Exercise 9.
Let $G = (V, E)$ be an undirected graph with $m$ edges, and let $C$ be a set of three colors. A 3-coloring assigns to every node on $V$ one color from $C$ arbitrarily. A proper 3-coloring has the property that, for every edge $uv \in E$, the colors of $u$ and $v$ are distinct. Not every graph has a proper 3-coloring, and deciding if there is one is an NP-complete problem. We may still 3-color a graph as good as possible: We call an edge $uv \in E$ satisfied if the colors of $u$ and $v$ are distinct.

Devise an algorithm that satisfies at least $2m/3$ edges for sure. It can be a deterministic polynomial-time algorithm, or a randomized algorithm running in expected polynomial time. (Do not forget to prove that your algorithm has the desired properties.)

Exercise 10.
This is of interest for choosing good splitters: Let $S$ be a set of $n$ distinct elements where a total order is defined. (Think of a set of numbers). We choose three random elements from $S$ and take the median of these three. What is the probability that the returned element has rank $k$ (for every $k = 1, \ldots, n$)? Show and explain your calculation, not only the final result.