Simple and Compositional Reification of Monadic Embedded Languages

Functional Pearl

Josef Svenningsson Bo Joel Svensson
Chalmers University of Technology
{josefs,joels}@chalmers.se

Abstract

When writing embedded domain specific languages in Haskell, it is often convenient to be able to make an instance of the `Monad` class to take advantage of the do-notation and the extensive monad libraries. Commonly it is desirable to compile such languages rather than just interpret them. This introduces the problem of monad *reification*, i.e. observing the structure of the monadic computation. We present a solution to the monad reification problem and illustrate it with a small robot control language. Monad reification is not new but the novelty of our approach is in its directness, simplicity and compositionality.

Categories and Subject Descriptors D.3.2 [*Programming Languages*]: Language Classifications—Applicative (functional) languages; D.3.4 [*Programming Languages*]: Processors—Code generation

Keywords Compiling, embedded language, reification, monads

1. Introduction

Benny is a computer science student working in a project involving programming robots in a low-level imperative language. However, Benny has a budding interest in functional programming using Haskell and has read "The Haskell school of expression" [6]. He gets the idea to implement a language for robot control embedded in Haskell. Benny realises that the capabilities of the robot hardware are very similar to those of the robots that Hudak evaluates graphically in a grid world. There is however a very important difference; the embedded language needs to be compiled into some form that is understood by the robot.

Guided by the capabilities of the target robot, Benny designs an API for robot programming. The robot can perform operations such as *move* that steps the robot forward and *turn* left and right. The robot also has the capability to execute program loops and conditionals and it has a forward facing `sensor` with which it can query the world.

ICFP '13, September 25–27,2013, Boston, MA, USA.

ACM 978-1-4503-2326-0/13/09. . . $15.00.
http://dx.doi.org/10.1145/2500365.2500611

The API that Benny designs is simple:

```
move      :: Program ()
turnLeft  :: Program ()
turnRight :: Program ()
sensor    :: Program Bool
cond      :: Program Bool
             -> Program ()
             -> Program ()
             -> Program ()
while     :: Program Bool
             -> Program ()
             -> Program ()
```

Now, Benny wants to express robot programs using the Haskell *do* notation. The motivation behind this is the imperative look and feel of the operations he identified and the potential to use all the control structures in the *Control.Monad* library. For this, a `Monad` instance is needed.

To be able to create a first-order representation of a computation described using monads, Benny needs to solve the problem of *reifying* monads. This is the story of how Benny came up with a particularly simple solution to this problem.

1.1 Example programs

Benny is quite happy with his language design and decides to write some programs to try it out.

The first program is called `sMove`, which implements a safe move operation. This operation can be executed by the robot even though it is facing an obstacle, without risk of harming robot or obstacle.

```
sMove :: Program ()
sMove = cond sensor turnRight move
```

The second program moves the robot forward until it stands directly in front of an obstacle such as a wall.

```
moveToWall :: Program ()
moveToWall = while ((liftM not) sensor) move
```

Based on these examples, Benny is quite satisfied with the design of his language and turns to implementing it.

1.2 Implementation and data structures

Enthused by the prospect of compiling these programs to the target language and seeing some robot action, Benny goes to work on the data structures.

Since the language is going to be compiled, Benny realises that the booleans in his language cannot be regular Haskell booleans. The booleans needs to be replaced by boolean typed expressions.

```
type Name = String

data BoolE = Lit Bool
           | Var Name
           | (:||:) BoolE BoolE
           | (:&&:)  BoolE BoolE
           | Not BoolE
```

Following this, the operations that should go into the `Program` data type feel straightforward.

```
data Program a where
  Move      :: Program ()
  TurnLeft  :: Program ()
  TurnRight :: Program ()
  Sensor    :: Program BoolE
  Cond      :: Program BoolE
               -> Program ()
               -> Program ()
  While     :: Program BoolE
               -> Program ()
               -> Program ()
```

Benny continues by naively adding constructors for `Return` and `Bind` to the `Program` data type.

```
data Program a where
  ...
  Return :: a -> Program a
  Bind   :: Program a
            -> (a -> Program b)
            -> Program b
```

Then he writes down the `Monad` instance.

```
instance Monad Program where
  return = Return
  (>>=)  = Bind
```

Proud of his accomplishments, Benny sends his Haskell module in an email to Prof. Björn. Benny knows that Björn is teaching an introductory Haskell course and should be able to provide feedback.

1.3 Problem statement

Meanwhile in Professor Björn's Office

Prof. Björn notices an email from Benny in his inbox and opens it.

> Dear Professor Björn
>
> I am CS student working in a robot control project. Usually, we program our robots in C but I have developed an interest in Haskell programming and thought it'd be natural to try implementing an EDSL. I know you teach an FP course and thought I would ask for your input. Attached to this mail is a file containing an outline of the data types I want to use. Does this look sensible to you?
>
> Thank you
> Benny

Prof. Björn opens the attachment and takes a look at the data type. He particularly notices the constructors `Return` and `Bind` and the `Monad` instance. He shakes his head at Benny's naiveté and writes an email back:

> Hello Benny
>
> I'm afraid your implementation of the monadic primitives can never work. The constructor `Return` can take any arbitrary value that has nothing to do with the language you're designing. These values may be strings, binary trees or higher-order functions from zygomorphisms to histomorphisms. The same problem goes for `Bind`; it has to be able to handle arbitrary values. It cannot possibly work! This is a very difficult problem and such a naive solution is bound to fail.
>
> Prof. Björn
> Dept. of Computer Science and Engineering
> Chalmers University of Technology

While waiting for feedback from Björn, Benny has carried on trying to implement a compiler for his language.

When Björn's email does arrive, Benny is confused. He feels he has already managed to compile the `Program` data type into a representation closer to that which is executed by the robot hardware.

He writes an apprehensive email back to Prof. Björn.

> Hello Prof. Björn
>
> Thanks for your feedback. But I think it does work! Attached to this email you find a file containing my attempt at compiling the `Program` data type into a representation closer to that which is executed by our robots.
>
> I have also attached two example programs that you can compile and then run in the graphical simulator (see figure 1)
>
> Benny

2. Compilation of the monadic robot EDSL

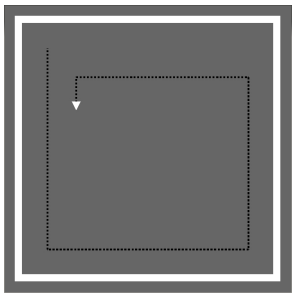
Björn receives Benny's latest email

Björn looks through the `Compiler` module and finds a data type describing a first-order representation of the robot language.

```
data Prg = PMove
         | PTurnRight
         | PTurnLeft
         | PSensor Name
         | PCond BoolE Prg Prg
         | PWhile Name Prg Prg
         | PSeq Prg Prg
         | PSkip
         | PAssign Name BoolE
```

From the `Prg` data type, Björn presumes that the `PSeq` constructor will be the result of compiling `Bind` and that `PSensor` binds a variable which can be used in `PAssign` and `PWhile`. The conclusion is that the representation looks sensible, but he is very interested in seeing the `Program a -> Prg` transformation. The compile function assumes a splittable name supply with the interface described in figure 2.

```
spiralIn :: Int -> Program ()
spiralIn 0 = return ()
spiralIn n = do
  replicateM_ 2 $ do
    replicateM_ n move
    turnLeft
  spiralIn (n-1)
```



```
followWall :: Program ()
followWall =
  while (return true) $
    cond checkLeft sMove $
             do turnLeft
                move

checkLeft :: Program BoolE
checkLeft = do
  TurnLeft
  s <- Sensor
  TurnRight
  return s
```

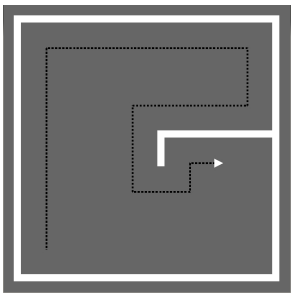


Figure 1. The `spiralIn` and `followWall` programs sent from Benny to Björn in an email

```
newNameSupply :: NameSupply
split2 :: NameSupply ->
          (NameSupply,NameSupply)
split3 :: NameSupply ->
          (NameSupply,NameSupply,NameSupply)
supplyValue :: NameSupply -> Int
```

Figure 2. The interface of the splittable name supply used in the implementation of the compile functions

```
runCompile :: Program a -> Prg
runCompile prg = snd $ compile s prg
  where
    s = newNameSupply

compile :: NameSupply -> Program a -> (a, Prg)
compile s Move = ((),PMove)
compile s TurnRight = ((),PTurnRight)
compile s TurnLeft  = ((),PTurnLeft)
compile s Sensor    = (Var nom,PSensor nom)
  where
    v   = supplyValue s
    nom = "v" ++ show v
compile s (Cond b p1 p2) =
    ((),bp `PSeq` PCond b' p1' p2')
  where
    (s1,s2,s3) = split3 s
    (b',bp)    = compile s1 b
    (a1,p1')   = compile s2 p1
    (a2,p2')   = compile s3 p2
compile s (While wp prg) = ((),PWhile nom nwp prg')
  where
    (s1,s2,s3) = split3 s
    (b,wp')    = compile s1 wp
    (c,prg')   = compile s2 prg
    nom        = "v" ++ (show $ supplyValue s3)
    nwp        = (wp' `PSeq` PAssign nom b)
compile s (Return a) = (a,PSkip)
compile s (Bind pa f) = (b, prg1 `PSeq` prg2)
  where
    (s1,s2)  = split2 s
    (a,prg1) = compile s1 pa
    (b,prg2) = compile s2 (f a)
```

Björn studies the code in disbelief, saves the module and tries it out on some examples. The code does indeed seem to work and Björn's disbelief is replaced with enthusiasm; this appears to be a simple and convenient way to reify monads.

Björn sends an email to Benny, inviting him to a meeting.

3. Meeting in Björn's office

BJÖRN: Hello, come in.

BENNY: Thanks.

BJÖRN: So, about this robot language! You gave me a bit of a surprise there. I was entirely sure that what you did was impossible.

BENNY: Oh, But I just did the first thing that came to mind. There was no deep thought behind it.

BJÖRN: The problem is [*Björn approaches his whiteboard*] when you try to reify the bind constructor of your representation.

```
Bind :: Program a -> (a -> Program b) -> Program b
```

BJÖRN: You need to come up with an element of type `a` to pass to the function.

BENNY: I didn't realise it was problem.

BJÖRN: But it is! Your solution is that you are careful about the return types of all operations in your language. They are either of unit type or some type which can be guaranteed to be reified. [*Björn scribbles on his whiteboard*]

```
Move :: Program ()

Sensor :: Program BoolE
```

BJÖRN: This is different from how such constructs are normally implemented. If I had implemented the language I would have given `Sensor` the type `Program Bool` so that I could write an evaluator of type `Program a -> a`, or perhaps `Program a -> M a`. But then adding a monad to the language and reifying it becomes much harder.

BENNY: Yes, now that you mention it, I have seen that style used for defining DSLs.

BJÖRN: Having constructors like `Sensor` return `BoolE` instead of `Bool` is a crucial part of why your method works. It allows the `compile` function to be written in such a way that it can generate the return values statically and pass them as arguments to `Bind`. In a sense you're doing evaluation at the same time as compilation. There's just enough evaluation to remove the monadic constructs. It's a very neat trick!

BENNY: Thank you.

BJÖRN: Another way to think about your trick is that you are using a writer monad to evaluate your language and produce the first-order syntax tree as a side effect. Then you could have used a state monad transformer on top of that for the fresh name supply. But that's a stylistic choice.

BENNY: You seem to be making a big deal out of this. I just did what I thought was the most straightforward thing to do.

3.1 Related work

BJÖRN: Others have solved this problem before but I have never seen a solution as simple as yours and was quite surprised that it works. For example, in this paper [*Björn shows Benny the paper [7]*], the authors use a continuation monad to be able to reify the monadic constructs of their language. If they

were to implement your robot language they would use the same `Program` data type as you do. However, they would not expose that to the user of the language. Instead they would create a type like this:

```
data P a =
  P (forall r. ((a -> Program r) -> Program r))
```

BJÖRN: The type `P` is a continuation monad so the monad instance comes naturally. Operations on `P` are defined using `Bind` like this:

```
while :: P BoolE -> P () -> P ()
while c b =
  P (\k -> While (runP c) (runP b) `Bind` k)

cond :: BoolE -> P () -> P () -> P ()
cond b t e =
  P (\k -> Cond b (runP t) (runP e) `Bind` k)

runP :: P a -> Program a
runP (P f) = f (\a -> Return a)
```

BENNY: I see. Continuations are quite magical to me. I could never have come up with that technique. How do they deal with transforming higher-order programs to first-order programs?

BJÖRN: That's a good question. The transformation to first order programs is dealt with by a library called Syntactic. It is described in a separate paper [*Björn pulls out the paper [2] from a pile of papers*]. It's hard to do an apples to apples comparison between Syntactic and your technique. Syntactic solves a much bigger problem than what you're doing so it is naturally more complicated.

BENNY: Ok.

BJÖRN: There are also the papers [5, 8] which solves the problem in a manner that is more similar to yours. They also have explicit constructors `Bind` and `Return` but give them a slightly different type which guarantees that all applications of `Bind` are *normalised*, so that all `Bind`s are right-associated. Unexpectedly, this allows them to make instances of the `Monad` class but at the same time constrain the arguments of `Bind`. Their technique is more complicated than yours, but just as with the case of Syntactic, they solve a more general problem.

However, there is one particular thing I like about your technique.

BENNY: What is that?

BJÖRN: Your technique is compositional.

3.2 Compositionality

BENNY: What does it mean that my technique is compositional?

BJÖRN: The compile function is compositional because there is one case for each constructor and each case deals with exactly one constructor. In particular, the constructors `Return` and `Bind` are handled completely separately from all other language constructs.

BENNY: And that is good ?

BJÖRN: Absolutely! Compositional definitions are nice because they imply that there is no weird semantical interaction between the constructs as they are defined independently. But it also means that it should be possible to factor out the constructors `Return` and `Bind` into a data type of their own. Then, it could be combined with other data types using techniques like Data Types á la Carte [9] or CompData [3]. This way, a language can be designed piece by piece.You can then select the set of pieces required for a particular task or that suits a particular brand of robots.

BENNY: Oh! Interesting.

3.3 The Monad laws

BJÖRN: There is still one problem with your method though.

BENNY: What's that?

BJÖRN: When we make instances of the monad class we expect certain laws to hold. [*Björn writes the laws on his whiteboard*]

```
m >>= return    = m
return a >>= f  = f a
(m >>= f) >>= g = m >>= \a -> f a >>= g
```

BJÖRN: These laws clearly don't hold for your instance. Take the first law for instance. The left hand side will have extra `Bind` and `Return` constructors compared to the right hand side.

BENNY: Hmmm. I hadn't really thought about that. But adding an extra return at the end of a computation shouldn't make a difference in my implementation.

BJÖRN: So, you're saying that your implementation actually obeys the first monad law?

BENNY: Well, at least when I run my programs I will never see any difference between the left hand side and the right hand side.

BJÖRN: Aha, so what you're saying is that if we compare the semantics of programs rather than comparing the programs themselves then we get some useful laws. [*Björn scribbles some new laws on the whiteboard*]

```
eval (m >>= return)    = eval m
eval (return a >>= f)  = eval (f a)
eval ((m >>= f) >>= g) = eval (m >>= \a -> f a >>= g)
```

BJÖRN: These laws are morally the same as the monad laws, especially if we don't let the user of the robot language ever compare terms in the language.

BENNY: Yes, that captures my intuition very well.

BJÖRN: Ok, good. Can you prove these equations?

BENNY: No, I don't have any experience proving programs correct.

BJÖRN: Well, it shouldn't be that difficult. Let me see what I can come up with.

[*Björn scribbles frenetically on a piece of paper for a couple of minutes.*]

Aha! Your technique is quite general. It can reify monads into any kind of structure which is a monoid. Return translates into the monoid unit and bind translates into the monoid operation.

For example, in your compilation function it is important that the semantics of `PSkip` is the identity of the semantics of `PSeq` and that `PSeq` is associative.

BENNY: Ok, that sounds good. I take that as meaning the method is quite general?

BJÖRN: Yes, requiring a monoid is a very mild restriction.

Look at the time! This was interesting, I got quite carried away. We must round off but please get back to me if you make any more progress on your robot language.

BENNY: Thanks very much for your time. Bye!

BJÖRN: Thank you.

4. Composing reifiable monadic languages

At Benny's computer

Benny was really intrigued by the idea of compositionally building embedded languages and after having read the papers Björn showed him he decides to try his own approach to the problem.

```
data (e1 :+: e2) x a
  = InjL (e1 x a) | InjR (e2 x a)
infixr :+:

class sub :<: sup where
  inj :: sub x a  -> sup x a

instance f :<: f where
  inj = id

instance (f :<: (f :+: g)) where
   inj = InjL

instance (f :<: h) => (f :<: (g :+: h)) where
  inj = InjR . inj
```

Figure 3. Data types from CompData for composing languages.

Inspired by CompData he starts out by adding the code in figure 3 to his file. The data type `:+:` is used for composing languages, and the `:<:` typeclass provides coercions so that the programmer doesn't have to worry about using the right sequence of the `InjL` and `InjR` constructors to inject terms into the composed language.

Benny then starts to add data types for the different language constructs in his robotic language. The different operations for the robot are pleasingly easy to add.

```
data MoveOp x a where
  Move :: MoveOp x ()

data TurnOp x a where
  TurnLeft  :: TurnOp x ()
  TurnRight :: TurnOp x ()

data SensorOp x a where
  Sensor :: SensorOp x BoolE

data CondOp x a where
  Cond :: x BoolE -> x () -> x () -> CondOp x ()

data WhileOp x a where
  While :: x BoolE -> x () -> WhileOp x ()
```

In order to test out these definitions, Benny next turns to implementing a compiler. He realises that the compiler now needs to be implemented as a class with one instance per compilable sublanguage.

```
runCompile :: Compile f => f a -> Prg
runCompile prg = snd $ compile s prg
  where s = newNameSupply

class Compile f where
  compile :: NameSupply -> f a -> (a, Prg)

instance Compile (MoveOp x) where
  compile _ Move = ((), PMove)

instance Compile (TurnOp x) where
  compile _ TurnLeft = ((), PTurnLeft)
  compile _ TurnRight = ((), PTurnRight)

instance Compile (SensorOp x) where
  compile s Sensor = (Var nom, PSensor nom)
    where v   = supplyValue s
          nom = "v" ++ show v
```

```
instance Compile x => Compile (CondOp x) where
  compile s (Cond b p1 p2) =
    ((),bp `PSeq` PCond b' p1' p2')
    where (s1,s2,s3) = split3 s
          (b',bp)    = compile s1 b
          (a1,p1')   = compile s2 p1
          (a2,p2')   = compile s3 p2

instance Compile x => Compile (WhileOp x) where
  compile s (While wp p) = ((),PWhile nom nwp p')
    where (s1,s2,s3) = split3 s
          (b,wp')    = compile s1 wp
          (c,p')     = compile s2 p
          nom        = "v" ++ (show $ supplyValue s3)
          nwp        = wp' `PSeq` PAssign nom b

instance (Compile (e1 f), Compile (e2 f))
  => Compile ((e1 :+: e2) f) where
  compile s (InjL a) = compile s a
  compile s (InjR a) = compile s a
```

But when Benny tries to add the monadic operations he finds that they are quite resistant to a compositional treatment. After much struggle he comes up with the following data type definition:

```
data Mops f x a where
  Oper   :: f x a -> Mops f x a
  Return :: a -> Mops f x a
  Bind   :: x a -> (a -> x b) -> Mops f x b
```

The recursion is provided by another data type that Benny calls `MonadExp`.

```
data MonadExp f a = In (Mops f (MonadExp f) a)
```

Benny thinks of the `MonadExp` data type as a representation of a monadic language parameterised over operations `Oper` that are constructed using the constructors of some type `f`. The monad instance for this data type is only slightly more complicated compared to the earlier, non-compositional setting.

```
instance Monad (MonadExp f) where
  return = In . Return
  (>>=) a f = In (Bind a f)
```

Benny can now write the type of his robotic language, built from independent pieces. It is noteworthy that the monadic expressions have to be added on top of all the other language constructs.

```
type Robot = MonadExp (MoveOp :+: TurnOp  :+:
                       CondOp :+: WhileOp :+:
                       SensorOp)
```

Given the definition of his language Benny can now write an injection function which helps writing smart constructors for all the different robot operations.

```
inject :: (sub :<: f)
       => sub (MonadExp f) a
       -> MonadExp f a
inject a = In $ Oper $ inj $ a

move :: (MoveOp :<: f) => MonadExp f ()
move =  inject Move

turnL :: (TurnOp :<: f) => MonadExp f ()
turnL = inject TurnLeft

turnR :: (TurnOp :<: f) => MonadExp f ()
turnR = inject TurnRight

sensor :: (SensorOp :<: f) => MonadExp f BoolE
sensor = inject Sensor
```

```
cond :: (CondOp :<: f)
     => MonadExp f BoolE
     -> MonadExp f () -> MonadExp f () -> MonadExp f ()
cond b p1 p2 = inject $ Cond b p1 p2

while :: (WhileOp :<: f)
      => MonadExp f BoolE
      -> MonadExp f () -> MonadExp f ()
while pb p = inject $ While pb p
```

Things are starting to fall into place. The only remaining bit is to complete the compiler for monadic expressions.

```
instance (Compile x, Compile (f x))
  => Compile (Mops f x) where
  compile s (Oper o) = compile s o
  compile s (Return a) = (a,PSkip)
  compile s (Bind m f) = (b,prg1 `PSeq` prg2)
    where (s1,s2)  = split2 s
          (a,prg1) = compile s1 m
          (b,prg2) = compile s2 (f a)

instance (Compile (f (MonadExp f)))
  => Compile (MonadExp f) where
  compile s (In a) = compile s a
```

Now, Benny has all the functions he needs to compile a simple example program.

```
test1 :: Robot ()
test1 = do
  move
  turnL
  move
  turnL
```

Compiling the program above gives the expected result.

```
PSeq PMove (PSeq PTurnLeft (PSeq PMove PTurnLeft))
```

Benny decides to contact Björn again to show him what he has done.

> Dear Prof. Björn
>
> I have attached a file which demonstrates that the monadic constructs can be factored out and compiled separately, just as you suggested.
>
> Thanks
> Benny

> Benny,
>
> Fascinating! You've managed to factor out the monadic operations so that they can be compiled once and for all. Users of your library don't have to be concerned at all with the semantics of bind and return. Very nice!
>
> I note that the data type for monadic operations cannot be composed as freely as the other operations, but instead has to be applied separately as a final step. Your solution is very similar to how variable binding is handled in the Syntactic library [1].
>
> Prof. Björn
> Dept. of Computer Science and Engineering
> Chalmers University of Technology

5. Epilogue

This paper shows a simple method of implementing monadic EDSLs. It shows a naïve approach to the monad reification problem which has the additional benefit of being compositional. In section 4, the language is reimplemented in an extensible way and the compile function is shown explicitly to be compositional by treating sub parts of the language separately.

The example language used throughout the story is small and quite limited. But the technique presented in this paper does scale up to larger languages and more complicated language constructs. It is currently used in the implementation of Obsidian, an EDSL for general purpose GPU programming [4]. Obsidian has language constructs which are considerably more advanced than the robot language is this paper. One example is the sequential for-loop which is higher-order (the type parameter `t` is not relevant to the current discussion):

```
SeqFor :: EWord32 -> (EWord32 -> Program t ())
       -> Program t ()
```

There does, however, seem to be restrictions on what kind of language constructs the reification technique presented in this paper can deal with. For instance, the functions from the `MonadPlus` class has resisted our attempts at adding them to an EDSL in the same way as we've added `Return` and `Bind`. The exact limits of the expressiveness of our method is currently unknown to us.

Acknowledgments

We would like to thank Emil Axelsson for very valuable help during the implementation of the methods used in this pearl. We thank Mary Sheeran for suggesting the names Björn & Benny. The ICFP reviewers provided many helpful suggestion which has improved the paper.

This research has been funded by the Swedish Foundation for Strategic Research (which funds the Resource Aware Functional Programming (RAW FP) Project) and by the Swedish Research Council.

References

[1] E. Axelsson. Syntactic. http://hackage.haskell.org/package/syntactic.

[2] E. Axelsson. A generic abstract syntax model for embedded languages. In *Proceedings of the 17th ACM SIGPLAN international conference on Functional programming*, pages 323–334. ACM, 2012.

[3] P. Bahr and T. Hvitved. Parametric compositional data types. In *MSFP*, pages 3–24, 2012.

[4] K. Claessen, M. Sheeran, and B. J. Svensson. Expressive Array Constructs in an Embedded GPU Kernel Programming Language. In *Proceedings of the 7th workshop on Declarative aspects and applications of multicore programming*, DAMP '12, pages 21–30, New York, NY, USA, 2012. ACM. ISBN 978-1-4503-1117-5.

[5] A. Farmer and A. Gill. Haskell DSLs for interactive web services. In *1st International Workshop on Cross-model Language Design and Implementation*, Sep 2012. (published on workshop website, http://workshops.inf.ed.ac.uk/xldi2012/).

[6] P. Hudak. *The Haskell School of Expression: Learning Functional Programming through Multimedia*. Cambridge University Press, New York, NY, USA, 1999. ISBN 0521643384.

[7] A. Persson, E. Axelsson, and J. Svenningsson. Generic monadic constructs for embedded languages. In *Proceedings of the 23rd international conference on Implementation and Application of Functional Languages*, IFL'11, pages 85–99, Berlin, Heidelberg, 2012. Springer-Verlag.

[8] N. Sculthorpe, J. Bracker, G. Giorgidze, and A. Gill. The constrained-monad problem. In *Proceedings of the 18th ACM SIGPLAN international conference on Functional programming*. ACM, 2013.

[9] W. Swierstra. Data types à la carte. *Journal of Functional Programming*, 18(4):423, 2008.