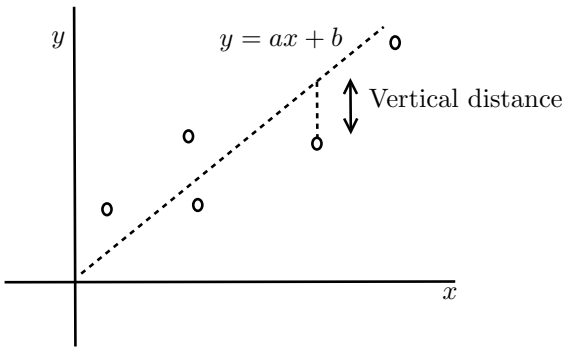
3 Exercise Set No. 3

1. Assume that a physicist takes n measurements of a variable which can be thought of as n points $(x_1, y_1), ..., (x_n, y_n)$ in a planar coordinate system. The physicist wants to find a line with the equation $y = ax + b$, that best fits the observed data, that is he wants to minimize the largest vertical distance from to the points (x_i, y_i).



   (a) Set up a linear program that models the problem: Denote the largest vertical distance by t. Write the optimization "$\min t$" with suitable constraints.(You may first use the absolute value function in the constraints and later remove it to obtain an LP.)

   (b) Find the dual program to the one above.

2. (a) Write the dual of the following optimization:

$$\begin{gathered} \max 7x_1 + 6x_2 + 5x_3 - 2x_4 + 3x_5 \\ \text{subject to} \\ x_1 + 3x_2 + 5x_3 - 2x_4 + 2x_5 \leq 4 \\ 4x_1 + 2x_2 - 2x_3 + x_4 + x_5 \leq 3 \\ 2x_1 + 4x_2 + 4x_3 - 2x_4 + 5x_5 \leq 5 \\ 3x_1 + x_2 + 2x_3 - x_4 - 2x_5 \leq 1 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{gathered} \tag{2}$$

   (b) Decide with the conditions of complementary slackness and only by paper whether $x^* = (0, \frac{4}{3}, \frac{2}{3}, \frac{5}{3}, 0)$ is an optimal solution to the linear program.