## 2 Exercise Set No. 2

A company is engaged in the production and sale of two kinds of hard liquor. It purchases intermediate-stage products in bulk, purifies them by repeated distillation, mixes them, bottles the product under its own brand names and sells it. One product is a bourbon, the other a blended whiskey. The problem is to decide how many bottles of each should be produced in the next production period.

As the companys products are very popular on the market, the production capacity is inadequate to produce all that it might sell. The bourbon requires 3 machine hours per bottle, while the blended whiskey requires 4 hours of machine time per bottle. There are 20,000 machine hours available in the production period. The direct operating costs, which are mainly for labor and materials, are SEK30 per bottle of bourbon and SEK20 per bottle of blended whiskey. The working capital available to finance these costs is SEK44000. Moreover, $40 \%$ of the sales revenues is made available to finance ongoing operations. The selling price is SEK50 for a bottle of bourbon and SEK45 for a bottle of blended whiskey.
(a) Set up a linear program in two variables $x_{1}$ and $x_{2}$ that maximizes the profit in the production period to come, subject to limitations on machine capacity and working capital.
(b) Sketch the feasibility region in the plane and give the coordinates of the vertices.
(c) Using CVX, calculate what the optimal production mix to schedule is and how large the companys profit can be made. Verify that the optimal point is located at a vertex.
(d) Write the optimization in the augmented form.
(e) Suppose that the company could spend some money to repair machinery and increase its available machine hours by 2000 hours (before production starts). Should the investment be made and if so, up to which price?
Hint: How does this change affect the linear program and your sketch?
(f) Suppose that there is no production limit and the company decides to instead reinvest $50 \%$ of the revenues in production. Show that in this case the problem is unbounded. What is the minimum reinvestment ratio for which the optimization is unbounded?

