Advanced Topics in Automata
Exercise 4

Submission: April 29, 2003 - Firm

Exercise

In this exercise we are going to study the properties of co-Büchi automata. As hinted from the name the co-Büchi accpetance condition is the dual of the Büchi condition. A run is accepting according to Büchi if it visits the set of accepting states infinitely often. A run is accepting according to co-Büchi if it visits the set of accepting states finitely often. We will see that co-Büchi automata exhibit some 'strange' behaviors.

Formally, a nondeterministic co-Büchi automaton on infinite words (NCW) is $A = \langle \Sigma, Q, Q_0, \delta, F \rangle$ where Σ is a finite alphabet, Q is a finite set of states, $Q_0 \subseteq Q$ is the set of initial states, $\delta : Q \times \Sigma \to 2^Q$ is the transition function, and $F \subseteq Q$ is the set of accepting states.

Given some word $w = w_0 w_1 \cdots \in \Sigma^\omega$, a run of A on w is an infinite sequence $r = q_0, q_1, \ldots \in Q^\omega$ such that $q_0 \in Q_0$ and forall i we have $q_{i+1} \in \delta(q_i, w_i)$. The run is accepting if $inf(r) \cap F = \emptyset$. A word w is accepted by an NCW A if there exists some accepting run of A on w. We denote the set of words accepted by A by $L(A)$.

A co-Büchi condition can be expressed as a Müller condition in an obvious way. In the previous exercise, when you proved that a Müller automaton can be converted into a Büchi automaton, you were supposed to use a construction that converts an NCW into an NBW. Check that you understand this claim (do not submit).

1. Show that NCW are not as strong as NBW. That is, find a language L and a Büchi automaton A such that $L(A) = L$. Show that there does not exist a NCW C such that $L(C) = L$.

2. Show that DCW and DBW are uncomparable. That is, there is a language L such that L can be recognized by a DCW and not by a DBW. There is a language L such that L can be recognized by a DBW and not by a DCW.

3. Show that NCW and DCW have the same expressive power. That is, for every NCW C there exists a DCW D such that $L(C) = L(D)$.

   Use the following construction. Given $C = \langle \Sigma, Q, Q_0, \delta, F \rangle$, construct the DCW $D = \langle \Sigma, 2^Q \times 2^Q, (Q_0, Q_0), \delta', 2^Q \times \{\emptyset\} \rangle$ where δ' is defined below.

$$\delta'((Q,S),\sigma) = \begin{cases} \{(T,T) \mid T = \delta(Q,\sigma)\} & S = \emptyset \\ \{(T,T') \mid T = \delta(Q,\sigma) \text{ and } T' = \delta(S,\sigma) \setminus F\} & S \neq \emptyset \end{cases}$$

4. An automaton is universal if we demand that in order for it to accept all the possible infinite runs on the word be accepting.

   Show that when considering as classes of languages all the following hold.

   - $UCW = NBW$ - that is, every language that is accepted by an NBW is accepted by a UCW and vice versa.
   - $UCW > NCW$ - that is, every language that is accepted by an NCW is accepted by a UCW. The opposite is not true.
   - $NBW > UBW$ - that is, every language that is accepted by an UBW is accepted by a NBW. The opposite is not true.

   Note that only the first includes some showing. The other two are conclusions from previous questions.

Food for thought

1. Give the best algorithm you can find for the membership problem. That is, given an NBW A and a word $w = \alpha\beta^\omega$ (where $\alpha, \beta \in \Sigma^*$), devise an algorithm that decides whether $w \in L(A)$.

   Analyze the time and space complexity of your algorithm.