Specification and Analysis of Contracts
Lecture 5
Deontic Logic

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Plan of the Course

1. Introduction
2. Components, Services and Contracts
3. Background: Modal Logics 1
4. Background: Modal Logics 2
5. **Deontic Logic**
6. Challenges in Defining a Good Contract language
7. Specification of 'Deontic' Contracts ($CL$)
8. Verification of 'Deontic' Contracts
9. Conflict Analysis of 'Deontic' Contracts
10. Other Analysis of 'Deontic' Contracts and Summary
Plan

1. Deontic Logic
   - Motivation
   - Deontic Logic Informally
   - Deontic Logic a Bit More Formally

2. Paradoxes in Deontic Logic
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2. Paradoxes in Deontic Logic
Why Deontic Logic?

- We have propose the use of ‘deontic’ e-contracts in the context of Service-Oriented Computing and Components
- Such contracts are based on deontic logic, which has many applications
- Deontic logic has been identified as a good specification language for information systems in general
  - Norms play a role in knowledge-based and intelligent systems
    - Databases
    - Legal expert systems
    - Electronic contracting
    - Fault tolerant systems
  - There is a need to capture the dynamic aspect of evolving computer systems
  - The ideas behind deontic logic can be used in the specification of long transactions
The Role of Deontic Logic in the Specification of Information Systems

- An information system (IS) is a system storing data about the real world.
- A conceptual model of an IS describes the properties of the data.
- Any property known to be true about the IS is an integrity constraint.
- For normal (hard) constraints we can use different logics:
  - Predicate logic: “all employees are persons”
  - Temporal logic: “the age of a person can never decrease”

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- What about desirable properties that can be violated? — exceptional (soft) constraints

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What about desirable properties that can be violated? — exceptional (soft) constraints.

Needs deontic logic.

Deontic Logic and Violations of Constraints

- Deontic logic is good to reason about ideal versus actual behavior.
- It uses operators for obligation, permission and prohibition and mechanisms to handle violations.
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Example

- In the context of a library “when a person $p$ borrows a book $b$, he should return it within 2 weeks” (syntax is not important)

\[ [(\text{borrow}(p, b))] O(\text{return}(p, b)) \leq 2 \text{ weeks} \]

- There is no control over the borrower on whether he will comply with this norm or not.
- We should add a mechanism to specify what happens in case the person does not return the book within 2 weeks.
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2. Paradoxes in Deontic Logic
Concerned with **moral** and **normative** notions
- *obligation*, *permission*, *prohibition*, *optionality*, *power*, *indifference*, *immunity*, etc

Focus on
- The logical consistency of the above notions
- The faithful representation of their intuitive meaning in law, moral systems, business organizations and security systems

Difficult to avoid **puzzles** and **paradoxes**
- Logical paradoxes, where we can deduce contradictory actions
- “Practical oddities”, where we can get counterintuitive conclusions

Approaches
- **ought-to-do**: expressions consider *names of actions*
  - “The Internet Provider *must send* a password to the Client”
- **ought-to-be**: expressions consider state of affairs (results of actions)
  - “The average bandwidth *must be* more than 20kb/s”
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Since Aristotle (384 BC–322 BC) there were some philosophers’ writing on obligation, permission and prohibition

Leibniz (1646–1716) related obligation, permission and prohibition with logical modalities of necessity, possibility and impossibility

Ernst Mally (1926) used the term *deontik* for his “Logic of the Will”
- Also called it: The logic of what ought to be
- No mention of Leibniz nor of relation between modal and normative notions

A lot of discussions in the late 1930s and early 1940s
- Jørgen Jørgensen and Alf Ross
The Beginnings

- It is accepted that the deontic logic was born as discipline from the following (independent) works
  - G.H. von Wright published the paper “Deontic Logic” (1951)
  - O. Becker (1952, in German)
  - J. Kalinowski (1953, in French)
- All 3 authors explored the analogy between normative and modal concepts
  - von Wright (1951)
    - Started by exploring the formal analogy between the modalities “possible”, “impossible” and “necessary” with the quantifiers “some”, “no” and “all”
    - Extended his study to the analogy with the normative notions (the 1951 paper)
  - A. Prior (1954) criticized von Wright’s paper
    - How to obtain derived obligations, i.e. conditional obligations?
    - von Wright’s answer by adding relative permission: $P(p/q)$: “it is permitted that $p$ on the condition that $q$”
  - Much more followed...
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Ought-to-do vs. Ought-to-be

- **Ought-to-do**: expressions consider *names of actions*
  - “One ought to close the window”

- **Ought-to-be**: expressions consider *state of affairs* (results of actions)
  - “The window ought to be closed”

Why is this so important?
Some things are easier to represent in one approach and others in the other.

“The average bandwidth must be more than 20kb/s”

Sergot’s example on the “strict University code”

The logical system may have some nicer properties in one or the other approach.

Paradoxes...
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Norms as prescriptions for conduct, are not true or false

- If norms have no truth-value, how can we reason about them and detect contradictions and define logical consequence?

According to von Wright: norms and valuations are still subject to logical view

Consequence: Logic has a wider reach than truth!

Prescriptive vs. descriptive view

Conditional norms

Meta-norms

How to represent what happens when an obligation is not fulfilled or a prohibition is violated?

Paradoxes

A lot more...
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2. Paradoxes in Deontic Logic
There are many formal systems for deontic logic

We will give a flavor of SDL (Standard Deontic Logic)

Usually called the Old System of Von Wright

- $P$: permission
- $O$: obligation
- $F$: prohibition
Standard Deontic Logic

- Takes different modal logics and makes analogies between “necessity” and “possibility”, with “obligation” and “permission”
- It turns out to be difficult!
  - Many of the rules in modal logic do not extrapolate to deontic logic
Standard Deontic Logic

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Example

In modal logic:

- If $\Box p$ then $p$ (if it is necessary that $p$, then $p$ is true)
- If $p$ then $\Diamond p$ (if $p$ is true, then it is possible)

The deontic analogs:

- If $O(p)$ then $p$ (if it is obligatory that $p$, then $p$ is true)
- If $p$ then $P(p)$ (if $p$ is true, then it is permissible)
SDL: Axiomatic System

Definition

SDL consists of the following axioms:

\[(K_O)\quad O(\varphi \Rightarrow \psi) \Rightarrow (O\varphi \Rightarrow O\psi)\]

\[(D_O)\quad \neg O \bot\]

\[(P)\quad P\varphi \leftrightarrow \neg O \neg \varphi\]

\[(F)\quad F\varphi \leftrightarrow O \neg \varphi\]

\[(Taut)\quad \text{the tautologies of propositional logic}\]

And two rules:

\[(N_O)\quad \frac{\varphi}{O\varphi}\]

\[(MP)\quad \frac{\varphi \quad \varphi \Rightarrow \psi}{\psi}\]
SDL: Semantics

- SDL has a **Kripke-like modal semantics** based on:
  - A set of possible worlds (with a truth assignment function of propositions per possible world)
  - An accessibility relation associated with the $O$-modality

- The accessibility relation points to **ideal** or **perfect deontic alternatives** of the current world

- To handle **violations** the semantics need to be extended
  - Many extensions have been proposed
Some Problems with Deontic Logic

- Problems to handle **violations** (exceptions, *contrary-to-duties*, *contrary-to-prohibitions*)
  - A **contrary-to-duty** (CTD) expresses what happen when an obligation is not fulfilled
  - A **contrary-to-prohibition** (CTP) defines what is to be done when a prohibition is violated

**Example**

**CTD:** You must send an acknowledgment within 10 minutes after receiving the message. If you don’t do that, you must pay double.

**CTP:** You are forbidden to send a message before having acknowledged the reception of the previous answer. If you don’t do that, you must pay double.
Some Problems with Deontic Logic

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- Paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, paradoxes, ...
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Paradoxes and Practical Oddities

- **Deontic paradoxes.** A paradox is an apparently true statement that leads to a contradiction, or a situation which is counter-intuitive
  - *The Gentle Murderer Paradox*
    1. It is obligatory that John does not kill his mother;
    2. If John does kill his mother, then it is obligatory that John kills her gently;
    3. John does kill his mother.

    It could be possible to infer that John is obliged to kill his mother (contradicting 1 above)

- **Practical oddities.** A situation where you can infer two assertions which are contradictory from the intuitive practical point of view, though they might not represent a logical contradiction

  Assume you have the following norms and facts:
  1. Keep your promise;
  2. If you haven’t kept your promise, apologize;
  3. You haven’t kept your promise.

  It could be possible to deduce that you are both obliged to keep your promise and to apologize for not keeping it
Paradoxes and Practical Oddities

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Example

1. It is obligatory that one mails the letter
2. It is obligatory that one mails the letter or one destroys the letter

In SDL these are expressed as:

1. $O(p)$
2. $O(p \lor q)$
Paradoxes
Ross’s paradox

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In SDL these are expressed as:

1. \( O(p) \)
2. \( O(p \lor q) \)

Problem

- In SDL one can infer that \( O(p) \implies O(p \lor q) \)
Example

1. You may either sleep on the sofa or sleep on the bed.
2. You may sleep on the sofa and you may sleep on the bed.

In SDL this is:

1. $P(p \lor q)$
2. $P(p) \land P(q)$
Paradoxes
Free Choice Permission Paradox

Example

1. You may either sleep on the sofa or sleep on the bed.
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In SDL this is:

1. $P(p \lor q)$
2. $P(p) \land P(q)$

Problem

- The natural intuition tells that $P(p \lor q) \Rightarrow P(p) \land P(q)$
- In SDL this would lead to $P(p) \Rightarrow P(p \lor q)$ which is $P(p) \Rightarrow P(p) \land P(q)$
- So $P(p) \Rightarrow P(q)$
- Thus: If one is permitted something, then one is permitted anything
Paradoxes
Sartre’s Dilemma

Example

1. It is obligatory I now meet Jones (as promised to Jones)
2. It is obligatory I now do not meet Jones (as promised to Smith)

In SDL this is:

1. \(O(p)\)
2. \(O(\neg p)\)
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Problem

- In natural languages the two obligations are intuitive
- But the logical formulae are inconsistent when put together (in conjunction) in SDL
- In SDL, $O(p) \Rightarrow \neg O(\neg p)$, and we get a contradiction
Example

1. It ought to be the case that Jones helps Smith who has been robbed
2. It ought to be the case that Smith has been robbed

And one naturally infers that:

Jones helps Smith who has been robbed if and only if Jones helps Smith and Smith has been robbed

In SDL the first two are expressed as:

1. $O(p \land q)$
2. $O(q)$
Paradoxes

The Good Samaritan Paradox

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And one naturally infers that:

Jones helps Smith who has been robbed if and only if Jones helps Smith and Smith has been robbed

In SDL the first two are expressed as:

1. \( O(p \land q) \)
2. \( O(q) \)

Problem

- In SDL one can derive that \( O(p \land q) \Rightarrow O(q) \) which is counter-intuitive in natural languages

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Example

1. It is obligatory that John does not kill his mother
2. If John does kill his mother, then it is obligatory that John kills her gently
3. John does kill his mother

In SDL these are expressed as:

1. \(O(\neg p)\)
2. \(p \Rightarrow O(q)\)
3. \(p\)
Example

1. It is obligatory that John does not kill his mother
2. If John does kill his mother, then it is obligatory that John kills her gently
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In SDL these are expressed as:

1. $O(\neg p)$
2. $p \Rightarrow O(q)$
3. $p$

Problem

- When adding a natural inference like $q \Rightarrow p$, one can infer that $O(p)$ (contradicting 1 above)
Paradoxes
Chisholm’s Paradox

Example

1. John ought to go to the party
2. If John goes to the party then he ought to tell them he is coming
3. If John doesn’t go to the party then he ought not to tell he is coming
4. John does not go to the party

In SDL these are expressed as:

1. \( O(p) \)
2. \( O(p \Rightarrow q) \)
3. \( \neg p \Rightarrow O(\neg q) \)
4. \( \neg p \)
Paradoxes
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Example

1. John ought to go to the party
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In SDL these are expressed as:

1. \( O(p) \)
2. \( O(p \implies q) \)
3. \( \neg p \implies O(\neg q) \)
4. \( \neg p \)

Problem

- In SDL one can infer \( O(q) \land O(\neg q) \) (due to statement 2)
Paradoxes: Diagnosis of the Problems

Part of the problems arise from the following 4 confusions [MWD96]

**Why paradoxes in deontic logic?**

1. **Confusion between ought-to-do and ought-to-be**
   - Take a *pragmatic* point of view: difficult to get a paradox-free logic of norms, ethics, and morality

2. **Confusion between the formal interpretation and the natural language**
   - Example, the logical *or* is usually understood as a *choice*

3. **Confusion between ideality and actuality**
   - Needs a good treatment of exceptions, CTD’s, CTPs, etc

4. **Confusion between normative notions for abstract contexts (e.g. ethics) and those needed in concrete practical applications**
   - In practical applications: not interested on the philosophical problems
   - A concrete application helps getting rid of most paradoxes

Reminder

- We want to use deontic e-contracts to specify and reason about contracts in software systems (e.g., components, services)
- We need a formal system to relate the normative notions of obligation, permission and prohibition
- We want to represent (nested) “exceptions”: Can we represent and reason about what happens when an obligation is not fulfilled or a prohibition is violated?
- We want to avoid the philosophical problems of deontic logic (restrict its use to our application domain)