Advanced Algorithms 2014. Exercises 5-6

Exercise 5.
Given $m$ finite sets and a number $k$, we wish to select $k$ “representative” elements from each set, in such a way that all these $mk$ representatives are distinct. (Note that the given sets are in general not disjoint; this special case would be trivial.) More precisely, the problem is to find a solution or to verify that no solution can exist.

Is this an easy or hard problem? Give either a polynomial-time algorithm (with time bound) or a polynomial-time reduction from a known NP-complete problem.

Exercise 6.
Let $Q$ be a set of $n$ unit squares in a grid, that is, squares of side length 1 whose corners have integer coordinates. A domino is a $2 \times 1$ rectangle. The task is to place as many as possible dominos onto $Q$, under the following conditions: Every domino must cover two adjacent squares of $Q$, and the dominos must not overlap. However we may rotate the dominos, i.e., place them horizontally or vertically. As a “practical” motivation, imagine that you have to store boxes with side lengths $2 \times 1$ on a floor where many squares are already occupied unsystematically by other heavy boxes which are not easy to move. The given set $Q$ of squares on the floor is the available space.

An optimal placement of dominos can be computed in polynomial time. How?

Hint: Reduce the problem to some other problem known to be solvable in polynomial time. (Avoid the trap to provide some questionable greedy algorithm without optimality proof.)