inspect the result will the current label be raised. These language features make LIO’s dynamic IFC approach more permissive than previous static

1 Using SafeHaskell (Terei et al., 2012) we ensure that untrusted code executes in this sublanguage.
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approaches (Pottier & Simonet, 2002; Li & Zdancewic, 2010; Russo et al., 2008), while still providing similar security guarantees (Sabelfeld & Russo, 2009).

Unlike other language-based work, LIO also bounds the current label with a current clearance. The clearance of a region of code may be set to impose an upper bound on the floating current label within the region. Hence, clearance can be understood as a discretionary access control mechanism restricting which data a subcomputation can access. By limiting sensitive data access to a “need to know” basis, clearance can reduce opportunities for code to leak data through covert channels—after all, code that cannot access sensitive data cannot leak it.

LIO Furthermore addresses two limitations common to most dynamic fine-grained systems: the lack of exception handling facilities and inability to recover from IFC monitor failures. (Breeze (Hriţcu et al., 2013) and an early, unpublished, version of LIO (Stefan et al., 2012b) are the two notable exceptions, further discussed in Section 7.) Our “mostly coarse-grained” dynamic IFC approach makes it easy to reason about leaks due to exceptional control flow. In particular, we need only reason about exception propagation across toLabeled blocks since the current label is only restored, or “lowered,” at these points; by treating computations executed by toLabeled as being of a separate context, the solution becomes clear: exceptions should not propagate beyond outside the toLabeled block. Equipped with exception-handling facilities, LIO encodes all IFC violations as catchable exceptions. This has the important consequence of allowing untrusted code to recover from IFC violations that would otherwise be considered fatal and leave the program in a stuck state.

This paper extends an earlier conference version (Stefan et al., 2011b) with these exception-handling facilities, an implementation of a real-world conference review web application called λChair, and formal proofs mechanized in the Coq theorem prover. The main contributions of this work are as follows.

• We propose a design point for IFC systems in which most values in lexical scope are protected by a single, mutable, current label. In our system, programmers can associate labels with values they care about by encapsulating them with the label constructor. Such values are similar to labeled values in fine-grained systems, but different in a crucial way: our encapsulation is explicitly reflected by types in a way that prevents implicit flows. In a similar way, our calculus (and Haskell implementation) provides labeled mutable references.

• Unlike other language-based work, our IFC model provides a notion of clearance which is used to provide a form of discretionary access control on code, i.e., restricting code to access data it “needs to know.”

• We present a simple dynamic, yet safe, exception-handling mechanism and encode IFC monitor failures using exceptions. Exceptions are crucial to making LIO a practical IFC system; real-world applications cannot “stop the world” on an IFC violation attempt.

• We prove information flow, access control, and isolation security properties of our design. A large part of our formalization is encoded in Coq. We remark that while our formal description of LIO is Haskell-centric, this is not a fundamental restriction—our formalism can be generalized to other programming languages.
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- We describe a Haskell implementation of the IFC calculus in Haskell. LIO can be implemented entirely as a library, demonstrating both the applicability and simplicity of the approach. This has the added benefit of not imposing the burden of learning a new programming language on developers—they simply need to understand a new API. Moreover, developers can use many existing compilers, tools and libraries. Our library, applications built on top of it (including λChair), and Coq proofs are available at http://labeled.io.

This paper is organized as follows. Section 2 describes the core information flow control LIO calculus. In Sections 3–5, we extend the core with clearance, mutable references, and exception-handling facilities. The security guarantees of the full calculus are given in Section 6. Related work is described in Section 7. We conclude in Section 8.

2 Core dynamic information flow control LIO calculus

IFC systems track and restrict the propagation of information according to a security policy. The core policy enforced by LIO, and most other IFC systems, is non-interference. Non-interference guarantees confidentiality (Goguen & Meseguer, 1982), by preventing sensitive information from being leaked to public entities, and integrity (Biba, 1977), by preventing unreliable information from flowing into critical operations.

In this section, we detail the core design of LIO and discuss the design trade-offs of a library-driven, mostly coarse-grained approach using the λChair conference review system as a driving example. In λChair, authenticated users can read any paper and can normally read any review. This reflects the normal practice in conference reviewing where, for example, every member of the program committee can see submissions and their reviews, and can participate in related discussion. In λChair, users can be added dynamically and assigned to review specific papers. Importantly, we use IFC to ensure that only assigned reviewers can write reviews for any given paper and that committee members in conflict with a paper cannot access the related discussions.

We incrementally describe the semantics of LIO using an extended simply-typed λ-calculus. First, we describe a pure base calculus. This calculus is then extended with labels (Section 2.2), labeled computations (Section 2.3), and labeled values (Section 2.4). Further leveraging labeled values we extend the calculus with toLabeled blocks to address label creep (Section 2.5). Finally, we extend this core with other features such as clearance, references and exceptions (Sections 3–5).

2.1 Base calculus for pure terms

Our semantics build on a pure, base calculus. The formal syntax of this base calculus is given in Fig. 1. Syntactic categories \( v \), \( t \), and \( \tau \) represent values, terms, and types, respectively. Values include primitives (Booleans \( \text{True}, \text{False} \); and unit \( () \))

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<tr>
<th>Values ( v := )</th>
<th>( \text{True} \mid \text{False} \mid () \mid \lambda x.t )</th>
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<tr>
<td>Terms ( t := )</td>
<td>( v \mid x \mid t_1 t_2 \mid \text{fix } t \mid \text{if } t_1 \text{ then } t_2 \text{ else } t_3 )</td>
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<tr>
<td>Types ( \tau := )</td>
<td>( \text{Bool} \mid () \mid \tau_1 \rightarrow \tau_2 )</td>
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Fig. 1. Formal syntax for values, terms, and types.
and functions \((\lambda x.t)\). Terms constitute values \(v\), variables \(x\), function applications \(t_1 t_2\), the standard fixpoint operator \(\text{fix} t\), and conditionals \(\text{if } t_1 \text{ then } t_2 \text{ else } t_3\).

Fig. 2 shows the reduction rules for these pure terms using structural operational semantics (Winskel, 1993). The relation \(t_1 \rightarrow t_2\) represents a single evaluation step of pure term \(t_1\) to term \(t_2\); we write \(\rightarrow^*\) for the reflexive and transitive closure of \(\rightarrow\). We say that \(t_1\) reduces to \(t_2\) in one step.

Substitution \(\{ t_2 / x \} t_1\) is defined in the usual way, homomorphic on all operators, renaming bound names to avoid capture. The reduction rules for these terms are self-explanatory and very much the same as those of standard \(\lambda\)-calculus—we do not explain them further.

We solely remark that our semantics does not model the sharing in lazy evaluation, as implemented by Haskell; modeling full lazy evaluation is beyond the scope of this paper and has no impact on our termination- and timing-insensitive security guarantees.

### 2.2 Security lattice

To enforce security policies, like most modern dynamic IFC systems, LIO associates labels with objects. Labels encode confidentiality and integrity data policies which are propagated alongside the information they protect. In turn, the system mandatorily enforces these individual policies when objects are read or written.

Labels are elements of a set \(\ell\) that forms a security lattice \((\ell, \sqsubseteq, \sqcap, \sqcup)\), with partial order \(\sqsubseteq\) (pronounced “can flow to”), binary join \(\sqcup\), and binary meet \(\sqcap\) (Denning, 1976). The \(\sqsubseteq\) relation is used by IFC systems when governing the allowed flows between differently labeled entities.\(^2\) For example, LIO only allows data labeled \(l_d \in \ell\) to be written to a channel labeled \(l_c \in \ell\) if \(l_d \sqsubseteq l_c\) holds true. The binary join is used to label computation results that depend on two objects by encoding the restrictions imposed by their labels, i.e., for

\(^2\) Decentralized IFC (DIFC) extends IFC with the decentralized label model of Myers and Liskov (Myers & Liskov, 1997), in which computations execute with a set of privileges, that can be used to loosen the restrictions imposed by the \(\sqsubseteq\) relation. LIO supports privileges and the DIFC model in full. However, since our formalisation is limited to the system without privileges, we omit this from the presentation. The interested reader is referred to the library documentation for more information.
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labels $l_A, l_B \in \ell$, the join $l_A \sqcup l_B$ is the smallest element such that $l_A \sqsubseteq l_A \sqcup l_B$ and $l_B \sqsubseteq l_A \sqcup l_B$. Dually, the binary meet $l_A \sqcap l_B$ encodes the intersection of the restrictions imposed by $l_A$ and $l_B$; the meet is primarily used when labeling objects that we expect to be read by entities labeled $l_A$ or $l_B$. Fig. 3 shows how information flows in this simple bounded lattice.

In LIO, labels are typed values. However, unlike most existing IFC systems (Zeldovich et al., 2006; Myers & Liskov, 2000; Krohn et al., 2007; Hriţcu et al., 2013), LIO is polymorphic in the label format. We solely require that the label type provide definitions for lattice relations $\sqsubseteq$, $\sqcup$, and $\sqcap$. In Haskell, this amounts to making the label type an instance of the typeclass `Label`:

```haskell
class Eq ℓ ⇒ Label ℓ where
  (\sqsubseteq) :: ℓ → ℓ → Bool
  (\sqcup) :: ℓ → ℓ → ℓ
  (\sqcap) :: ℓ → ℓ → ℓ
```

As an example, consider the definition of the typical 2-point lattice $\ell_2 = \{\text{Public}, \text{Secret}\}$, where $\text{Public} \sqsubseteq \text{Secret}$ and $\text{Secret} \not\sqsubseteq \text{Public}$:

```haskell
data ℓ_2 = Public | Secret deriving (Eq, Ord)

instance Label ℓ_2 where
  x \sqsubseteq y = x \leq y
  x \sqcup y = \text{max } x \text{ y}
  x \sqcap y = \text{min } x \text{ y}
```

Here, we simply use the `Ord` functions ($\leq$, `min`, and `max`), as defined by the compiler, to define the lattice operations. Of course, in real-world applications developers can define more complex label formats, such as the DLM (Myers & Liskov, 1997), HiStar (Zeldovich et al., 2006), Flume (Krohn et al., 2007), or DCLabels (Stefan et al., 2011a). Since such label definitions are typically provided by trusted code, LIO simply assumes that labels form a lattice, i.e., we do not verify that labels form a partially ordered set with a well-defined least upper bound and greatest lower bound. However, in certain cases, static analysis (e.g., in the form of refinement types (Rondon et al., 2008)) can be used to verify that provided definitions are well-defined.

To model labels, we extend our calculus to make labels first-class. Instead of modeling typeclasses, for simplicity, we assume that our calculus is polymorphic in the label type $\ell$. With this in mind, we extend the syntactic categories of Fig. 1 as shown on the right (Fig. 4). Here, values are extended with labels—metavariables $l$ and $c$ span over such values; types are extended with the label type $\ell$; and, terms are extended with label operations.

The reduction rules for these label operations are straightforward, and given in Fig. 5. In the rest of the paper, we sometimes use a more lax notation to describe label operations, e.g., $l_1 \sqsubseteq l_2$ in place of $l_1 \sqsubseteq l_2 \Rightarrow \text{True}$. The rules for the label operations $\sqsubseteq$, $\sqcap$, and $\sqcap$ rely on the label-specific implementation of these operators, as used in the premise of
rule (L\textsc{op}); we use the partial function $\llbracket \cdot \rrbracket_\ell$, which maps terms to values, to denote this. For example, instantiating our calculus to $\ell_2$, $\llbracket \text{Public} \sqcup \text{Secret} \rrbracket_{\ell_2} = \text{Secret}$, $\llbracket \text{Secret} \sqsubseteq \text{Public} \rrbracket_{\ell_2} = \text{True}$, etc. We highlight that our evaluation rules reduce the left operand first. Reducing the right operand first does not affect the semantics—we chose left-to-right evaluation solely because it matches the implementation of the labels used in \texttt{\textlambda}Chair (see (Stefan \textit{et al.}, 2011a)).

\subsection*{2.3 Restricting Haskell to safe IFC subset with the L\textsc{io} monad}

As previously mentioned, every object in an IFC system must be labeled. Importantly, this includes the \textit{current execution context} whose label we call the \textit{current label}.\footnote{More generally, every thread in the system is labeled. However, since we are focusing on a single-threaded system, we refer to the main thread context as the current execution context and its label as the current label.} The current label serves a role similar to the program counter ($\text{pc}$) in static IFC systems (Volpano \textit{et al.}, 1996). Namely, it restricts the current computation from performing side-effects which might compromise confidentiality. For instance, if the current label is $l_{\text{cur}}$, L\textsc{io} restricts the computation from writing to entities labeled $l_e$ unless $l_{\text{cur}} \sqsubseteq l_e$.

To accomplish this, L\textsc{io} provides a monad called L\textsc{io}. The L\textsc{io} monad encapsulates Haskell’s I\textsc{o} monad as to allow for L\textsc{io} computations to perform (restricted) I\textsc{o}. The monad also encapsulates the current label $l_{\text{cur}}$, which is retrieved with the \texttt{getLabel} function. The relevant parts of the definition are given below. By convention, we use $\ell$ for type variables that are expected to be instantiated by a label. The library is polymorphic over $\ell$ for greater flexibility, but in any normal program, every occurrence of $\ell$ will be instantiated by the same label type. Hence, it is more intuitive to think of $\ell$ as representing a particular (though unspecified) label type. Below we give the interface for this monad. We omit the definitions for simplicity.

\begin{verbatim}
data LIO \ell \tau
  instance Monad (LIO \ell)
  getLabel :: Label \ell \Rightarrow LIO \ell \ell
\end{verbatim}

We remark that the definitions for the monadic \texttt{return} and \texttt{bind} ($\gg=$) are trivial—a reference to the current label is simply threaded through the computation—and no security checks are performed by these combinators. Instead, library functions (e.g., \texttt{readFile}) use the current label to perform security checks (so as to enforced IFC) before executing any underlying I\textsc{o} actions. Taking this approach, the L\textsc{io} library provides a collection of L\textsc{io} actions that are similar to the I\textsc{o} actions available in standard Haskell libraries, but additionally enforce IFC—at a high level, this allows us to use L\textsc{io} in place of I\textsc{o}. \hfill\Box
RETURN
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid \text{return } t \rangle \xrightarrow{0} \langle l_{\text{cur}}, c_{\text{cur}}. m \mid LIO^{TCB} t \rangle \]

BIND
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid t_1 \rangle \xrightarrow{n} \langle t'_{\text{cur}}, c'_{\text{cur}}. m' \mid LIO^{TCB} t' \rangle \]
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid t_1 \gg t_2 \rangle \xrightarrow{n} \langle t'_{\text{cur}}, c'_{\text{cur}}. m' \mid t_2 t' \rangle \]

LIOPure
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid t_1 \xrightarrow{-} t'_1 \rangle \]
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid t_1 \xrightarrow{0} \langle l_{\text{cur}}, c_{\text{cur}}. m \mid t'_1 \rangle \]

GETLABEL
\[ \langle l_{\text{cur}}, c_{\text{cur}}. m \mid \text{getLabel} \rangle \xrightarrow{0} \langle l_{\text{cur}}, c_{\text{cur}}. m \mid \text{return } l_{\text{cur}} \rangle \]

Fig. 7. Semantics for the LIO monad.

To ensure security, all side-effecting computations must be encoded in LIO. Importantly, the visibility of the LIO value constructor is limited to the LIO trusted computing base (TCB) so as to guarantee that “untrustworthy” (and potentially malicious) code cannot perform arbitrary I/O.\(^4\) Henceforth, we assume that all computations are in the LIO monad.

To formally describe the behavior of the LIO monad, we extend the syntactic categories of our calculus as shown on the right (Fig. 6). Our extension simply adds monadic actions (LIO\(^{TCB} t\)) to values, monadic operations to terms, and a type for LIO computations. We note that the LIO\(^{TCB}\) constructor is not part of the surface syntax, i.e., programs that use LIO\(^{TCB}\) are not considered valid.\(^5\)

We explicitly distinguish pure-term evaluation from top-level monadic-term evaluation. Specifically, an LIO program is a configuration—spanned over by metavariable k—of the form \(\langle l_{\text{cur}}, c_{\text{cur}}. m \mid t \rangle\), where \(l_{\text{cur}}\) is the current label, \(c_{\text{cur}}\) is the current clearance (explained in Section 3), \(m\) is the memory store (see Section 4), and \(t\) is the monadic term under evaluation. The reduction \(\langle l_{\text{cur}}, c_{\text{cur}}. m \mid t \rangle \xrightarrow{n} \langle t'_{\text{cur}}, c'_{\text{cur}}. m' \mid t' \rangle\) represents a single evaluation step from term \(t\), with current label \(l_{\text{cur}}\), current clearance \(c_{\text{cur}}\), and memory \(m\), to term \(t'\), with current label \(l'_{\text{cur}}\), current clearance \(c'_{\text{cur}}\), and memory \(m'\). For the moment, we ignore the clearance and memory in the configuration. Index \(n\) in the transition relation counts the number of executed toLabeled actions; this is an artifact of the proof technique and not relevant to the semantics. We write \(\xrightarrow{n}^*\) for the reflexive and transitive closure of \(\xrightarrow{n}\). The reduction rules for the core LIO operations are given in Fig. 7. The rules for return and \((\gg=)\) are trivial and standard—all IFC checks are performed by the non-proper

\(^4\) To accomplish this, we use Safe Haskell (Terei et al., 2012): the module in which the data type is defined is marked Unsafe, while the modules that expose IFC-enforcing LIO actions are marked Trustworthy. This ensures that LIO can safely execute arbitrary LIO actions by simply marking their enclosed module Safe; Safe Haskell prevents Safe code from depending on Unsafe modules.

\(^5\) For simplicity, we do not use additional syntactic categories to distinguish between values and terms that are part of the surface syntax from those that are not. In Section 6, we define a safe predicate for making such a distinction.
morphism of \( LIO \). Similarly, the \((LIOPURE)\) rule specifies that if we have a top level pure term, it should be evaluated to completion, i.e., until it reduces to a monadic term. Rule \((GETLABEL)\) defines the \( LIO \) library function for retrieving the underlying current label, further discussed below.

2.3.1 Coarse-grained labeling with the current label

To soundly reason about IFC, every value must be labeled. However, and in contrast to other language-based systems (e.g., Jif (Myers & Liskov, 2000), FlowCaml (Simonet, 2003), Breeze (Hrițcu et al., 2013) etc.) in which every value is explicitly labeled, the values in our calculus are not associated with labels (see Fig. 1–6). This is a direct consequence of taking a library-based approach: we cannot explicitly label every Haskell value without modifying the language runtime. Instead, and like several IFC operating systems (Efstathopoulos et al., 2005; Zeldovich et al., 2006), we take a coarse-grained approach and use the current label to protect all values in scope, i.e., in \( LIO \), the current label \( l_{\text{cur}} \) is the label on all “unlabeled” values in the current execution context. Since we use the current label to restrict the current computation from performing arbitrary side-effects, this also ensures that the confidentiality (and integrity) of all values in scope is preserved.

In addition to ensuring that every value is labeled, this coarse-grained labeling approach has two other interesting consequences. First, it does not force developers to explicitly label every piece of data. This eliminates the need to clutter code with labels, reason about the security implications of every value, or define a special default label. Instead, developers only explicitly label data they care about (as detailed in Section 2.4).

Second, and more importantly, it eliminates the implicit flows problem by construction (Sabelfeld & Myers, 2003). This problem arises when information can be leaked through the program control flow. An example of an implicit flow is given in Fig. 8 (written in a hypothetical alternative \( LIO \) language without explicit labels). Here, secret bit \( b \) is leaked into public reference \( x \) according to the program control flow, i.e., what code—which assignment (to public reference \( x \))—is executed depends on the secret \( b \).

To prevent such leaks, language-based approaches rely on the program counter label to reflect the sensitivity of the branch condition within each branch and, in turn, disallow such unsafe assignments. In Haskell, and thus \( LIO \), branch conditions have type \( \text{Bool} \)—they are not explicitly labeled values. Rather, the branch condition is (conceptually) labeled by the current label, which is common across both branches. As a consequence, control flow cannot be used to leak sensitive information: regardless of the branch taken, the current label prevents writes to public entities. Consider implementing the attack in Fig. 8 with \( LIO \). Since the branch condition \( b_{\text{Secret}} \) is not explicitly labeled, it is protected by \( l_{\text{cur}} \). But since \( b_{\text{Secret}} \) is secret, we must have \( l_{\text{cur}} \equiv \text{Secret} \), meaning any subsequent writes (within the branches or after) to public references are disallowed since \( l_{\text{cur}} \not\subseteq \text{Public} \). In Section 4, we further expand on the semantics of mutable references.

```plaintext
if b_{\text{Secret}}
  then x_{\text{Public}} := 1_{\text{Public}}
else x_{\text{Public}} := 0_{\text{Public}}
```

Fig. 8. Implicit flows problem.
2.3.2 A floating current label

The current label protects all data in scope by serving as an upper bound label on all values. To preserve this invariant, when reading sensitive data, we can either disallow reads from entities more sensitive or raise the current label to protect the newly read data. Like other coarse-grained systems we take the latter approach and raise the current label to “float” above the labels of all the entities from which data has been read.

Raising the current label allows computations to flexibly read data, at the cost of being more limited in where they can subsequently write. Concretely, a computation with current label $l_{cur}$ can read data labeled $l_d$ by raising its current label to $l'_{cur} = l_{cur} \sqcup l_d$, but can thereafter only write to entities labeled $l_e$ if $l'_{cur} \sqsubseteq l_e$. For example, LIO allows a public computation to read secret data by raising $l_{cur}$ from Public to Secret. Importantly, the new current label prevents the computation from subsequently writing to public entities. Some static IFC systems, such as Jif (Myers & Liskov, 2000), are even more permissive in allowing public writes after reading secret data if no secret data is actually being leaked. In Section 2.5, we present a method that can be used to safely restore the current label, making our dynamic IFC system equally permissive.

2.4 Explicitly labeling values

$\lambda$Chair associates a label with the logged-in user’s browser and each submitted paper as a way to ensure that users only see papers dictated by their role—author or committee member. When reading a paper from the database system, the label of the HTTP request handler, or controller, is raised. In turn, the system enforces that a response only be sent to the user’s browser when the controller label can flow to the browser label.

Suppose that the $\lambda$Chair database contains two papers, as shown in Fig. 9, submitted by Alice and Bob (neither of whom is part of the committee). When Alice wishes to see the index of all papers she submitted, the controller must read from the database only data whose labels can flow to the browser label $l_{Alice}$. Otherwise, the controller will reach a state in which the current label is above the browser label (e.g., $l_{Alice} \sqcup l_{Bob}$) and it will no longer be allowed to respond to the user. In language-based IFC systems (Myers et al., 2001; Simonet, 2003), this is typically not a concern because values returned from the database can be individually and explicitly labeled. As a result, the controller would be able to compare the label of the value retrieved from the database and the browser label, only using the retrieved value if its label flows to the browser label.

To avoid being overly restrictive, LIO similarly provides Labeled values. A labeled value protects an arbitrary term with a strict, explicit label, irrespective of the current label. We define such values as follows.

$$\text{data Labeled } \ell \tau$$

As before, we restrict the value constructor to the TCB. However, to allow non-TCB code to create and manipulate labeled values, we provide a safe, IFC-abiding, interface. This is particularly important since labeled values are protected by their explicit labels—untrusted
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\[
\begin{align*}
\text{LABEL} & \quad \ell \sqsubseteq l \implies l \sqsubseteq c_{\text{cur}} \implies \text{True} \\
\langle \ell_{\text{cur}}, c_{\text{cur}}, m \mid \text{label } t \rangle & \xrightarrow{0} \langle \ell_{\text{cur}}, c_{\text{cur}}, m \mid \text{return}(\text{Labeled}^{\text{LCB}} t) \rangle \\
\text{UNLABEL} & \quad l \sqsubseteq \ell' \sqsubseteq c_{\text{cur}} \implies \text{True} \\
\langle \ell_{\text{cur}}, c_{\text{cur}}, m \mid \text{unlabel } t \rangle & \xrightarrow{0} \langle \ell_{\text{cur}}, c_{\text{cur}}, m \mid \text{unlabel } t' \rangle \\
\text{LABELOF} & \quad \text{labelOf}(\text{Labeled}^{\text{LCB}} t) \xrightarrow{0} \langle \ell'_{\text{cur}}, c_{\text{cur}}, m \mid \text{return } t \rangle
\end{align*}
\]

Fig. 10. Semantics for labeled values.

code should not be allowed to bypass the label and arbitrarily inspect (or modify) the protected value. This interface for creating and inspecting labeled values is given below.

\[
\begin{align*}
\text{label} & :: \text{Label } \ell \Rightarrow \ell \rightarrow \tau \rightarrow \text{LIO } \ell \ (\text{Labeled } \ell \ \tau) \\
\text{unlabel} & :: \text{Label } \ell \Rightarrow \text{Labeled } \ell \ \tau \rightarrow \text{LIO } \ell \ \tau \\
\text{labelOf} & :: \text{Label } \ell \Rightarrow \text{Labeled } \ell \ \tau \rightarrow \ell
\end{align*}
\]

To describe the semantics of these functions, we extend the values, terms and types of our calculus as shown in Fig. 11. (As with LIO^{LCB}, we do not consider the Labeled^{LCB} constructor part of the surface syntax.) The reduction rules for the new terms are given in Fig. 10; rule (LABELCTX), (UNLABELCTX), and (LABELOFCTX) reduce terms until they have appropriate structures to trigger rules (LABEL), (UNLABEL), and (LABELOF), respectively. We ignore parts of these rules that involve the current clearance \(c_{\text{cur}}\) until Section 3.

The label function is used to explicitly label terms. As rule (LABEL) shows, the function associates the supplied label \(l\) with term \(t\) by wrapping the term with the Labeled^{LCB} constructor. It first asserts that the new label (\(l\)) used to protect \(t\) is at least as restricting as the old label (the current label, \(\ell_{\text{cur}}\)), i.e., \(\ell_{\text{cur}} \sqsubseteq l \implies \text{True}\).

The dual of label, unlabel, unwraps such explicitly labeled values. As given by rule (UNLABEL), unlabel takes a labeled value Labeled^{LCB} \(l \ t\) and returns the wrapped term \(t\). Since the returned term is no longer explicitly labeled by \(l\) and is, instead, protected by the current label, \(\ell_{\text{cur}}\) must be at least as restricting as \(l\). To ensure this, the current label
is raised from \( l_{\text{cur}} \) to \( l_{\text{cur}} \uplus l \)—this captures the fact that the remaining computation might depend on \( t \) and highlights that the current label always “floats” above the labels of the values observed by the current computation.

Finally, we provide the \( \text{labelOf} \) function as a way to inspect the label of a labeled value. As detailed by the (\text{LABEL}O\text{F}) reduction rule, \( \text{labelOf} \) takes a labeled value \( \text{Labeled}^{\text{TCB}} t \) and simply returns the label \( l \) protecting term \( t \). Since the label of a \( \text{Labeled} \) value is strict, \( \text{labelOf} \) does not require an additional context rule for reducing the label. Unlike \( \text{unlabel} \), \( \text{labelOf} \) also does not raise the current label—\( \text{labelOf} \) is part of the pure calculus. Indeed, this allows code to check the label of a labeled value before deciding to \( \text{unlabel} \) it (and thereby raise the current label). This design decision has an important consequence: regardless of the current label (and clearance) of the configuration, \( \text{labelOf} \) always succeeds—hence labels are effectively “public.”

Example 1 (Fetching papers for reviewers)

Turning to our \( \lambda \text{Chair} \) use case, we now consider some of the core functions that are used by the top-level request handler (controller). In particular, we show how to fetch papers for a given reviewer using a simple underlying database system. The specific label type used by \( \lambda \text{Chair} \) is \( \text{DClabe}l \), as defined in (Stefan et al., 2011a). Hence, we define a type alias for the \( \text{LIO} \) monad with the label instantiated to \( \text{DClabe}l \):

\[
\text{type} \ DC \ \tau = \text{LIO} \ \text{DClabe}l \ \tau
\]

This database system operates on labeled papers. As defined below, a paper is simply a record with several fields, including the (unique) paper id (\( \text{paperId} \)), the paper itself (\( \text{pdf} \)), labeled reviews, etc.

\[
\text{data} \ Paper = \text{Paper} \{ \text{paperId} :: \text{Id}, \text{pdf} :: \text{PDF}, \text{reviews} :: [\text{LabeledReview}], \ldots \}
\]

\[
\text{type} \ \text{LabeledPaper} = \text{Labeled} \ \text{DClabe}l \ \text{Paper}
\]

Among other operations, the database system provides a \( \text{fetchPapers} \) function which is used to get the list of all such papers:

\[
\text{fetchPapers} :: DC \ [\text{LabeledPaper}]
\]

For simplicity, we omit the implementation details of \( \text{fetchPapers} \) and only remark that it relies on TCB code to wrap an underlying \( \text{IO} \)-based database system API and explicitly label the fetched papers.

While simple, the \( \text{fetchPapers} \) function is sufficient for fetching a given reviewer’s papers. Note that if the controller simply unlabels the papers returned by \( \text{fetchPapers} \), the current label may be raised to a point where the computation cannot respond back to the user, i.e., the current label may not flow to the browser label. This situation, for example, happens when the current user is not part of the committee and another author’s paper is

---

\( \text{footnote} \) 6 In fact, labels are actually protected by the current label, but since code can always observe an object’s label at the current label, this is akin to being public.

\( \text{footnote} \) 7 We elide the details of labeled reviews used in the actual \( \lambda \text{Chair} \) implementation and simplify some of the application details (e.g., the generic database system API). The interested reader is referred to the code documentation at \text{http://labeled.io} for more details.
unlabeled—\( \lambda \text{Chair} \) prevents such data from being sent (leaked) back to the user’s browser. Hence, we need to make sure that the controller only reads data that the end-user can see.

To this end, we define \( \text{fetchPapersFor} \):

\[
\text{fetchPapersFor} :: \text{User} \to \text{DC} \ [\text{LabeledPaper}]
\]

\[
\text{fetchPapersFor user} = \text{do}
\]

\[
\text{let browserLabel} = \text{userToLabel user}
\]

\[
\text{lpapers'} = \text{filter} (\lambda \text{lpaper} \to \text{labelOf lpaper} \sqsubseteq \text{browserLabel}) \text{lpapers}
\]

\[
\text{mapM unlabeled lpapers'}
\]

This function fetches the papers, filters the ones the user is allowed to read by comparing the paper's label with the user’s browser label—itself computed with function \( \text{userToLabel} \)—and unlabels them. At this point, the controller can compose the HTML page containing the paper information and safely respond to the user.

In addition to providing a simple illustration of how labeled values are used in LIO, this simple example serves to illustrate the importance of labeled values. Specifically, by providing labeled values in the language, we can implement core functionality such as \( \text{fetchPapersFor} \) in the untrusted LIO application code; without labeled values such functionality would otherwise have to be implemented in the trusted database layer or database system itself. Indeed, building on this observation, we can, for example, extend \( \lambda \text{Chair} \) to implement an in-memory database which solely uses the aforementioned database system as a persistence layer, i.e., it solely relies on the actual database system to keep the papers persistent.

### 2.5 Addressing label creep with \( \text{toLabeled} \)

In conference systems, it is often that case that some reviews are superseded by others, papers change titles, submissions are withdraw, etc. Hence, the \( \lambda \text{Chair} \) database system provides functions for updating (or deleting) existing papers. For instance, \( \text{updatePaper} \) is used to update the paper with the supplied paper id with the new labeled paper. The type for this function is given below.

\[
\text{updatePaper} :: \text{Id} \to \text{LabeledPaper} \to \text{DC} ()
\]

Similar to \( \text{fetchPapers} \), this function relies on TCB code to communicate with the actual database system; from a security stance, it is only interesting to note that the function always ensures that the current computation can overwrite the existing paper (by performing a \( \sqsubseteq \)-check with the current label and label protecting the existing paper).

Suppose we wish to implement a function that performs a partial update, i.e., an update wherein only part of the paper object is updated. This is useful, for example, when a user only updates the abstract of the paper and leaves other parts such as the underlying PDF intact. Indeed, sending a PDF file to perform a “full” update is not practical. An implementation of such a partial update function is given below.
Here, we assume that the type `PartialPaper` encodes a partial paper (e.g., by using a `Maybe` type for each of the fields in `Paper`) and function `merge` simply merges the content of the new partial paper and existing paper. The underlying `fetchPaperById` database function behaves as expected: it returns the labeled paper corresponding to the id.\footnote{Note that this has the implication that id’s are effectively public. However, since the number of elements in the database is public (as revealed by the length of the list returned by `fetchPapers`), this is not surprising.}

Unfortunately, this implementation has the drawback of always raising the current label to the label of the paper being updated. This can result in a scenario where actions that follow a partial update fail (e.g., writes to less sensitive entities), solely because the current label is overly restricting. Raising the current label to a point where the computation can no longer perform certain useful side-effects is known as label creep (Sabelfeld & Myers, 2003). Though label creep does not compromise security, since the current label still protects all data in lexical scope, it hinders functionality. In this example, this is particularly unappealing since `partialUpdatePaper` does not return any information about the existing paper, which would, in turn, be used in the response to the user—it simply writes back to the database. Ideally, `partialUpdatePaper` should not raise the current label, despite performing a computation on sensitive data.

In general, being able to perform computations on sensitive data without raising the current label is crucial to building practical applications. To this end, LIO provides the `toLabeled` function that executes an LIO action and subsequently restores the current label:

\[
toLabeled :: \text{Label } \ell \Rightarrow \ell \to \text{LIO } \ell \to \text{LIO } (\text{Labeled } \ell \tau)
\]

Intuitively, if the current label at the point `toLabeled l t` gets executed is `l_{\text{cur}}`, `toLabeled` executes `t` and restores the current label to `l_{\text{cur}}`, i.e., `toLabeled` provides a separate context in which `t` is evaluated. Of course, returning the result of `t` directly would allow for trivial leaks of sensitive data. Hence, `toLabeled` labels the result of `t` with `l`. This design decision effectively states that the result of `t` is protected by label `l`, as opposed to the current label at the point `t` completed. Of course, `toLabeled` requires that the result of `t` not be more sensitive than `l`.

More formally, we extend terms with the `toLabeled` primitive: \( t ::= \cdots | \text{toLabeled } t_1 t_2 \) and give the precise semantics in Fig. 12. In both rules, the current label and clearance are preserved. Rule (`\text{TO Labeled Ctx}`) simply reduces the label argument. Rule (`\text{TO Labeled}`) specifies the non-trivial case. As noted above, the label `l` is used to label the result of `t`.
Hence, the rule first ensures that we are not trying to create a labeled value below the current label (or above the current clearance, see Section 3), i.e., \( l_{\text{cur}} \sqsubseteq l \leadsto \text{True} \). The rule then completely reduces \( t \) to an LIO value.\(^9\) If the current label \( l'_{\text{cur}} \) at the time of completion is below the provided upper-bound \( l \), then “transferring protection” of the result \( t' \) from \( l'_{\text{cur}} \) to \( l \) is safe and we thus simply return the result, labeled with \( l \). Observe that if \( l'_{\text{cur}} \sqsubseteq l \leadsto \text{False} \), then labeling the result \( t' \) with \( l \) might result in a leak, e.g., if \( t' \) actually contains information above \( l \). In Section 5, we consider the cases where these conditions do not hold. We finally remark that the (TO Labeled) increments the index \( n \) to indicate that toLabeled was executed at least once.

**Example 2 (Partially updating papers)**

Returning to our partial update \( \lambda \)Chair example, we can now use toLabeled in a straightforward way to implement partialUpdatePaper. This new implementation is given below.

```haskell
partialUpdatePaper :: Id \rightarrow \text{PartialPaper} \rightarrow \text{DC} ()
partialUpdatePaper i new = do
  lold \leftarrow \text{fetchPaperById} i
  lnew \leftarrow \text{toLabeled} (\text{labelOf} lold) $ do
    old \leftarrow \text{unlabel} old
    -- Merge the new (partial) paper and existing paper:
    return $ \text{merge} \text{new} \text{old}
    -- Perform actual update:
    \text{updatePaper} i \text{inew}
```

This implementation is almost identical to the original one. It only differs in wrapping the part of the code that is computing on sensitive data with toLabeled. Specifically, it wraps the part of the code that unlabels the existing paper and performs the merge. (Since toLabeled returns a labeled value, we no longer need to explicitly label the merged paper—we simply return it.) The current label within the toLabeled blocks is raised to the join of the current label and the label of the existing paper (labelOf lold) by function unlabel. Importantly, however, the current label before and after calling partialUpdatePaper remains the same.

\(^9\) By using big-step semantics, we do not need to rely on the use of trusted functions that (save and) restore the current label and clearance.
2.5.1 An alternative semantics for toLabeled

Naturally, one may ask why toLabeled demands that we provide the label of the result as an argument, as opposed to simply using the final current label of the executed computation. Indeed, an early version of LIO had such an implementation:

\[
\text{toLabeled}': \text{Label} \ell \Rightarrow \text{LIO} \ell \tau \rightarrow \text{LIO} \ell \text{Labeled} \ell \tau
\]

However, different from the version of LIO as presented in this paper, inspecting the label of labeled values with labelOf raised the current label to join of the current label and label of the value. The semantics for this alternative function

\[
\text{labelOf}': \text{Label} \ell \Rightarrow \text{Labeled} \ell \tau \rightarrow \text{LIO} \ell \ell
\]

is given below.

\[
\text{LABELOOf}^* \\
\begin{array}{c}
\ell_{\text{cur}} \sqcup l \rightsquigarrow \ell'_{\text{cur}} \\
\ell'_{\text{cur}} \sqsubseteq \ell_{\text{cur}} \\
\langle \ell_{\text{cur}}, c_{\text{cur}}, m \mid \text{LABELOOf}' \text{ (Labeled}^{\text{top}} l \ell) \rangle \xrightarrow{0} \langle \ell'_{\text{cur}}, c_{\text{cur}}, m \mid \text{return } l \rangle
\end{array}
\]

This difference is particularly important since information can otherwise be leaked by encoding it into the labels themselves (Russo & Sabelfeld, 2010; Buiras et al., 2014). To illustrate this point, consider the 3-point lattice \( \ell_3 = \{ \text{Public}, \text{Secret}, \text{TopSecret} \} \) and the following code that uses toLabeled’ and labelOf to leak the value of a secret Boolean.

\[
\text{leakBool} : \text{Labeled} \ell_3 \text{Bool} \rightarrow \text{LIO} \ell_3 \text{Bool}
\]

\[
\text{leakBool } \text{secretBool } = \text{do}
\]

-- Current label is Public
\[
\text{secretBool }' \leftarrow \text{toLabeled} ' \text{secretBool } \quad \text{-- Raise current label to Secret}
\]

-- Raise label to TopSecret if \( s \) is True
\[
\text{when } s \ (\text{raiseLabel TopSecret})
\]

-- Current label is Public
\[
\text{return } \text{labelOf } \text{secretBool }' \equiv \text{TopSecret}
\]

\[
\text{where } \text{raiseLabel } l = \text{label } l () \gg \text{unlabel}
\]

The key distinction between the two designs is what label is used to protect the label of a labeled value (Buiras et al., 2014). (Recall that in an IFC system every piece of data must be labeled—this include labels themselves.) In the early version of LIO (that with toLabeled’ and labelOf’), the label on the label of a value was the label itself. Hence, inspecting the label of a value required raising the current label. Importantly, however, toLabeled’ did not require programmers to supply an upper-bound label for the labeled result. In contrast, the current version of LIO considers the current label \( l_{\text{cur}} \) as the label protecting the labels of labeled values. In this system, inspecting the label of a value does not require raising the current label, and labelOf is, in turn, pure. Of course, the trade-off is that the label on the result produced by toLabeled must be provided a-priori. Our experience with building applications has shown that the ability to inspect labels outweighs the “burden” of specifying an upper-bound for toLabeled (see Giffin et al., 2012 for a description of an example system that uses LIO). In fairness, most of our applications are
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web-centric and while we believe this experience to extend to other domains, evaluating this trade-off for other kinds of applications is an interesting direction for future work.

3 Addressing covert channels with clearance

IFC systems do not typically restrict what data code can read, rather—and as we have done thus far—they only restrict where the code can write to once it has read the data. Similarly, code can always write to channels or create objects with arbitrary labels, as long as doing so does not leak information, i.e., code can always write to and allocate entities more sensitive than the current label. In many cases, however, it is useful to execute code with least privilege by limiting its access to the data/entities needed to perform the task (Saltzer & Schroeder, 1975). This principle not only simplifies security auditing, but also eliminates the opportunity for the code to leak sensitive data by exploiting covert channels (Lampson, 1973). LIO introduces the notion of clearance to language-based IFC systems (Stefan et al., 2011b), recently also adopted by Breeze (Hriţcu et al., 2013), as a means for restricting access to certain labeled entities.

3.1 Restricting data-access with clearance

The current clearance $c_{cur}$ is a label tracked by the LIO monad alongside the current label $l_{cur}$; in our formalization, the clearance appears as the second component of a program configuration $⟨l_{cur}, c_{cur}, m | t⟩$. LIO restricts access to certain labeled entities using the clearance in two different ways.

First, the clearance is used to restrict the reading of overly-sensitive data by enforcing that the current clearance always be an upper bound on the current label, i.e., for all valid program configurations $⟨l_{cur}, c_{cur}, m | t⟩$, it is the case that $l_{cur} ⊑ c_{cur} ⇝ True$. This restriction is enforced by the LIO interface. For example, unlabel as given in rule (UNLABEL) of Fig. 10 only unlabels the labeled value if raising the current label $l_{cur}$ will not result in a current label $l'_{cur}$ that is above the current clearance, i.e., $l'_{cur} ⊑ c_{cur} ⇝ True$. In a similar way, before reading from a file or reference (see Section 4), we ensure that raising the current label will not violate this guarantee.

The use of clearance to restrict code from reading certain entities is a form of discretionary access control. As a consequence, we can prevent malicious code form exploiting covert channels to leak overly-sensitive information by ensuring that it cannot read such data. As an example, suppose that the partial update function in $\lambda$Chair is implemented by a third-party developer (e.g., to implement a better merging function). If the developer is malicious, they can use the partial update function to leak the contents of a competing author’s paper through covert channels. Indeed, this is simple since the developer can create an account on the $\lambda$Chair platform and take on the role of an author, as to ensure that their malicious code is executed. A malicious version of partialUpdatePaper is given below.

```haskell
leakyPartialUpdatePaper :: Id → PartialPaper → DC ()
leakyPartialUpdatePaper i new = do
    -- Get all existing papers:
papers ← fetchPaperByld i
```
-- Leak information about some of the papers
mapM maybeLeak papers
-- Execute the normal partial update:
partialUpdatePaper i new
  where maybeLeak lpaper = toLabeled (labelOf lpaper)$ do
    paper ← unlabel lpaper
    -- If the paper has a specific author, leak it:
    when (paperAuthors paper ≡ ...) $ leakToCovertChannel paper

Here, we use function leakToCovertChannel to leak information about papers written by certain authors; otherwise the function behaves in the same way as the normal partialUpdatePaper code. The function leakToCovertChannel leaks (part of) the sensitive paper content through a covert channel. For instance, the code can leak information by diverging (or not) according to the paper content, i.e., through the termination covert channel (Askarov et al., 2008); alternatively, it can leverage the external timing covert channel (Agat, 2000) to leak the information by delaying the response according to the content, etc. Using clearance, we can prevent such leaks by setting the clearance to the label of the browser—in this case, the leakyPartialUpdate will fail to unlabel papers which the requesting user, i.e., the attacker, is not allowed to read. Since the code running on behalf of one user does not have access to another user’s data, it cannot leak it—it can only leak data it can already read.

The second role of clearance is to restrict code from writing to and allocating entities labeled above the clearance. For example, label as given in rule (LABEL) of Fig. 10 only creates a Labeled value if the label of the value is bounded by the clearance. Similarly, toLabeled as given in rule (TO Labeled) of Fig. 12 requires that the upper-bound of the result to be below the clearance. In a similar way, before creating or writing to a file or reference (see Section 4), we ensure that their label is below the current clearance.

The motivation for restricting reads with clearance is straightforward. However, as noted above, LIO also restricts the allocation and writing of entities to those whose sensitivity is below the current clearance. As in (Zeldovich et al., 2006), this addresses attacks in which malicious code duplicates sensitive data, e.g., by copying a file, only to read it later, when the system policy changes. Without clearance, reasoning about the consequence of a system policy change would be immensely difficult. In particular, without clearance we would have to reason about the full lifetime of data—if any code can copy data, when is it safe to change policy?

### 3.2 Making clearance first-class

To leverage clearance for isolation, as described above, we execute a term in a configuration that has initially set the desired clearance. Of course, in many application it is useful to be able to “drop” privileges and continue executing with least privilege (Saltzer & Schroeder, 1975). For example, in λChair when authenticating user requests, the clearance must be high enough to read credentials, but once the authentication is complete, having access to such information is unnecessary and dangerous: a simple bug in the code that generates an
HTML list of the user’s papers could potentially leak the credentials. Hence, we provide a means for inspecting and manipulating the clearance. Specifically, we provide:

\[\text{getClearance} :: \text{Label } \ell \Rightarrow \text{LIO } \ell \ell\]
\[\text{lowerClearance} :: \text{Label } \ell \Rightarrow \ell \rightarrow \text{LIO } \ell ()\]

The \text{getClearance} and \text{lowerClearance} functions are used to get and set the current clearance, respectively.

We add the primitives \text{getClearance} and \text{lowerClearance} to the syntactic category of terms \(t ::= \cdots | \text{getClearance} | \text{lowerClearance} t\) and formally describe its semantics in Fig. 13. The rules are mostly self-explanatory. We solely highlight that the premise in rule (\text{LOWERCLEARANCE}) requires the new current clearance \(c'_{\text{cur}}\) to be below the current clearance \(c_{\text{cur}}\) and above the current label. By lowering the clearance, code can effectively run with least privilege. Of course, allowing code to arbitrarily raise the clearance would trivially prevent us from confining untrusted code—hence code can only decide to access fewer entities.

However, recall from rule (\text{TO Labeled}) that \text{toLabeled} restores the current label and clearance. Hence, combined with \text{toLabeled}, we can use \text{lowerClearance} to execute a term \(t\), at a lower clearance, without lowering the current clearance:

\[\text{withClearance} :: \text{Label } \ell \Rightarrow \ell \rightarrow \text{LIO } \ell \tau \rightarrow \text{LIO } \ell (\text{Labeled } \ell \tau)\]
\[\text{withClearance } c'_{\text{cur}} t = \text{toLabeled } c'_{\text{cur}} (\text{lowerClearance } c'_{\text{cur}} \gg> t)\]

This use of \text{toLabeled} addresses the dual to the label creep described in Section 2.5: by lowering the current clearance a program can reach a state where \(l_{\text{cur}} = c_{\text{cur}}\), at which point it cannot read or write to entities more-sensitive than \(l_{\text{cur}}\). More interestingly, this enables powerful security patterns. For instance, it allows arbitrary untrusted code to treat code it depends on as untrustworthy. It also allows for the structuring of programs in such a way that different components execute with least privilege and are isolated from one another. For example, in \(\lambda\text{Chair},\) we can wrap request handlers with \text{withClearance} to isolate requests based on the user (browser) label. In Section 6.2.2, we give a precise definition for isolation and show how LIO, when used in such a way, successfully isolates terms.
4 Mutable labeled references

Many applications rely on imperative data-structures, which are often implemented using mutable reference. LIO provides labeled alternatives to Haskell’s IORef’s (Peyton Jones, 2001). With mutable references, we can implement an efficient in-memory database for \( \lambda \)Chair. In particular, by modeling each paper as a reference, updating a paper becomes very cheap; it simply amounts to writing to a reference, as opposed to creating a large immutable data structure (that contains the rest of the papers).

The LIO reference API is given below.

\[
\text{data } \text{LIORef } \ell \tau \\
\text{newLIORef } :: \text{Label } \ell \Rightarrow \ell \rightarrow \tau \rightarrow \text{LIO } \ell (\text{LIORef } \ell \tau) \\
\text{readLIORef } :: \text{Label } \ell \Rightarrow \text{LIORef } \ell \tau \rightarrow \text{LIO } \ell \tau \\
\text{writeLIORef } :: \text{Label } \ell \Rightarrow \text{LIORef } \ell \tau \rightarrow \tau \rightarrow \text{LIO } ()
\]

While the implementation of secure references can vary, we simply wrap Haskell’s IORef’s. Intentionally, this API resembles the standard Haskell API for mutable references. The key difference is that the function for creating references takes an additional argument: the label of the reference.

To formally describe this API, we extend our calculus with references as shown in Fig. 14. Like \text{LabeledTCB}, the \text{LIORef}^\text{TCB} constructor is restricted to the TCB and is strict in its first argument. References are created with \text{newLIORef}, read with function \text{readLIORef}, and modified with \text{writeLIORef}. We overload the \text{labelOf} function to allow code to inspect the label of a reference.\(^{10}\)

We model the reference store as part of the memory configuration \( m \). The store is a map from addresses—spanned over by metavariable \( a \)—to labeled values.\(^{11}\) Since we do not provide any mechanisms for explicit deallocation or address inspection/comparison in the LIO API we can model the store as an infinitely-large map, while allowing the implementation to safely garbage collect unused references as necessary.\(^{12}\) In our formalization, this memory store appears as the third component of a program configuration \( \langle l_{\text{cur}}, c_{\text{cut}}, m \mid t \rangle \).

The reduction rules for references are given in Fig. 15. When creating a reference, as given by rule (\text{NEWLIOREF}), \text{newLIORef } \ell t t \) creates a labeled value that guards \( t \) with label \( l \) and stores it in the memory store at a new, fresh, address \( a \). Subsequently, the function

\(^{10}\) In our implementation, we use a type class \text{LabelOf} to define the \text{labelOf} function. Both \text{LIORef} and \text{Labeled} are instances of this class.

\(^{11}\) Since the label of a reference accompanies the address (both wrapped by the \text{LIORef}^\text{TCB} constructor), an alternative memory store that simply maps addresses to terms is sufficient—we chose the labeled-store approach to simplify the proof burden (see Section 6).

\(^{12}\) Non-opaque pointers could potentially be used to leak information (e.g., by freeing a reference in a secret context only to allocate a reference and inspect its address in a public context). Adapting LIO to deal with non-opaque pointers can be done as in (Hedin & Sands, 2006).
returns an $LIORef$ value that contains the reference label and address where the term is stored. (Like $Labeled^{TCB}$, the constructor $LIORef^{TCB}$ is not part of the surface syntax and thus cannot be abused by untrusted code.) Rule (READLIOREF) specifies the semantics for reading a labeled reference; reading the term stored at address $a$ simply amounts to unlabeling the value $m(a)$ stored at the underlying address. Function writeLIORef, specified by rule (WRI TELIOREF), updates the memory store with a new labeled term $t$ for the reference at location $a$, leaving the label intact. Note that in the latter three rules, we impose the restriction that the label of the reference $l$ be bound by the current label and clearance, i.e., $l_{cur} \subseteq l \leadsto True$ and $l \subseteq c_{cur} \leadsto True$. This ensures that we both preserve the confidentiality of data in scope and avoid reading/modifying entities above the clearance. It is worth remarking that if we consider the current label $l_{cur}$ as the dynamic version of the $pc$, our restriction that the label of the reference be above the current label ($l_{cur} \subseteq l \leadsto True$) when writing to the reference is similar to the one imposed by (Pottier & Simonet, 2002). Finally, the rule $labelOf$, given by (LABELOFLIOREF), is self-explanatory, we do not discuss it further.

Fig. 15. Semantics for monadic LIO terms related to references.
This is in contrast with the original semantics of exceptions as presented in (Stefan et al., 2012b), where an explicit label was associated with every thrown exception. In comparison to the treatment of exceptions in (Stefan et al., 2012b; Hriţcu et al., 2013), the approach of this paper is considerably simpler.

5 Exception handling

Like references, exceptional control flow is common in real-world applications. As already noted, LIO provides support for throwing and catching exceptions. Code can throw an exception using the throwLIO function and catch exceptions using catchLIO:

\[
\begin{align*}
\text{throwLIO} &: (\text{Exception } e, \text{Label } \ell) \Rightarrow e \to \text{LIO } \ell \tau \\
\text{catchLIO} &: (\text{Exception } e, \text{Label } \ell) \Rightarrow \text{LIO } \ell \tau \to (e \to \text{LIO } \ell \tau) \to \text{LIO } \ell \tau
\end{align*}
\]

This API is identical to that of standard Haskell, except that it operates in the LIO monad. Moreover, the semantics for these functions are standard.\(^{13}\) Nevertheless, we must consider the implication on security when they are used in concert with other LIO library functions—in particular, toLabeled.

In Fig. 16, we formally extend values with exceptions $\xi$ and a new LIO constructor $\text{LIO}^\text{tcb}$, terms with the exception handling functions (throwLIO and catchLIO), and types with Exceptions. For simplicity, we only consider a single exception type.

Fig. 17 gives the exception-related reduction rules. Function throwLIO, as given by rule (THROWLIO), raises an exception by simply lifting the exception term $t$ into the LIO monad with constructor $\text{LIO}^\text{tcb}$. Indeed, the role of the $\text{LIO}^\text{tcb}$ constructor is to distinguish between exceptional and non-exceptional monadic control flow. Building on this, we add a new reduction rule for bind ($\gg>$) that propagates exceptions; as shown by (BINDEX)

---

\(^{13}\) This is in contrast with the original semantics of exceptions as presented in (Stefan et al., 2012b), where an explicit label was associated with every thrown exception. In comparison to the treatment of exceptions in (Stefan et al., 2012b; Hriţcu et al., 2013), the approach of this paper is considerably simpler.
rule, bind re-throws the exception if the term under evaluation reduced to an exceptional monadic term \((LIO^{TCB}_X \cdot t)\). Otherwise, it behaves as before, according to rule (BIND).

The semantics for \(catchLIO\) is also straightforward. Since throwing an exception depends on the information present in the lexical scope, \(catchLIO\) must retain the current label to reflect this fact; observe that all the \(catchLIO\) reduction rules in Fig. 17 leave the context intact. Rule (CATCH) specifies the case where the term does not raise an exception and reduces to a “normal” \(LIO\) value. Here, the value is simply returned. Rule (CATCHE) specifies the case where the term raises an exception. In this case, the exception handler \(t_2\) is applied to the exception \(t_1\). We note that our semantics are lazy in the exception value, much in the same way as Haskell; neither \(throwLIO\) nor \(catchLIO\) force the evaluation of the exception.

The reduction rules of Fig. 17 take the standard approach of propagating exceptions up the call stack until the nearest enclosing \(catchLIO\). Though necessary, this is not sufficient; without modifying \(toLabeled\), exceptions can be used to leak information. Consider the following function:

\[
condThrow :: Labeled \ell_2 \mathbb{Bool} \rightarrow LIO \ell_2 ()
\]

\[
condThrow secretBool \equiv \begin{aligned}
s & \leftarrow \text{unlabel} \ \text{secretBool} \\
\text{when} \ s & \left(\text{throwLIO} \ \xi\right)
\end{aligned}
\]

Suppose that \(condThrow\) is invoked with the current label \(Public\) and \(secretBool\) has label \(Secret\). Then, \(throwLIO\) raises exception \(\xi\) if the secret is \(True\); if the secret is \(False\) \(condThrow\) simply returns \(()\). This function alone cannot be used to leak the secret, since the current label at the end of \(condThrow\) is \(Secret\). However, by wrapping \(condThrow\) with \(toLabeled\), we can avoid raising the current label when the secret is \(False\) and thus leak the value into a public reference:

\[
\text{leakSecret} :: Labeled \ell_2 \mathbb{Bool} \rightarrow LIO \ell_2 \mathbb{Bool}
\]

\[
\text{leakSecret} \ \text{secretBool} \equiv \begin{aligned}
\text{-- Create public reference:} \\
\text{publicRef} & \leftarrow \text{newLIORef} \ \text{Public} \ \text{True} \\
\text{toLabeled} \ \text{Secret} & \leftarrow \text{catchLIO} \ (\begin{aligned}
\text{toLabeled} \ \text{Secret} & \leftarrow \text{condThrow} \ \text{secretBool} \\
\text{writeLIORef} \ \text{publicRef} \ \text{False} & \quad \text{-- Write only if no exception is thrown} \\
(\lambda \_ & \rightarrow \text{return} ()
\end{aligned})
\end{aligned}
\]

\[
\text{-- Read direct leak of secret:} \\
\text{readLIORef} \ \text{publicRef}
\]

Assume that this function is invoked with a \(Public\) current label. First, the function creates a public reference \(publicRef\) initialized to \(True\). Then, if the secret is \(True\), the exception thrown by \(condThrow\) escapes the innermost \(toLabeled\) block up to the \(catchLIO\), which invokes the handler. At this point the current label is \(Secret\), since \(condThrow\) raised the label to read the secret. However, the outer \(toLabeled\) restores the current label to \(Public\). This allows us to read the \(publicRef\), which is still \(True\). By contrast, if the secret is \(False\), \(condThrow\) simply returns \(()\); the enclosing \(toLabeled\) ensures that the current label remains \(Public\). At this point, we write \(False\) into the public reference. Finally, we again
read and return the reference contents. In both cases the returned value corresponds to the secret boolean.

This code illustrates that the standard propagation of exceptions up the call stack until reaching the nearest enclosing catchLIO is not sufficient. Hence, LIO propagates exceptions up to the nearest catchLIO or toLabeled. Intuitively, the correct semantics for toLabeled are as before with the added requirement that all exceptions be caught by it: regardless of how the computation enclosed by toLabeled terminates—with an exception or value—a Labeled value must always be returned.

Formally, we extend values with another Labeled constructor \( v ::= \cdots | \text{Labeled}^{\text{CB}} X t, \) that encodes the fact that \( t \) is an exception. The additional rule for toLabeled is given by (TO-LABELEX) in Fig. 18: if term \( t \) raises an exception (that is not caught) \( \text{LIO}^{\text{CB}} X t', \) we wrap the exception by the new Labeled constructor. When unlabeling such a labeled value, as given by (UNLABELEX), LIO simply propagates the exception. Of course, unlabel raises the current label, as to ensure that information from the point of the throw cannot be leaked. Finally, (LABELOF2) gives the additional rule for labelOf, which allows programs to inspect the label of Labeled values wrapping exceptions. Note that we do not allow code to distinguish between Labeled\(^{\text{CB}}\) and Labeled\(^{\text{CB}}\); doing so would allow for trivial leaks.

With these modifications in place, we highlight that the actions in leakSecret following the toLabeled block will always be executed, even if an exception is raised inside condThrow. Intuitively, we close the leak due to exception propagation by simply assuring that the execution of (possibly public) actions following a toLabeled block does not depend on the abnormal termination of a computation wrapped by toLabeled. In a similar manner, we can address leaks due to the timing and non-termination behavior of the enclosed computation (Stefan et al., 2012a).

### 5.1 Recovering from monitor failures

Our reduction rules given thus far in Fig. 2–18 do consider cases where label checks fail. Like other dynamic IFC systems (e.g., (Askarov & Sabelfeld, 2009; Sabelfeld & Russo, 2009; Austin & Flanagan, 2009; Austin & Flanagan, 2010; Devriese & Piessens, 2011)), this implies aborting the program execution when a monitor failure occurs. For practical
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systems, this approach is usually not convenient: we cannot halt the system when a λChair request handler is about to violate IFC.

As we previously alluded to, LIO and Breeze (Hriţcu et al., 2013) differ from most other dynamic IFC systems in using exceptions to encode monitor failures. For example, when the security conditions in rule (UNLABEL) are not met, we throw an exception:

\[
\text{UNLABEL} \quad \frac{\ell_\text{cur} \not\subseteq l \leadsto l' \quad \ell'_\text{cur} \subseteq c_\text{cur} \leadsto \text{False}}{
\ell_\text{cur}, c_\text{cur}, m \mid \text{unlabel (Labeled}^{\text{TCB}}_l l) \xrightarrow{0} \ell_\text{cur}, c_\text{cur}, m \mid \text{throwLIO} \xi_{\text{IFC}}}
\]

Here, \(\xi_{\text{IFC}}\) is simply an exception containing information about the failure. In the same way, we provide reduction rules dual to those in Fig. 10–18 that simply throw exceptions when a security condition is not met. We do not discuss these rules further since they are straightforward. The only interesting case is a particular failure of \(\text{toLabeled}\), given below.

\[
\text{toLabeled} \quad \frac{\ell_\text{cur} \subseteq l \leadsto \text{True} \quad l \subseteq c_\text{cur} \leadsto \text{True}}{
\ell_\text{cur}, c_\text{cur}, m \mid \text{toLabeled} l \xrightarrow{a+1} \ell'_\text{cur}, c'_\text{cur}, m' \mid \text{return (Labeled}^{\text{TCB}}_l l \xi_{\text{IFC}})}
\]

Here, the enclosed term \(t\) raises the current label \(l'_\text{cur}\) above the upper bound \(l\). By simply throwing an exception we would potentially be leaking information about data more sensitive than \(l'_\text{cur}\). (Malicious code can “throw” an exception by raising the current label above the upper-bound imposed by \(\text{toLabeled}\), reintroducing the attack from the previous section.) As mentioned before, \(\text{toLabeled}\) must return a labeled value. Therefore, we return a labeled value that contains an exception that encodes the monitor failure; at the point of \(\text{unlabel}\), this “delayed” exception is raised.

By encoding monitor failures with exceptions, as opposed to stopping the program, LIO allows untrusted code to catch exceptions and safely recover from attempted IFC violations. Consider, for instance, the following function that unlabels a \(\text{Labeled}\) value and returns a \(\text{Maybe}\) value to indicate the success or failure of the operation:

\[
\text{safeUnlabel} :: \text{Label} \ell \Rightarrow \text{Labeled} \ell \tau \rightarrow \text{LIO} \ell (\text{Maybe} \tau)
\]

\[
\text{safeUnlabel} lv = \text{catchLIO (do v ← unlabel lv -- Fails if labelOf lv \not\subseteq c_\text{cur} \leadsto \ast \text{False } return (Just v) )} \quad (\lambda \_ \rightarrow \text{return Nothing})
\]

If the label of \(lv\) is above the current clearance, the LIO \(\text{unlabel}\) throws an exception, which is handled by simply returning \(\text{Nothing}\). If the label of \(lv\) is below the clearance, the current label is raised and the unlabeled result is returned, wrapped by \(\text{Just}\) instead. More generally, and as discussed in (Hriţcu et al., 2013), this is a very useful feature: it not only treats code in an egalitarian fashion, but it allows us to integrate untrusted code in an application without having to worry that the code will halt the system by causing a monitor failure. Indeed, exceptions also allows us to prevent denial of service attacks of this form.
6 Security guarantees

In this section, we show that programs written in LIO satisfy non-interference and a form of discretionary access control (DAC). Informally, non-interference states that secret values cannot be leaked by LIO programs, while DAC ensures that computations cannot bypass the restrictions imposed by clearance to access or create arbitrary data. Before delving into the details of these security guarantees, we first highlight some notational difference with the previous sections and describe the extent of our mechanization in Coq.

Notation  To allow for incremental introduction of concepts, in the previous section we used \( LIO^{TCB} \) and \( LIO^{TCB}_X \) constructors to respectively denote non-exceptional and exceptional monadic LIO terms that have been executed to the point of containing no more side effects. In this section, we use a single constructor that additionally takes a boolean argument to indicate whether the value is an exception or not: term \( LIO^{TCB}_b t \) corresponds to \( LIO^{TCB} t \) if \( b = \text{true} \) and \( LIO^{TCB}_X t \) if \( b = \text{false} \). Similarly, we use \( \text{Labeled}^{TCB}_b \), with \( b \in \{\text{true}, \text{false}\} \), instead of the \( \text{Labeled}^{TCB} \) and \( \text{Labeled}^{TCB}_X \) constructors.

Mechanized proofs  We formalized a large subset of the calculus, described in Section 2, using the Coq theorem prover. The mechanized subset omits references and the reduction rules corresponding to monitor failures described in Section 5.1. Moreover, the Coq implementation uses a concrete four-point lattice similar to that shown in Fig. 3. For this subset, we mechanized the propositions, lemmas, theorems, and proofs given below; we distinguish the non-mechanized parts of the proofs with the symbol \( \triangleright \). We leave the extension to the full calculus with an abstract lattice to future work.

6.1 Non-interference

In this section, we prove that LIO satisfies non-interference using the term erasure technique from (Li & Zdancewic, 2010; Russo et al., 2008). Intuitively, the term erasure technique allows us to show that a program satisfies non-interference by showing that the behavior of the program with all the sensitive data (classified above \( l \)) “erased” cannot be distinguished by an attacker (at observation level \( l \)) from the behavior of the original program.

To model such programs, we extend our calculus and reduction rules with erased terms, denoted by a new non-terminal \( \bullet \), as follows:

\[
\begin{align*}
\text{HOLE} & \quad \text{HOLELIO} \\
\text{t ::= } & \cdots \mid \bullet \quad \vdash \bullet : \tau \\
& \bullet \leadsto \bullet \\
& \{\bullet, \bullet, \bullet | \bullet\} \xrightarrow{n} \{\bullet, \bullet, \bullet | \bullet\}
\end{align*}
\]

Intuitively, an erased term can have any type. Moreover, an erased term or configuration, the latter represented by \( \{\bullet, \bullet, \bullet | \bullet\} \), always reduces to itself. We use a meta-level erasure function \( \varepsilon_l(\cdot) \) to replace all terms more sensitive than the attacker’s observation level \( l \) with \( \bullet \). To an attacker, terms and configurations above their observation level appear as \( \bullet \); the new reduction rules also ensure that no information can be learned from the reduction of such terms (by effectively diverging).

Fig. 19 gives the definition of the erasure function for values, terms, memories, and configurations. For most values, the erasure function is simply the identity function, since most
values are not heterogeneously labeled. Similarly, for most terms, the function is simply applied homomorphically (e.g., \( \varepsilon_l(\text{if } \text{True} \text{ then } t_2 \text{ else } t_3) = \text{if } \text{True} \text{ then } \varepsilon_l(t_2) \text{ else } \varepsilon_l(t_3) \)).

There are only three interesting cases. First, when erasing a \( \text{Labeled}_{TCB} l_1 t \) value, we erase the term \( t_2 \) protected by label \( l_1 \) to \( \bullet \) when the label does not flow to \( l \); otherwise we simply apply the function homomorphically. Second, we aggressively erase values that are about to be labeled with \( \text{Label} \). We note that such aggressive erasure would not be correct for \( \text{toLabeled} \), which also returns a labeled value, since \( \text{toLabeled} \) takes a monadic \( \text{LIO} \) action that may produce side-effects observable to the attacker. Finally, we erase a whole configuration to \( \langle \bullet, \bullet, \bullet | \bullet \rangle \) when the current label is not below \( l \); this ensures that the attacker cannot observe anything about sensitive configurations.

The addition of \( \bullet \) and corresponding reduction rules completes our calculus and semantics definition. We now prove several general properties for this calculus, followed by two key properties needed for the non-interference theorem: simulation and determinacy of our monadic reduction relation and a new relation that erases sensitive terms.

Our first lemma states that values are in normal form, i.e., values do not reduce.

**Lemma 1 (Values do not reduce)**

- For any \( v \), there is no \( t \) such that \( v \leadsto t \).
- For any \( l_{\text{cur}}, c_{\text{cur}}, m, v, n \), there is no \( k \) such that \( \langle l_{\text{cur}}, c_{\text{cur}}, m | v \rangle \xrightarrow{n} k \).

**Proof**

The first case follows by induction on the pure term reduction relation. The second case follows by induction on the structure of \( v \).

Though straightforward, this lemma is helpful when distinguishing terms that terminate since, as in most sequential IFC calculi, our non-interference guarantee is termination insensitive, i.e., it only holds for terminating terms. And, recall that our calculus allows non-terminating terms with \( \text{fix} \).

The next proposition show that the erasure function is homomorphic over substitution and idempotent over terms, memories and configurations.

**Proposition 1 (Idempotence and distribution properties of the erasure function)**

1. Idempotent over terms: \( \varepsilon_l(t) = \varepsilon_l(\varepsilon_l(t)) \)
2. Idempotent over memory: \( \varepsilon_l(m) = \varepsilon_l(\varepsilon_l(m)) \)
3. Idempotent over configurations: \( \varepsilon_l(k) = \varepsilon_l(\varepsilon_l(k)) \)
4. Homomorphic over substitution: \( \varepsilon_l(\{ t_1 / x \} t_2) = \{ \varepsilon_l(t_1) / x \} \varepsilon_l(t_2) \)

Intuitively, the first three properties respectively state that multiple application of the erasure function does not affect the term, memory, or configuration once it has been erased. In other words, the erasure function should completely erase sensitive data encoded in a term.

The erasure function additionally distributes over the pure reduction relation.

**Proposition 2 (Erasure function distributes over the pure-term reduction relation)**

For any label \( l \), if \( t \xrightarrow{n} t' \) then \( \varepsilon_l(t) \xrightarrow{n} \varepsilon_l(t') \).

**Proof**

Straightforward induction on \( t \), using Lemma 1, and Proposition 1. \( \square \)

In other words, taking a step in the pure reduction and erasing the end term is the same as first erasing the term and taking a step. Intuitively this is stating that sensitive data does not affect the reduction of a pure term.

We now extend this intuition to simulation with a new reduction relation under which sensitive terms and configurations are erased. This new monadic-term reduction relation with erasure is defined as follows:

**Definition 1 (Reduction of pure and monadic terms with erasure)**

\[
\begin{align*}
    k & \xrightarrow{n} k' \\
    \Rightarrow k \xrightarrow{n \rightarrow_l} \varepsilon_l(k')
\end{align*}
\]

Configurations under this relations are evaluated in the same way as before, with the exception that, after one evaluation step, the erasure function is applied to the resulting configuration. In this manner, the relation guarantees that confidential data, i.e., data above level \( l \), is erased as soon as it is created.

Fig. 20 highlights the intuition behind the more general simulation result: erasing all sensitive data, i.e., data whose label is not below \( l \), and then taking a step in \( \xrightarrow{n \rightarrow_l} \) is the same as taking a step in \( \xrightarrow{n} \) and then erasing all the secret values in the resulting configuration. Observe that if configuration \( k \) leaks data labeled above \( l \) (such that it is observable at \( l \)), then erasing all sensitive data and taking a step in \( \xrightarrow{n \rightarrow_l} \) might not be the same as taking steps in \( \xrightarrow{n} \) and then erasing all the secret values in the resulting configuration—the data might have already been leaked.

First, we show that the current label after taking a step is always at least as restricting as the current label before taking the step.

**Proposition 3 (Monotonicity of the current label)**

If \( (l_{\text{cur}}, c_{\text{cur}}, m \mid t) \xrightarrow{n} (l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t') \) then \( l_{\text{cur}} \sqsubseteq l'_{\text{cur}} \).

**Proof**
Straightforward induction on \( t \), using the lattice-properties of labels (namely, reflexivity of \( \sqsubseteq \) and definition of \( \sqsubseteq \)).

This proposition not only reduces the number of cases we need to consider, but also reinforces our intuition that none of the LIO terms can lower the current label once sensitive data is incorporated in the context (and thereby allow for such data to be leaked). We note that since \textit{toLabeled} is defined in terms of a big-step it does not actually restore the current label; rather it executes a term in a separate context in a single step.

We now prove simulation of the monadic-term reduction relation. The proof follows by induction on the number of executed \textit{toLabeled} blocks, i.e., index \( n \) on the \( \stackrel{n}{\rightarrow} \) relation. These cases are further broken down into several simpler cases, according to the observational level of the attacker and current labels (before and after taking a step). To simplify presentation, these supporting statements are given in Appendix A.

\textbf{Lemma 2 (Single-step simulation without toLabeled)}

If \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \rightarrow \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle \) then \( \varepsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle) \rightarrow_1 \varepsilon_l(\langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle) \).

\textbf{Proof}

Straightforward case analysis on \( l_{\text{cur}} \subseteq l \) and \( l'_{\text{cur}} \subseteq l \). All cases follow directly from supporting Propositions 10, 11, and 12 given in Appendix A.

This base-case simulation corresponds to the scenario where no \textit{toLabeled} blocks are executed. The single-step simulation lemma for arbitrary terms follows by induction, using this lemma for the base case.

\textbf{Lemma 3 (Single-step simulation)}

If \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \stackrel{n}{\rightarrow} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle \) then \( \varepsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle) \rightarrow_1 \varepsilon_l(\langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle) \).

\textbf{Proof}

Straightforward case analysis on \( l_{\text{cur}} \subseteq l \) and \( l'_{\text{cur}} \subseteq l \) using Lemma 2 for the base case. The cases follow directly from the supporting propositions—Propositions 11, 12, and 14—given in Appendix A.

Having established the simulation between the standard reduction relation and the relation with erasure, we now solely need to show that the latter relation is deterministic to prove non-interference.

First, we show that the pure-term reduction relation is deterministic.

\textbf{Proposition 4 (Determinacy of pure-term reduction)}

If \( t \rightsquigarrow t' \) and \( t \rightsquigarrow t'' \) then \( t' = t'' \).

\textbf{Proof}

By induction on the pure-term reduction relation, using Lemma 1.

Since several reduction rules for the monadic-term reduction relation are given in terms of big-steps, we show that the big-step relation, i.e., relation wherein the end-terms are values, is deterministic:

\textbf{Proposition 5 (Determinacy of big-step monadic-term reduction)}

If \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \stackrel{n}{\rightarrow} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid \text{LIO}_{\text{TCB}}^{b'} t' \rangle \) and \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \rightarrow_1 \langle l''_{\text{cur}}, c''_{\text{cur}}, m'' \mid \text{LIO}_{\text{TCB}}^{b''} t'' \rangle \), then \( l'_{\text{cur}} = l''_{\text{cur}}, c'_{\text{cur}} = c''_{\text{cur}}, m' = m'', t' = t'', \) and \( b' = b'' \).
By induction on \( t \). Most cases follow by inversion of the first multi-step monadic-term reduction hypothesis. The \( \text{LIO} \), \( \text{return} \), and \( \text{throwLIO} \) cases further require the inversion of the second hypothesis.

This proposition is crucial to the non-interference theorem. Indeed, it can serve as a first sanity-check when extending the library with new primitives: adding \( \text{LIO} \) actions that are non-deterministic, such as \( \text{getTimeOfDay} \) would trivially break this statement. And, extending the system to consider a non-deterministic reduction relation is non-trivial. Indeed, it may require changing even the security condition (Zdancewic & Myers, 2003; Sabelfeld & Myers, 2003).

We now use these two propositions to show that the single-step monadic-term reduction relation is deterministic.

**Proposition 6 (Determinacy of monadic-term reduction)**

If \( k \xrightarrow{n} k' \) and \( k \xrightarrow{n} k'' \) then \( k' = k'' \).

**Proof**

By induction on the monadic-term reduction relation, using Proposition 4 and Lemma 1. We use Proposition 5 for the \( \text{BIND} \), \( \text{BINDEX} \), \( \text{TO\LABELED} \), \( \text{TO\LABELED\EX} \), \( \text{CATCH\LIO} \), and \( \text{CATCH\LIO\EX} \) cases.

From this, the determinacy of the relation with erasure follows in a straightforward way:

**Lemma 4 (Determinacy of monadic-term reduction with erasure)**

For any label \( l \), configurations \( k, k', \) and \( k'' \), and index \( n \), there exists an index \( n' \), such that if \( k \xrightarrow{n} l k' \) and \( k \xrightarrow{n'} l k'' \) then \( k' = k'' \).

**Proof**

By inversion of the hypotheses, using \( n \) as a witness, and Proposition 6.

Our proof uses \( n \) as a witness and thus shows that, if the number of executed \( \text{toLabeled} \) actions is the same, then the resulting configurations are equivalent. The statement of the lemma, however, allows for \( n' \) to differ from \( n \). This is because configuration \( k \) may be a secret computation or it may contain a secret subcomputation (in the form of a \( \text{toLabeled} \) block) that executes a different number of \( \text{toLabeled} \) actions according to a secret. Importantly, the end configurations will remain equivalent.

Before stating the non-interference theorem, we first define a \( \text{safe} \) function \( \varsigma \) to distinguish terms that are only composed of surface syntax. Fig. 21 gives the definition of this function for values, memories and configurations. For terms, we define \( \varsigma \) as the conjunction of its application to all the term components. Since the definition of \( \varsigma \) is straightforward, we only remark that our definition for memories is permissive in treating a non-empty memory \( m \) as safe when \( m \) only contains safe terms.

As in previous works on non-interference, we state non-interference as the preservation of \( l \)-equivalence, defined according to a syntactic equivalence relation \( \approx_l \).\(^{14}\) We define this

---

\(^{14}\) While considering syntactic \( l \)-equivalence is standard, a treatment of semantic \( l \)-equivalence would be an interesting research direction.
Applying Lemma 3 to the two hypotheses, we have:

Proof

Using $\epsilon$ to define homomorphically over the structure of the term.

$l$-equivalence relation as the equivalence kernel of the erasure function $\epsilon_l(\cdot)$ for configurations. That is, $k \approx_l k'$ iff $\epsilon_l(k) = \epsilon_l(k')$. Note that this equivalence relation precisely captures the power of an attacker: to an attacker at observation level $l$, two terms that are $l$-equivalent cannot be distinguished.

Theorem 1 (Non-interference)

For any label $l$, index $n$, and two configuration $k_1$ and $k_2$, such that $\xi(k_1)$ and $\xi(k_2)$, there exists an index $n'$, such that if $k_1 \approx_l k_2$, $k_1 \overset{n}{\rightarrow} \xi(k_1)$ and $k_2 \overset{n'}{\rightarrow} \xi(k_2)$, then $k_1 \approx_l k_2$.

Proof

Applying Lemma 3 to the two hypotheses, we have: $\epsilon_l(k_1) \overset{n}{\rightarrow} \epsilon_l(k_1')$ and $\epsilon_l(k_2) \overset{n'}{\rightarrow} \epsilon_l(k_2')$. From $k_1 \approx_l k_2$ and the definition of $\approx_l$, we trivially have $\epsilon_l(k_1) = \epsilon_l(k_2)$. Then, by Lemma 4, we have $\epsilon_l(k_1') = \epsilon_l(k_2')$. From the definition of $l$-equivalence, this is the same as $k_1' \approx_l k_2'$.

Our Coq proof uses types to eliminate degenerate cases, but this is not fundamental to the proof and we thus elide this detail.

The theorem states that if two configurations with possibly secret information, but indistinguishable to an attacker at level $l$, take a step, then the resulting configurations are also indistinguishable to the attacker. In other words, the attacker does not learn any sensitive information by observing configurations at lower sensitivity levels. Note, however, that the number of $\text{tolabeled}$ actions executed in each step may differ according to data the attacker cannot observe—we assume that the attacker cannot observe the index counts.

This non-interference statement is stronger than that considered in the conference version of this paper (Stefan et al., 2011b), which is stated in terms of a big-step. Specifically, this statement says that no information is leaked at any intermediate step, as opposed to solely stating that the result of two $l$-equivalent programs do not leak information. However, as in the conference version, this is a termination-insensitive result, i.e., we only make claims about the case where the configurations can each take a step and thus leaks due to non-termination are not captured. Indeed, a term that diverges in a sensitive context (a $\text{tolabeled}$ block) can be distinguished from one that terminates. Intuitively, if the attacker can even observe some timing information, such as the number of executed $\text{tolabeled}$ actions, she can learn secrets. Hence, by ensuring that no information from a sensitive subcomputation is visible to public contexts—this includes the termination behavior and number of steps taken by the sensitive computation—we can prove a much stronger property: termination-sensitive non-interference. Intuitively, this amounts to (1) executing each $\text{tolabeled}$ block in a separate thread and (2) ensuring that the forking of a
thread takes a single step such that that the public (parent) computation is not affected by the timing/termination behavior of the computation executing within the sensitive context. In (Stefan et al., 2012a), we modify LIO precisely in this way.

### 6.2 Discretionary access control and isolation

In this section, we show that LIO programs cannot write or allocate entities below the current label or read, write or allocate entities above their current clearance. Building on this, we then show how LIO can be used to isolate untrusted computations so as to ensure they can only access a particular part of memory and any faults are contained, i.e., faults in the untrusted code are not percolated into the outer context.

#### 6.2.1 Discretionary access control

In the previous section, we showed that the current label after taking a step is always at least as restricting as the current label before taking the step. The dual holds for clearance; the current clearance after taking a step is always at most as restricting as the current clearance before taking the step.

**Proposition 7 (Monotonicity of the current clearance)**

If \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle \) then \( c'_{\text{cur}} \sqsubseteq c_{\text{cur}} \).

**Proof**

By induction on \( t \), using the lattice-properties of labels (namely, reflexivity of \( \sqsubseteq \)) and the fact that only (LOWERCLEARANCE) modifies the clearance (for which the statement holds trivially).

This proposition states that the current clearance monotonically decreases as the program progresses. In other words, the program can give up access to certain entities as it progresses, but not conversely. This statement is the clearance equivalent of Proposition 3, which states that once a computation reads confidential data, it cannot lower its current label to write to entities less sensitive.

Before delving into our access control guarantees, we first define two store modifiers:

\[
\begin{align*}
l \preceq m & = \{ (a, \text{Labeled}^\text{CB} b \mid l') : (a, \text{Labeled}^\text{CB} b \mid l') \in m \text{ and } l \sqsubseteq l' \} \\
m \preceq l & = \{ (a, \text{Labeled}^\text{CB} b \mid l') : (a, \text{Labeled}^\text{CB} b \mid l') \in m \text{ and } l' \sqsubseteq l \}
\end{align*}
\]

\[
l_1 \preceq m \preceq l_2 = l_1 \preceq m \cap m \preceq l_2
\]

Symbol \( l \preceq m \) denotes the subset of \( m \) containing all the references whose label is above or equal to \( l \). Similarly, \( m \preceq l \) contains the references whose label is below or equal to \( l \). Operator \( l_1 \preceq m \preceq l_2 \) encompasses the subset of \( m \) containing all the reference whose labels are between the labels \( l_1 \) and \( l_2 \). Finally, we introduce the complement of the described subsets as \( l \preceq m, m \preceq l \), and \( l_1 \preceq m \preceq l_2 \), respectively.

---

15 When considering privileges, these access restrictions give the code containing the privilege the discretion to access certain entities below the current label and above the current clearance.
Lemma 5 (No write-access below current label)

Given a term $t$ and memory $m$, such that $\zeta(t)$ and $\zeta(m)$, if the term reduces to a value according to $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$, then $l_{\text{cur}} \preceq m = l_{\text{cur}} \preceq m'$.

Intuitively, this lemma states that the partitions, of the initial and final memory stores, that (may) contain references with labels below $l_{\text{cur}}$ are identical, i.e., the computation could not have modified or created references below $l_{\text{cur}}$. Note, however, that the lemma does not state that term $t$ cannot read from a reference below the current label. A corollary of this lemma states that any labeled values created by $t$ are labeled above $l_{\text{cur}}$.

A similar, though slightly stronger, access control statement holds for clearance.

Lemma 6 (No access above current clearance)

Given term $t$ and memory $m$, such that $\zeta(t)$ and $\zeta(m \preceq c_{\text{cur}})$, if the term reduces to a value according to $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$, then $m \preceq c_{\text{cur}} = m' \preceq c_{\text{cur}}$.

In other words, the partition of memory above the initial current clearance remains inaccessible throughout the program execution, i.e., the computation could not have modified or created references above $c_{\text{cur}}$. A corollary of this lemma states that any labeled values created by $t$ are labeled below $c_{\text{cur}}$. As shown in Appendix A, computations also cannot read data above the clearance; this allows us to execute a term $t$ with an alternative memory—one where references above the clearance are arbitrarily modified—without affecting its behavior.

From these two lemmas, we can further state that the current computation is restricted to modifying references whose labels are between the current label and clearance:

Proposition 8 (Memory writes bounded by current label and clearance)

Given term $t$ and memory $m$, such that $\zeta(t)$ and $\zeta(m \preceq c_{\text{cur}})$, if the term reduces to a value according to $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$, then $l_{\text{cur}} \preceq m \preceq c_{\text{cur}} = l_{\text{cur}} \preceq m' \preceq c_{\text{cur}}$.

Proof

Directly from Lemma 5 and 6. $\square$

6.2.2 Isolation

Using the above access control properties of LIO, we now show how terms can be executed in isolation. To this end, we first define an `isolate` function, similar to the `withClearance` of Section 3:

\[
\text{isolate} :: \text{Label} l \Rightarrow l \Rightarrow \text{LIO} l (\cdot) \Rightarrow \text{LIO} l (\cdot)
\]

\[
\text{isolate l c t = toLabeled c (lowerClearance c >> raiseLabel l >> t) >> return (\cdot)}
\]

\[
\text{where raiseLabel l = label l (\cdot) >> unlabel}
\]

This function executes a term $t$ in a context where the initial current label and clearance are $l$ and $c$, respectively. While simple, this isolation function can be used to ensure that the untrusted term $t$ can only modify a specific portion of memory and indeed, behave, as if it executes in a separate context:

Lemma 7 (Single term isolation)
If $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid \text{isolate } l c t \rangle \xrightarrow{n+1} \langle l_{\text{cur}}, c_{\text{cur}}, m' \mid \text{LIO}\text{^{\text{core}}} (\top) \rangle$, then $l \leq m \leq c = l \leq m' \leq c$, $m' = (l \leq m' \leq c) \cup (l \leq m \leq c)$, and $(l, c, m \mid t) \xrightarrow{n+1} \langle l', c', m' \mid \text{LIO}\text{^{\text{core}}} (\top) \rangle$.

Here, the memory equations simply state that term $t$ could only have modified the part of the memory store $m$ that is between $l$ and $c$. Regardless of whether $t$ terminates by raising the current label, lowering the current clearance, and/or throwing an exception, the isolate function ensures that this “fault” is not propagated to the outer computation.

Indeed, this can directly be used to address the poison pill attacks described in (Hriţcu et al., 2013). Unfortunately, like the non-interference theorem, this lemma assumes that term $t$ terminates.

By wrapping different terms with isolate and using disjoint labels for their corresponding current labels and clearances, we can guarantee that the terms will execute in isolation, on disjoint parts of the memory. Such a term isolation theorem, for two terms, is given below.

**Theorem 2 (Term isolation)**

Assume fresh(·) deterministically creates objects that are globally unique. Given safe terms $t_1$ and $t_2$, memory $m$, and labels $l_1, c_1, l_2, c_2$, bounded by $l_{\text{cur}}$ and $c_{\text{cur}}$, such that $l_1 \subseteq c_1$, $l_2 \subseteq c_2$, $l_1 \not\subseteq l_2$, $l_2 \not\subseteq l_1$, $c_1 \not\subseteq c_2$, and $c_2 \not\subseteq c_1$, if $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid \text{isolate } l_1 c_1 t_1 \gg \text{isolate } l_2 c_2 t_2 \rangle \xrightarrow{n+1} \langle l_{\text{cur}}, c_{\text{cur}}, m' \mid \text{LIO}\text{^{\text{core}}} (\top) \rangle$, then $\langle l_1, c_1, m \mid t_1 \rangle \xrightarrow{n_1} \langle l'_1, c'_1, m_1 \mid \text{LIO}\text{^{\text{core}}} (\top) \rangle$, $\langle l_2, c_2, m \mid t_2 \rangle \xrightarrow{n_2} \langle l'_2, c'_2, m_2 \mid \text{LIO}\text{^{\text{core}}} (\top) \rangle$, $n = (n_1 + 1) + (n_2 + 1)$, $m = m_1 \cap m_2$, and $m' = m_1 \cup m_2$.

Intuitively, the theorem states that the behavior of terms $t_1$ and $t_2$ (under the supplied context labels) is not affected by isolate function. Importantly, it also states that the two terms operate on disjoint parts of the memory—indeed the behavior of $t_2$ is the same as executing it with initial memory $m$, as opposed to $m_1$, the memory after term $t_1$ was executed. In the context of $\lambda$Chair, this isolation property is especially important since it allows us to ensure that requests running on behalf of different users run in isolation.

### 7 Related Work

Heintze and Riecke (Heintze & Riecke, 1998) consider security for lambda-calculus where lambda-terms are explicitly annotated with security labels, for a type-system that guarantees non-interference. One of the key ideas behind their work is to provide an operator that raises the security label of a term. This is similar to raising the current label when manipulating labeled values whose labels are above the current label. Indeed, this notion of a floating current label dates back to the High-Water-Mark security model (Landwehr, 1981) of the ADEPT-50 in the late 1960s, which was later adopted by Asbestos (Efstathopoulos et al., 2005), HiStar (Zeldovich et al., 2006), and Flume (Krohn et al., 2007) IFC Operating Systems.

Abadi et al. (Abadi et al., 1999) develop the dependency core calculus (DCC) based on a hierarchy of monads to guarantee non-interference. In their calculus, they define a monadic type that protects the confidentiality of pure values at different security levels. Our LIO and Labeled types serve a similar role. However, since LIO has the guarantee that code cannot create labeled values below the current label and or above the current clearance,
the *Labeled* type is not a monad—we must inspect the current label and clearance before a new labeled value can be created (e.g., by applying a function to the protected value). Nevertheless, we can use *unlabel* and *toLabeled* in the *LIO* monad to achieve the dynamic equivalent functionality of DCC’s (non-standard) typing rules for the bind operator. Tse and Zdancewic (Tse & Zdancewic, 2004) translate DCC to System F and show that non-interference can be stated using parametricity. Unfortunately, like DCC, they rely on a non-standard typing rule for bind—they provide several definitions for this operator and rely on GHC’s `UndecidableInstance` extension (which lifts type conditions of (Sulzmann et al., 2007)) to resolve the correct bind. Crary et al. (Crary et al., 2005) present a monadic calculus for non-interference for programs with mutable state. While inspired by these works, we do not take a domain-specific approach to extend the Haskell type system or modify the Haskell runtime; rather, we take a dynamic, label-polymorphic, and library approach to IFC. Importantly, our implementation does not rely on any non-standard constructs—this reduces the task of understanding IFC enforcement to understanding the LIO API.

Harrison and Hook show how to monadically encode abstract operating systems called *separation kernels* (Harrison, 2005). The idea behind this work is to first partition a program into multiple processes, each associated with a separate domain (label), running in isolation. Inter-process communication is allowed through a kernel that meditates the message exchange according to a security policy (e.g., non-interference). To formally reason about separation kernels, the authors use a monad-layering approach, modeling state with the *State* monad, concurrency with the *Resumption* monad, etc. This approach is orthogonal to our approach; we use monads in a trivial fashion and primarily as a way to implement the calculus semantics as a library. In (Stefan et al., 2012a), we describe the concurrent version of LIO, which unlike (Harrison, 2005), considers termination sensitive non-interference.

The seminal work by Li and Zdancewic (Li & Zdancewic, 2006) presents an implementation of information-flow security for Haskell. Instead of modifying the language runtime, they take a library-based approach by encoding IFC-constrained computations using arrows (Hughes, 2000) (a generalization of monads). This work was extended by Tsai et al. (Tsai et al., 2007) to consider concurrency and side-effecting computations. Russo et al. (Russo et al., 2008) show an alternative library-based approach that eliminates the need for arrows; they, instead, describe a monadic library that encodes static IFC. This library relies on monadic types to track information-flow in pure and side-effecting computations. Morgenstern and Licata (Morgenstern & Licata, 2010) extend this idea to implement an authorization- and IFC-aware programming language in Agda. However, and as is the case with many static systems (Sabelfeld & Russo, 2009), their library is less permissive. Nevertheless, this library is a closely related work. In particular, we note that the SecIO library (Russo et al., 2008) has functions that serve the static counterpart of some of the core LIO functions (e.g., like *unlabel*, they provide a function that maps pure labeled values into monadic computations; like *toLabeled*, they provide a function that allows safely writing to public entities after reading secret data).

Another closely related work is that of (Devriese & Piessens, 2011); this work uses monad transformers and parametrized monads (Atkey, 2006) to enforce non-interference, both dynamically and statically. Different from our work, they focus on modularity (separating IFC enforcement from underlying user API), using type-class level tricks. Unfor-
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unfortunately, like the work on separation kernels, this requires programmers to first partition their code to fit the new programming model, whereas the usage of LIO strives to be very close to Haskell’s existing IO libraries.

Hedin and Sabelfeld present a dynamic information-flow monitor for a core JavaScript with exceptions (Hedin & Sabelfeld, 2012). In their calculus, they associate a security level with every exception. This is similar to our approach in associating the current label with exceptions thrown by \texttt{throwLIO}. Their semantics diverge from standard JavaScript in disallowing public exceptions from being thrown in secret contexts and, to address this permissiveness issue, they provide a non-standard construct that can be used to upgrade the label of an exception. Unfortunately, IFC violations (which may arise when an upgrade is not performed) are fatal.

Our initial treatment of exceptions was presented in the unpublished manuscript (Stefan et al., 2012b). While the semantics are mostly the same as those presented in this paper, there are some subtle differences. In particular, in the original work, exceptions had an associated explicit label—the current label at the time of a \texttt{throwLIO}. At the time of a \texttt{catchLIO}, the current label is then raised to the join of the exception label and current label. Unfortunately, these semantics were largely complicated by the implementation. Specifically, the LIO monad was implemented as a state monad with IO as the base monad and the current label and clearance as the monad state. Since the monad state may change according to the computation control flow, it was necessary that exceptions carry the additional state information to ensure that the current label is not arbitrarily lowered. However, by removing this implementation consideration, we were able to simplify the semantics to those presented in this paper and also simplify the implementation—the key insight is that the current label and clearance are global to the computation and thus the state monad needs to only contain a reference to these labels. Indeed, this simplification reduced the complexity to the interaction of exceptions and \texttt{toLabeled}.

In parallel with our initial work on exceptions, Hritcu et al. presented the Breeze IFC language (Hritcu et al., 2013). Breeze explored the design space of IFC and exceptions. Not only do they consider various calculi with exceptions, but, like our work, also address the issue of treating IFC monitor failures as recoverable failures. We refer the interested reader to the Breeze paper for a very comprehensive comparison of Breeze and LIO. Here, we only remark that, like Breeze, we delay the propagation of exceptions raised in \texttt{toLabeled} blocks (in Breeze, these are called brackets). Indeed, our semantics for exceptions are very similar to their calculus $\lambda_{\text{throw+D}}$. Both of these calculi differ from our original presentation (Stefan et al., 2012b) in hiding exceptions raised in a \texttt{toLabeled} block where the current label is above the supplied upper bound, see rule (TO\_LABELED\_FAIL).

Different from most language-based IFC systems, LIO relies on the notion of clearance to restrict information leakage due to covert channels. Bell and La Padula (Bell & La Padula, 1976) formalized clearance as a bound on the current label of particular users’ processes. In the 1980s, clearance became a requirement for high-assurance secure systems purchased by the US Department of Defense (Department of Defense, 1985). HiStar (Zeldovich et al., 2006) re-cast clearance as a bound on the label of any resource created by the process (where raising a process’s label is but one means of creating a something with a higher label). We adopt HiStar’s more stringent notion of clearance, which prevents software
Sequential LIO

from copying data it cannot read and facilitates bounding the time during which possibly untrustworthy software can exploit covert channels.

8 Summary

We presented LIO, an IFC system that explores a new design point in language-based information flow security. LIO takes a coarse-grained labeling approach, inspired by IFC OSes. In particular, LIO only associates a single, mutable, label—the current label with all the values in context (lexical scope) and dictates how information flows to/from the context. Compared to typical language-based IFC systems, where labels are explicitly associated with values, this design approach is amendable to a fast, library implementation. But, to allow programmers to handle differently labeled data, LIO provides an abstract data type, Labeled, that encapsulates a term and its explicit label. (In a similar way we provide mutable labeled references.) Labeled values serve the dual purpose of addressing label creep—the raising of the current label as increasingly sensitive data is incorporated into the context—by encapsulating the result of sensitive sub-computation, as executed by toLabeled. Unlike other language-based work, our IFC system also implements clearance as a means for restricting the kinds of data a computation can read/write to; LIO relies on this form of discretionary access control to address covert channels: code cannot leak data it cannot read. Finally, LIO provides exception handling constructs which we additionally use to encode monitor failures (from which untrusted code can recover).

We proved several security theorems for LIO. First, we showed that LIO programs, which may perform complex side-effects (e.g., mutate variables and throw exceptions), satisfy non-interference, i.e., LIO programs satisfy data confidentiality and integrity. Second, we showed that clearance is a form discretionary access control. And, finally, we showed how, using incomparable labels and toLabeled, two terms can execute in isolation, operating on disjoint parts of memory.

We implemented LIO as a Haskell library, using Safe Haskell to ensure that untrusted code executes in the LIO monad, i.e., our IFC sub-language. To illustrate the expressiveness of LIO, we described the core of a conference review system, λChair, that uses IFC to enforce high-level security policies. In addition to λChair, we (and others) have used LIO to implement several other web applications, some of which are in production use. We found the library-based approach to be very effective, both in terms of deployment (at the time of this writing, the library has thousands of downloads) and design (the interface matured as a result of several iterations).

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A Detailed proofs

In this section, we provide expand the proof details for the results in Section 6.

Proposition 1 (Idempotence and distribution properties of the erasure function)

1. Idempotent over terms: $\epsilon(t) = \epsilon(\epsilon(t))$
2. Idempotent over memory $\epsilon^e$: $\epsilon^e(m) = \epsilon(\epsilon^e(m))$
3. Idempotent over configurations: $\epsilon^c(k) = \epsilon(\epsilon^c(k))$
4. Homomorphic over substitution: $\epsilon^c(\{ t_1 / x \} t_2) = \{ \epsilon^c(t_1) / x \} \epsilon^c(t_2)$

Proof

The first property follows by induction on term $t$; all cases follow trivially from the inversion of the induction hypothesis. The second and third properties follow from the definition of the erasure function for memories and configurations and first property. The fourth property follows by induction on $t_2$; most cases follow directly from the induction hypothesis and definition of substitution.

Since a number statements rely on several inversion and distribution properties for the erasure function, we give these below.
Proposition 9 (Inversion properties of the erasure function)

1. Labeled values:
   - If \( l_1 \not\sqsubseteq l \) then \( \text{Labeled}^\text{TCB}_b l_1 \bullet = \epsilon_l(\text{Labeled}^\text{TCB}_b l_1 t) \) for any \( t \).
   - If \( l_1 \sqsubseteq l \) then \( \text{Labeled}^\text{TCB}_b l_1 \epsilon_l(t) = \epsilon_l(\text{Labeled}^\text{TCB}_b l_1 t) \).

2. Monadic values: \( \text{LIO}^\text{TCB}_b \epsilon_l(t) = \epsilon_l(\text{LIO}^\text{TCB}_b t) \).

3. Configurations:
   - If \( l_{\text{cur}} \not\sqsubseteq l \) then \( \langle \bullet, \bullet, \bullet | \bullet \rangle = \epsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle) \) for any \( c_{\text{cur}}, m \) and \( t \).
   - If \( l_{\text{cur}} \sqsubseteq l \) then \( \langle l_{\text{cur}}, c_{\text{cur}}, \epsilon_l(m) | \epsilon_l(t) \rangle = \epsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle) \).

Proof

All properties follow directly from the definition of the erasure function. \( \square \)

This proposition states that, in certain cases, we can invert the application of the erasure function to labeled values, \( \text{LIO} \) values, and configurations.\(^{16} \)

Simulation

Our simulation lemma follows by induction on the number of executed \( \text{toLabeled} \) blocks. The two lemma, Lemma 2 and 3, rely on several supporting propositions. We give these below.

Our first base-case simulation proposition considers the case when both the starting and end configuration labels can flow to the attacker observation level. In other words, the current term \( t \) does not raise the current label (e.g., with \( \text{unlabel} \)) nor does it execute any \( \text{toLabeled} \) blocks.

Proposition 10

For any label \( l \), such that \( l_{\text{cur}} \not\sqsubseteq l \) and \( l'_{\text{cur}} \not\sqsubseteq l \), if \( \langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle \overset{0}{\rightarrow} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' | t' \rangle \) then \( \epsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle) \overset{0}{\rightarrow} \epsilon_l(\langle l'_{\text{cur}}, c'_{\text{cur}}, m' | t' \rangle) \)

Proof

By induction on \( t \). Most cases follow directly from inversion of the first \( 0 \longrightarrow \) reduction hypothesis or Lemma 1. The \( \gg \gg \gg \) and \( \text{catchLIO} \) cases follow from the definition of the single- and multi-step relations, using Propositions 9 and a supporting proposition (not given here) whose statement is the multi-step version of this proposition. The terms for which there is a context reduction rule (e.g., \( \text{label} \), \( \text{unlabel} \), etc.), we further rely on Proposition 2. \( \square \)

The next proposition considers the case when initial configuration cannot be observed by the attacker, i.e., the initial current label does not flow to the attacker label.

Proposition 11

For any label \( l \), such that \( l_{\text{cur}} \not\sqsubseteq l \), if \( \langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle \overset{n}{\rightarrow} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' | t' \rangle \) then it is also the case that \( \epsilon_l(\langle l_{\text{cur}}, c_{\text{cur}}, m | t \rangle) \overset{n}{\rightarrow} \epsilon_l(\langle l'_{\text{cur}}, c'_{\text{cur}}, m' | t' \rangle) \)

Proof

\(^{16} \) We note that, while we can prove inversion for all terms (and cases), we only need properties 1 and 2 to prove the more interesting simulation property.
We break the proof into two cases:

- Case $l_{\text{cur}} \subseteq l$: follows from Proposition 3.
- Case $l_{\text{cur}} \nsubseteq l$: follows trivially from the single-step reduction rule of an erased configuration, (HOLE).

Here, the simplicity of the proof allows us to consider the case where the number of executed $\text{toLabeled}$ blocks is any natural $n$.

The more interesting case—when the current label is raised by the current term $t$—is given below. As shown below, only $\text{unlabel}$ actually raises the current label, hence, we can directly consider the simulation for an arbitrary number $n$ of executed $\text{toLabeled}$ blocks. However, we must consider the case when $\text{unlabel}$ is executed as part of a bigger action (e.g., in $\gg=$ or $\text{catchLIO}$).

**Proposition 12**

For any label $l$, such that $l_{\text{cur}} \subseteq l$ and $l'_{\text{cur}} \nsubseteq l$, if $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$ then $\varepsilon_{l}(\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle) \xrightarrow{n} \varepsilon_{l}(\langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle)$

**Proof**

By induction on $t$. Most cases follow directly by inversion of the $\xrightarrow{n}$ reduction rule and $l_{\text{cur}} \subseteq l$ hypothesis. The remaining cases are:

- Case $\text{unlabel} t_1$: Breaks down into the three reduction rules for $\text{unlabel}$:
  - Case $(\text{unlabel} \text{CTX})$: Follows directly from the definition of the $\xrightarrow{n} \gamma$ reduction rule and rule $(\text{unlabel} \text{CTX})$, using Propositions 1 and 2.
  - Case $(\text{unlabel})$: Both sub-cases (where the label of the value being unlabel can and cannot flow to $l$) follow directly from the definition of the $\xrightarrow{n} \gamma$ reduction rule and rule $(\text{unlabel})$, using Propositions 1 and 9.
  - Case $(\text{unlabel} \text{EX})$: Same as the $(\text{unlabel})$ case, but using the definition of $(\text{unlabel} \text{EX})$ instead.
- Case $t_1 \gg= t_2$: Straight forward induction on $t_1$, using Propositions 1 and 9.
- Case $\text{catchLIO} t_1 t_2$: Straight forward induction on $t_1$.

These supporting statements are used to prove the base-case simulation, Lemma 2, where no $\text{toLabeled}$ blocks are executed. However, all but one of the above supporting propositions consider the more general case, where any number of $\text{toLabeled}$ blocks are executed. We need to extend Proposition 10 to arbitrary terms to prove the inductive case.

To do this, however, we must first show simulation for the big-step reduction relation holds (since $\text{toLabeled}$ is defined in terms of a big-step), if the starting and end current labels can flow to the attacker label.

**Proposition 13**

For any label $l$, such that $l_{\text{cur}} \subseteq l$, if $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid \text{LIO}^{\text{CB}}_{b} t' \rangle$ then $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid \varepsilon_{l}(t) \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid \text{LIO}^{\text{CB}}_{b} \varepsilon(t') \rangle$

**Proof**
By induction on $t$, most cases follow by inversion of the first $\frac{n}{n'}$ reduction hypothesis and the resulting $\frac{n}{n} \to$ hypothesis. This leaves us with the terms that reduce to LIO values: LIO, return, and throwLIO. The first follows by inversion and Lemma 1. The latter two follow directly from the definition of the $\frac{n}{n}\to$ and $\frac{n}{n'}\to$ reduction relations.

Using this proposition, the general version of Proposition 10 follows:

**Proposition 14**

For any label $l$, such that $l_{\text{cur}} \subseteq l$ and $l'_{\text{cur}} \subseteq l$, if $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$ then $\varepsilon(l_{\text{cur}}, c_{\text{cur}}, m \mid t) \xrightarrow{n} \varepsilon(l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t')$.

**Proof**

We break this into two cases:

- **Case $n = 0$**: Trivially from Lemma 2.

- **Case $n = n' + 1$**: By induction on $t$. Most cases follow trivially by inversion. The remaining cases are:
  - **Case $t_1 \gg t_2$**: By inversion we break this down into the two sub-cases corresponding to the reduction rules (BIND) and (BINDEX). Both cases follow directly from the definition of the $\frac{n}{n} \to$ reduction rule, using Propositions 1, 9, and 13.
  - **Case toLabeled $t_1 t_2$**: By inversion we break this down into the three sub-cases corresponding to the reduction rules:
    - **Case (toLabeledCTX)**: Trivially by inversion.
    - **Case (toLabeled)**: Both sub-cases (where the label of the result can and cannot flow to $l$) follow directly from definition of the $\frac{n}{n} \to$ reduction rule and rule (toLabeled), using Propositions 1, 9, and 13.
    - **Case (toLabeledEX)**: Like the (toLabeled) case, but using the definition of (toLabeledEX) instead.
  - **Case catchLIO $t_1 t_2$**: By inversion we have two cases corresponding to (catchLIO) and (catchLIOEX), both of which follow in the same way as the $\gg$ case.

Directly, the single-step simulation lemma, Lemma 3, for arbitrary terms follows.

**Discretionary access control and isolation**

First, we give the proof for Lemma 5, which states that the current computation cannot write to references below the current label:

**Lemma 5 (No write-access below current label)**

Given a term $t$ and memory $m$, such that $\zeta(t)$ and $\zeta(m)$, if the term reduces to a value according to $\langle l_{\text{cur}}, c_{\text{cur}}, m \mid t \rangle \xrightarrow{n} \langle l'_{\text{cur}}, c'_{\text{cur}}, m' \mid t' \rangle$, then $l_{\text{cur}} \preceq m = l_{\text{cur}} \preceq m'$.

**Proof**

Observe that $t$ can only modify $m$ by creating a new reference or writing to an existing reference with (NEWLIOREF) and (WRITELIOREF), respectively. Both of these rules require the label of the (potentially new) reference to be above $l_{\text{cur}}$. Hence we know that
the memory below the current label will remain unchanged if $t$ takes a single step. Using Proposition 3 we can directly extend this to an arbitrary number of steps. □

The somewhat dual statement, Lemma 6 states that the current computation cannot read or write to references above the current clearance (or create labeled values labeled as such):

**Lemma 6 (No access above current clearance)$^*$**

Given term $t$ and memory $m$, such that $\zeta(t)$ and $\zeta(m \preceq c_{\text{cur}})$, if the term reduces to a value according to $\langle \iota_{\text{cur}}, c_{\text{cur}}, m | t \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m' | t' \rangle$, then $m \preceq c_{\text{cur}} = m' \preceq c_{\text{cur}}$.

**Proof**

Observe that $t$ can only modify $m$ by creating a new reference or writing to an existing reference with (NEWLIOREF) and (WRITELIOREF), respectively. Both of these rules require the label of the (potentially new) reference to be below $c_{\text{cur}}$. Hence we know that the memory above the current clearance will remain unchanged if $t$ takes a single step. Using Proposition 3 we can directly extend this to an arbitrary number of steps. □

Indeed, since no memory above the current clearance can be accessed we can simply replace that part of the memory with arbitrary references:

**Proposition 15 (Reduction is independent of memory above clearance)$^*$**

If $\langle \iota_{\text{cur}}, c_{\text{cur}}, m_1 | t \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m'_1 | \text{LIO}^{\text{TCB}} t' \rangle$, $\zeta(t)$, and $m_1 \preceq c_{\text{cur}} = m_2 \preceq c_{\text{cur}}$, then $\langle \iota_{\text{cur}}, c_{\text{cur}}, m_2 | t \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m'_2 | \text{LIO}^{\text{TCB}} t' \rangle$ and $m'_1 \preceq c_{\text{cur}} = m'_2 \preceq c_{\text{cur}}$.

**Proof**

Follows in the same way as the proof for Lemma 6. □

Before delving into the term isolation proof we first give two supporting propositions. First, a straightforward property for bind:

**Proposition 16 (Term evaluation is oblivious to memory above clearance)$^*$**

The reductions $\langle \iota_{\text{cur}}, c_{\text{cur}}, m | t_1 \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m' | \text{LIO}^{\text{true}} \rangle$ and $\langle \iota_{\text{cur}}, c_{\text{cur}}, m | t_2 \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m' | \text{LIO}^{\text{true}} \rangle$ hold iff $\langle \iota_{\text{cur}}, c_{\text{cur}}, m | t_1 \gg t_2 \rangle \xrightarrow{n^*} \langle \iota'_{\text{cur}}, c'_{\text{cur}}, m' | \text{LIO}^{\text{true}} \rangle$ holds and $n = n_1 + n_2$.

**Proof**

Directly from definition of bind. □

Second, we give simple memory equivalence when store modifiers are used:

**Proposition 17 (Equivalence of memory subsets)$^*$**

For labels $l_1, c_1, l_2$, and $c_2$, such that $l_1 \subseteq c_1, l_2 \subseteq c_2, l_1 \not\subseteq l_2, l_2 \not\subseteq l_1, c_1 \not\subseteq c_2$, and $c_2 \not\subseteq c_1$, if $l_1 \preceq m \preceq c_1 = l_2 \preceq m' \preceq c_2$ then $l_2 \preceq m \preceq c_2 = l_2 \preceq m' \preceq c_2$.

**Proof**

Since the labels are incomparable, it is easy to show that $(l_2 \preceq m \preceq c_2) \subset (l_1 \preceq m \preceq c_1)$ and $(l_2 \preceq m' \preceq c_2) \subset (l_1 \preceq m' \preceq c_1)$, from which the statement trivially holds. □

From these, the term isolation theorem follows is a mostly straightforward way.

**Theorem 2 (Term isolation)$^*$**
Assume fresh(·) deterministically creates objects that are globally unique. Given safe terms \( t_1 \) and \( t_2 \), memory \( m \), and labels \( l_1, c_1, l_2, \) and \( c_2 \), bounded by \( l_{\text{cur}} \) and \( c_{\text{cur}} \), such that \( l_1 \lneq c_1 \), \( l_2 \lneq c_2 \), \( l_1 \lneq l_2 \), \( l_2 \lneq l_1 \), \( c_1 \lneq c_2 \), and \( c_2 \lneq c_1 \), if \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid \text{isolate } l_1 c_1 t_1 \rangle \)

isolate \( l_2 c_2 t_2 \)\( \xrightarrow{n_2} \langle l_{\text{cur}}, c_{\text{cur}}, m' \mid \text{LIO}^{\text{TCB}}(\cdot) \rangle \) then \( \langle l_1, c_1, m \mid t_1 \rangle \xrightarrow{m_1} \langle l_1', c_1', m_1 \mid \text{LIO}^{\text{TCB}}(\cdot) \rangle \),

\( \langle l_2, c_2, m \mid t_2 \rangle \xrightarrow{n_2} \langle l_2', c_2', m_2 \mid \text{LIO}^{\text{TCB}}(\cdot) \rangle \), \( n = (n_1 + 1) + (n_2 + 1) \), \( m = m_1 \cap m_2 \), and \( m' = m_1 \cup m_2 \).

**Proof**

From Proposition 16, we have \( \langle l_{\text{cur}}, c_{\text{cur}}, m \mid \text{isolate } l_1 c_1 t_1 \rangle \xrightarrow{n_1+1} \langle l_{\text{cur}}, c_{\text{cur}}, m_1 \mid \text{LIO}^{\text{TCB}}_{\text{true}}(\cdot) \rangle \) and \( \langle l_{\text{cur}}, c_{\text{cur}}, m_1 \mid \text{isolate } l_2 c_2 t_2 \rangle \xrightarrow{n_2+1} \langle l_{\text{cur}}, c_{\text{cur}}, m' \mid \text{LIO}^{\text{TCB}}_{\text{true}}(\cdot) \rangle \).

Applying Lemma 7 to the first reduction we have \( m_1 = (l_1 \leq m_1 \leq c_1) \cup (l_1 \leq m \leq c_1) \),

\( \langle l_1, c_1, m \mid t_1 \rangle \xrightarrow{n_1} \langle l_1', c_1', m_1 \mid \text{LIO}^{\text{TCB}}_{\text{true}}(\cdot) \rangle \), and \( l_1 \leq m \leq c_1 = l_1 \leq m_1 \leq c_1 \). From Proposition 17 and \( l_1 \leq m \leq c_1 = l_1 \leq m_1 \leq c_1 \) we have \( l_2 \leq m \leq c_2 = l_2 \leq m_1 \leq c_2 \).

Applying Lemma 7 to the second reduction we have \( m' = (l_2 \leq m' \leq c_2) \cup (l_2 \leq m_1 \leq c_2) \),

\( \langle l_2, c_2, m_1 \mid t_2 \rangle \xrightarrow{n_2} \langle l_2', c_2', m_1 \mid \text{LIO}^{\text{TCB}}_{\text{true}}(\cdot) \rangle \), and \( l_2 \leq m_1 \leq c_2 = l_2 \leq m' \leq c_2 \). From Proposition 17 and \( l_2 \leq m_1 \leq c_2 = l_2 \leq m' \leq c_2 \) we have \( l_1 \leq m_1 \leq c_1 = l_1 \leq m' \leq c_1 \).

Further applying Proposition 15 we have \( \langle l_2, c_2, m \mid t_2 \rangle \xrightarrow{n_2} \langle l_2', c_2', m_2 \mid \text{LIO}^{\text{TCB}}_{\text{true}}(\cdot) \rangle \) and \( m \leq c_2 = m_2 \leq c_2 \). From Proposition 8 we have \( l_2 \leq m \leq c_2 = l_2 \leq m_2 \leq c_2 \).

Finally, \( m = m_1 \cap m_2 \) and \( m' = m_1 \cup m_2 \) follow from simple equation manipulation. □