DSLs of Mathematics, Theorems and Translations

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In this talk, we present some of the ideas behind the course on DSLs of Mathematics (DSLM), currently in preparation in Chalmers.

We view mathematics as a rich source of examples of DSLs. For example, the language of group theory, or the language of probability theory, embedded in that of measure theory. The idea that the various branches of mathematics are in fact DSLs embedded in the "general purpose language" of set theory was (even if not expressed in these words) the driving idea of the Bourbaki project, which exerted an enormous influence on present day mathematics.

In DSLM, we consequently develop this point of view, aiming to show computer science students that they can use the tools from software engineering and functional programming in order to deal with the classical continuous mathematics they encounter later in their studies.

In this talk, we'll start with the simple example of the standard development of a calculus of derivatives. This can be seen as a DSL whose semantics are given in terms of limits of real sequences. We can try to give alternative semantics to this language, in terms of complex numbers. This leads to the notion of holomorphic function, and to an essentially different calculus than in the real case.

Our second example is that of extending the language of polynomials to power series. This DSL can also be interpreted in various domains: real numbers, complex numbers, or intervals.

In the case of complex numbers, a fundamental theorem creates a bridge between the DSL of derivatives and that of power series, through the identity of holomorphic and (regular) analytic functions. This leads to the discussion of translation between DSLs, an aspect which is fundamental in mathematics, but has been somewhat neglected by computer science. Thus, we believe that a closer examination of the DSLs of mathematics can also be relevant for practical software engineering.

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