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Examples and Results from a BSc-level Course on Domain Specific Languages of Mathematics

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Introduction

Domain-Specific Languages of Mathematics (DSLsofMath)

- Undergraduate course developed at Chalmers, taught since 2016.
- Goal: Encourage students to approach mathematical domains from a functional programming perspective.

Course focus

- Make functions and types explicit
- Explicit distinction between syntax and semantics
- Types as carriers of semantic information
- Organize the types and functions in DSLs
- Make variable binding and scope explicit

Example - The limit of a function

We say that $f(x)$ **approaches the limit** *L as x* **approaches** *a, and we write*

$$\lim_{x \to a} f(x) = L,$$

if the following condition is satisfied:
for every number $\epsilon > 0$ there exists a number $\delta > 0$, possibly depending on ϵ, such that if $0 < |x - a| < \delta$, then x belongs to the domain of f and

$$|f(x) - L| < \epsilon$$

- Adams & Essex, Calculus - A Complete Course

Limit of a function - continued

$$\lim_{x \to a} f(x) = L,$$

if

$$\forall \epsilon > 0$$

$$\exists \delta > 0$$

such that if

$$0 < |x - a| < \delta,$$

then

$$x \in Dom\, f \wedge |f(x) - L| < \epsilon$$

Limit of a function - continued

First attempt at translation:

$$lim\ a\ f\ L = \forall(\epsilon > 0)(\exists(\delta > 0)(P\ \epsilon\ \delta))$$

where

$$P\ \epsilon\ \delta = (0 < |x - a| < \delta) \Rightarrow (x \in Dom\, f \wedge |f(x) - L| < \epsilon))$$

Limit of a function - continued

Finally:

$$lim\ a\ f\ L = \forall(\epsilon > 0)(\exists(\delta > 0)(\forall x(P\ \epsilon\ \delta\ x)))$$

where

$$P\ \epsilon\ \delta\ x = (0 < |x - a| < \delta) \Rightarrow (x \in Dom\, f \wedge |f(x) - L| < \epsilon))$$

Course material

- A DSL for arithmetic expressions and complex numbers
- Logic and calculational proofs
- Types in Mathematics
- Compositional Semantics and Algebraic Structures
- Polynomials and Power Series
- Higher-order Derivatives and their Applications
- Matrix algebra and linear transformations
- Exponentials and Laplace

Course context

- Semi-compulsory course, spring of second year in CSE programme
- Students struggle with math-heavy courses in third year
- Students do well with (functional) programming
- Can a functional programming perspective help to clarify the mathematics?

Course results

- 2016: 28 students, pass rate: 58%
- 2017: 43 students, pass rate: 68%
- 2018: 39 students, pass rate: 89%

CSE program

	Fall	Spring
Year 1	Compulsory courses	Compulsory courses
Year 2	Compulsory courses	DSLsofMath OR ConcProg
Year 3	TSS + Control	...

Results in subsequent courses

	PASS	IN	OUT
TSS pass rate	77%	57%	36%
TSS mean grade	4.23	4.10	3.58
Control pass rate	68%	45%	40%
Control mean grade	3.91	3.88	3.35

Group sizes: PASS 34, IN 53, OUT 92 (145 in all)

Results in previous courses

	PASS	IN	OUT
Pass rate for first 3 semesters	97%	92%	86%
Mean grade for first 3 semesters	3.95	3.81	3.50
Math/physics pass rate	96%	91%	83%
Math/physics mean grade	4.01	3.84	3.55

Group sizes: PASS 34, IN 53, OUT 92 (145 in all)

Future work

- Working with earlier and later courses, can these ideas be useful in their curriculum?
- Better tool support in the course, proof systems?
- More rigorous empirical evaluation of course efficacy

Thanks!

`https://github.com/DSLsofMath/DSLsofMath`