4 Exercise Set No. 4

A company is involved in producing an automotive navigation equipment. As a part of its service, the company provides the fastest rout between two given locations. For this purpose, it utilizes a road map. The map is essentially a digraph $G = (V, A)$, where each arc $a \in A$ denotes a one-way road (two way roads need two arcs) and each vertex is a junction between a number of roads. There is also one vertex for the starting point and one for the termination point. Figure 1 shows an example of such a road map.

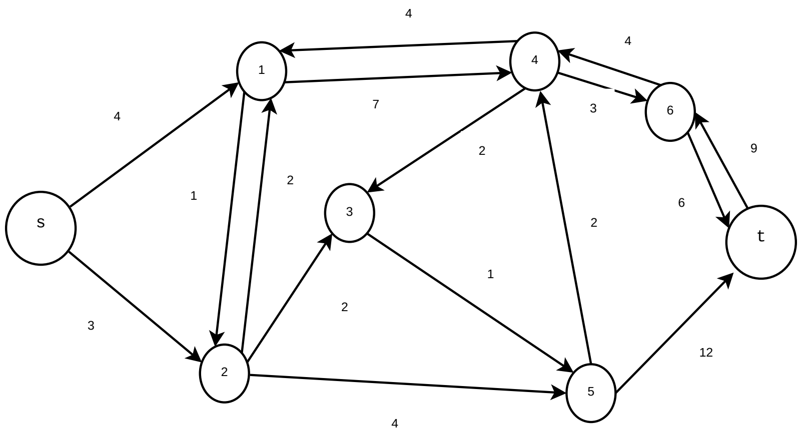

Figure 1: The map between the starting and the final locations.

To estimate the traveling time for each path, the company receives data from a real-time traffic analysis center, which reports a traveling time estimate for each one-way road. These estimates are shown as positive weights on the graph (see Figure 1). The aim is to find a path in the digraph, which has the smallest total traveling time.

In this exercise we use linear programming to solve the above problem. Taking a specific path, for each arc $a \in A$ define $x_a \in \mathbb{Z}^{\geq 0}$ as the number of times that a occurs in the path. Notice that x_a is zero if an arc is not included in the path. The set $\{x_a \mid a \in A\}$ is the set of integer variables in the optimization.

1. For the example in Figure 1, write an ILP to find the fastest rout. Write a set of linear constraints for the variables $\{x_a\}$ to guarantee that they refer to a path.

   Hint: A set of arcs in G is a path from the starting vertex to the terminal one if and only if:

   - The number of ingoing arcs to the starting vertex is one less than the outgoing arcs.

- The number of ingoing arcs to the terminal vertex is one more than the outgoing arcs.
- The number of ingoing arcs to any other vertex is equal to the outgoing arcs.

Write down this conditions in terms of the variables $\{x_a\}$.

2. Relax the optimization to an LP optimization and solve the relaxed LP optimization with CVX. Hand in you CVX code, the optimal solution and the optimal value. Is the answer integral? Draw the fastest rout in Figure 1.

3. Change from 12 to 11 the estimated time for the road between nodes 5 and t. Run CVX again. Is the solution integral?