Maintenance of random logical networks

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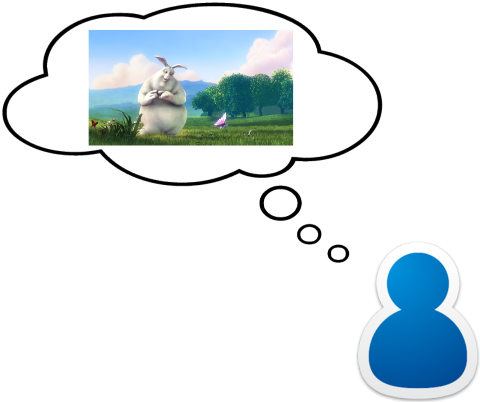
DCS seminar, CHALMERS

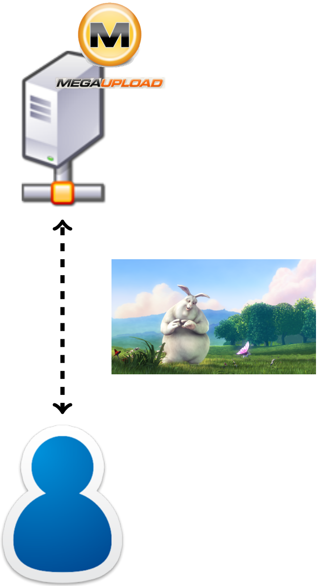
October 4, 2017

Plan

1. **A Quick Example of P2P Networks**
2. A New Model of Evolution
3. A Concrete Example: Uniform k-out Random Graphs
4. A More General Question



Motivation: modelling of P2P networks 1/2



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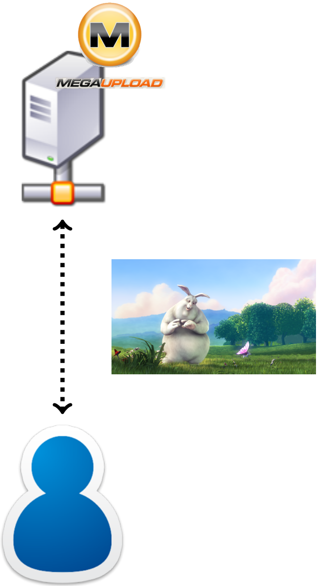

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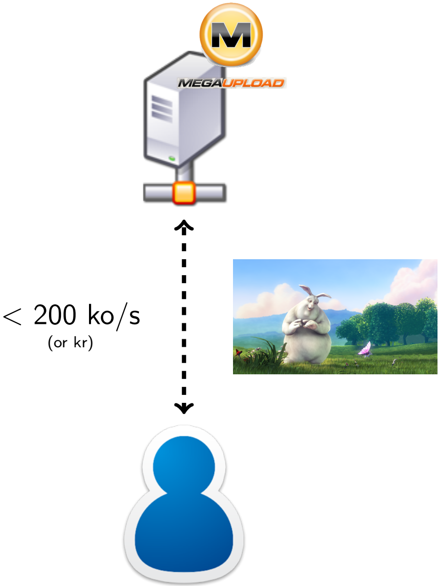

Motivation: modelling of P2P networks 1/2

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(or kr)

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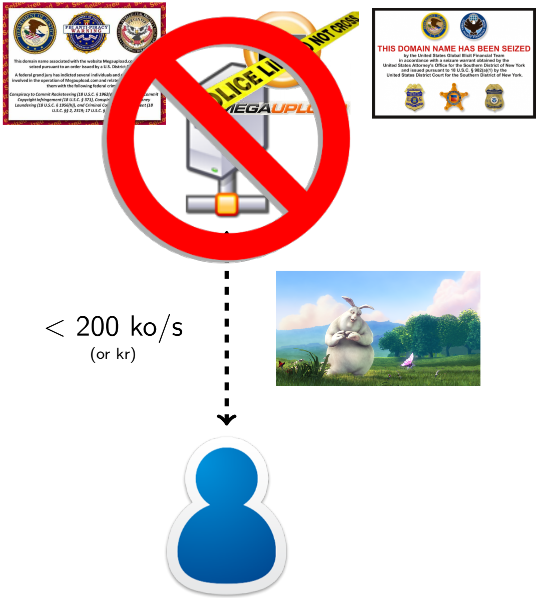
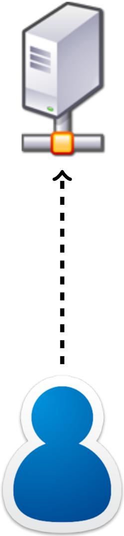
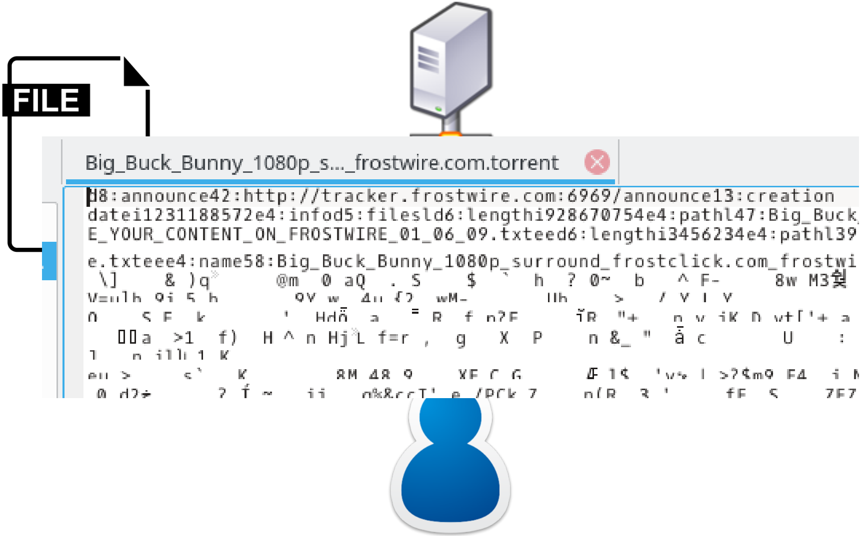


Motivation: modelling of P2P networks 2/2

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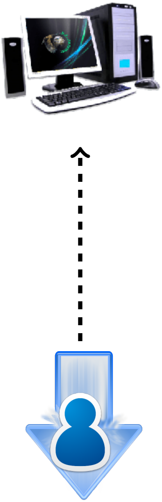

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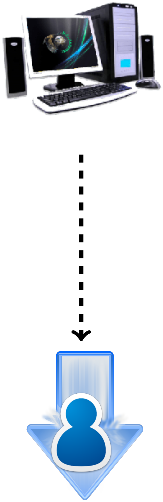

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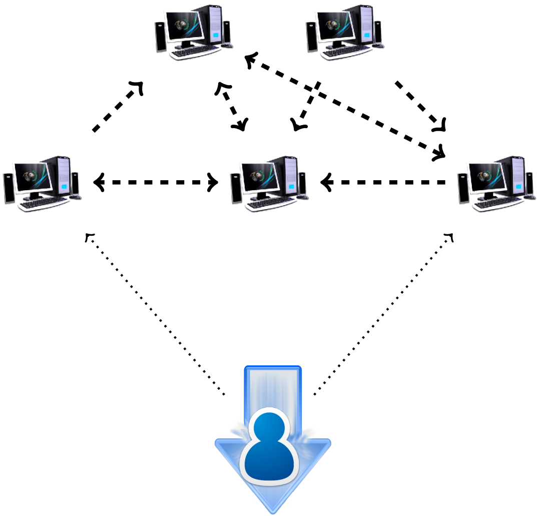

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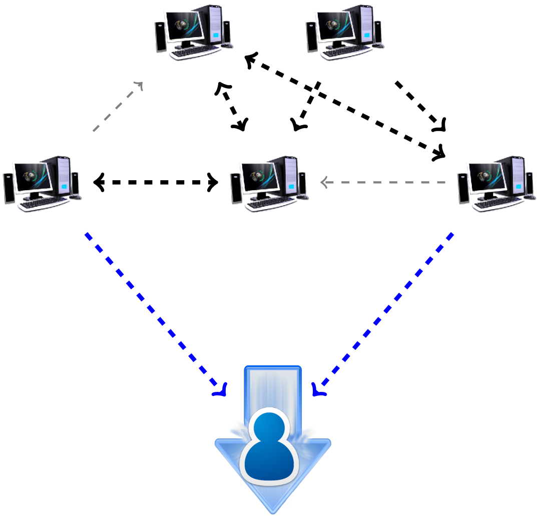

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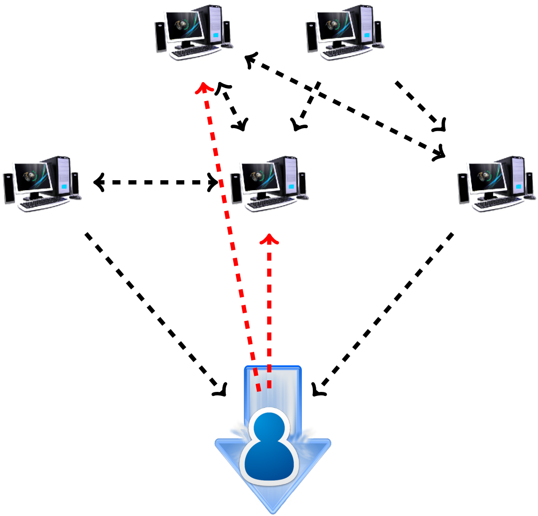

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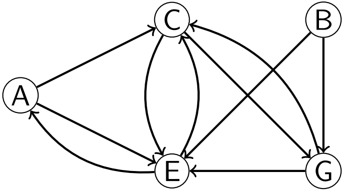
Introduction: Logical, Decentralized, Dynamic networks



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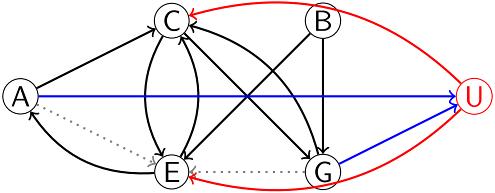

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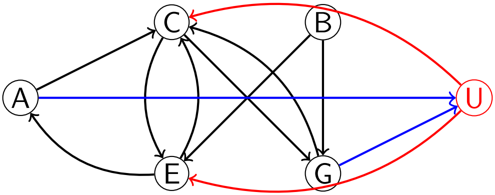

Why should we look for good models ?

- analysis of the evolution of some concrete networks
- analysis of distributed algorithms running over such networks
- simulations of distributed algorithms operating over such networks (including adaptive algorithms working over dynamic networks)

Our model of evolution

Constrain the evolution to always stick to the target model

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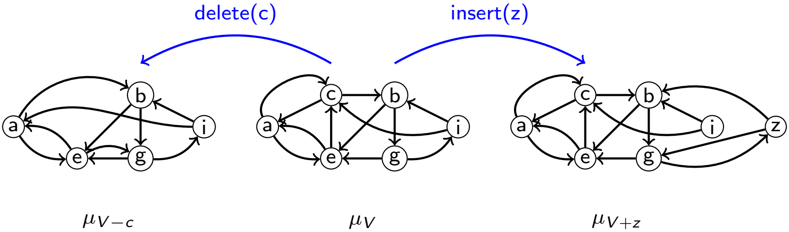

Context : evolution of $\mathrm{P2P}$ networks

- **In the literature**, some *good properties* of the network are maintained over time, but implies :
  - difficulties in update alogrithms' conception leading to complex procedures
  - dynamicity modelled by a probabilistic process (Poisson) : **non realistic** [Pouwelse *et al*, 2005]
  - difficult analysis without those hypothesis (analysis under simplistic update models : insertion only, fifo)

Typical good properties

- small degree, small diameter, high connectivity, etc

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- **Our solution** : *randomness preservation*

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- **Our solution** : *randomness preservation*, answers those problems:
  - properties are always maintained
  - analysis is simplified
  - it is not influenced by an **adversarial** sequence of updates
  - no drift phenomena

Typical good properties

- small degree, small diameter, high connectivity, etc

Local update algorithms

- $\mathcal{LOCAL}$ model (synchronous, error-free, message passing)
- Two submodels: exact size of the network **known** or **unknown** to the participating nodes

Introduction: An Optimistic First Contact Model

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- **"First Contact":** Every node has access to a global primitive which samples uniformly a node over the entire network

$$\mathit{RandomVertex}()$$

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- Optimistic: uniformity, reusability, availability

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What about the cost model ?

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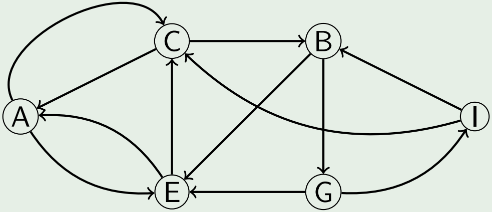
Uniform k-out graphs

Example of a 2-out graph



Uniform k-out graphs

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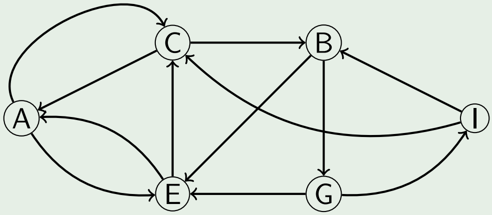

- Directed graphs with no loops and where each vertex has exactly k out-neighbours

Uniform k-out graphs

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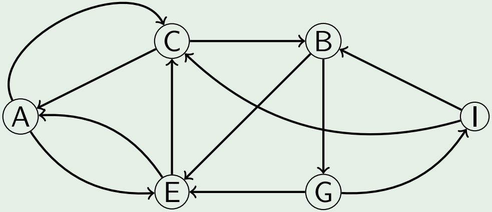
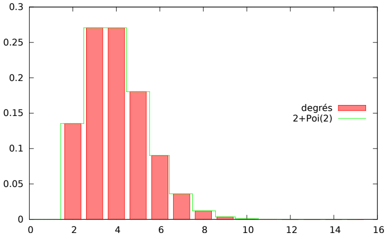


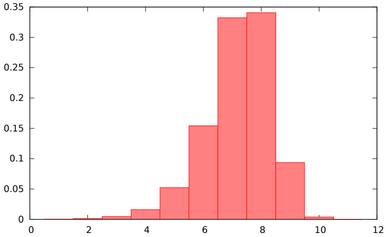
- Directed graphs with no loops and where each vertex has exactly k out-neighbours
- The uniform distribution over vertex set V is equivalent to:
  - For each $v \in V$, the outgoing neighbourhood of v is a uniform k-subset of $V - v$
  - All outgoing neighbourhood are independent

Why uniform k-out graphs ?

Figure : Some statistics of 2-out random graphs.



(a) Degrees distribution for $G^2_{10^7}$



(b) Distances distribution for $G^2_{10^4}$

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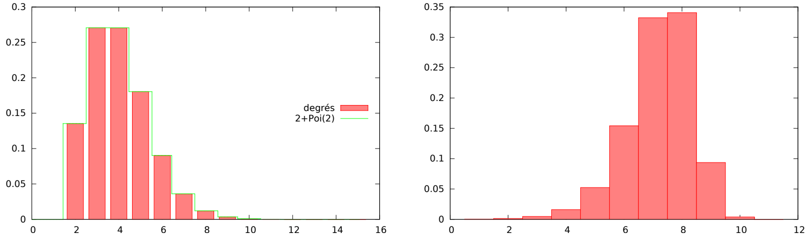
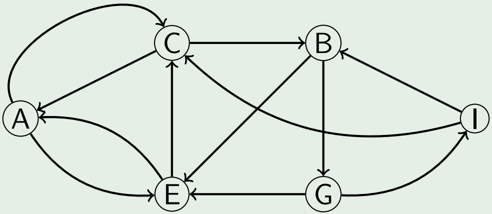


(a) Degrees distribution for $G^2_{10^7}$ (b) Distances distribution for $G^2_{10^4}$

The uniform distribution is associated with good properties similar to those sought for $\mathrm{P2P}$ networks (small degree/diameter, high connectivity)

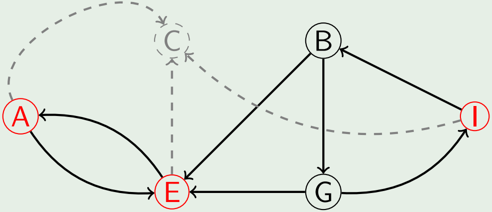
Maintenance of k-out graphs : deletion 1/2

Deletion of node C



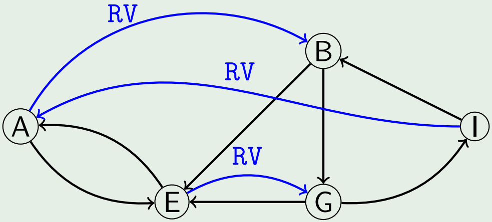
Node C wishes to leave the network.

Deletion of node C



Node C leaves the network, and 3 loosed edges are created.

Deletion of node C



Nodes A, E and I find a substitute node for node C using `RandomVertex()`.

In total, we need $k + \mathcal{O}(1/n)$ calls in average.

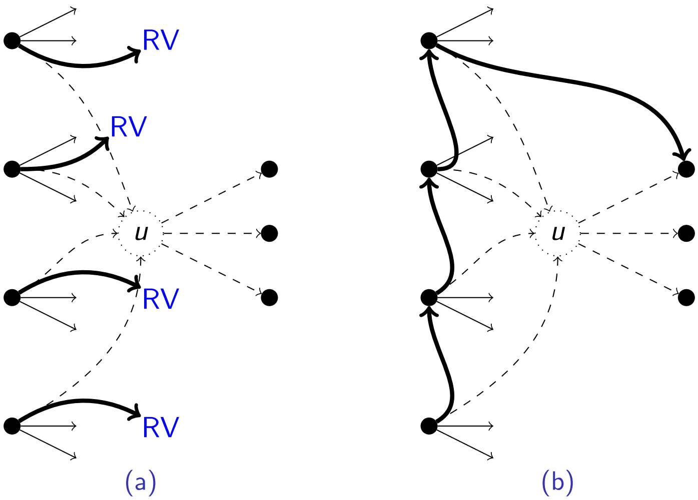
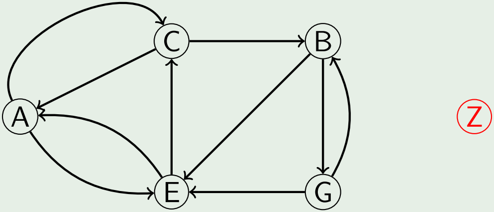


Figure : Typical deletion of node u in the two algorithms.

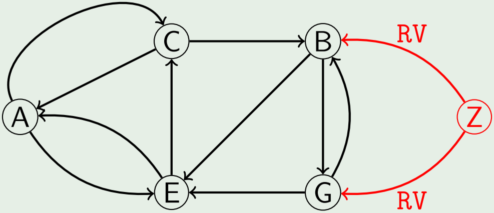
Maintenance of k-out graphs: insertion 1/2

Insertion of node Z



Node Z wishes to join the network.

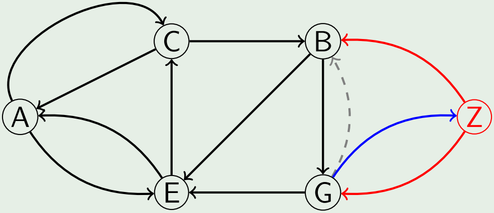
Insertion of node Z



Node Z chooses 2 distinct nodes as out-neighbours, using `RandomVertex()`.

In average, $k + \mathcal{O}(1/n)$ calls to the primitive are needed.

Insertion of node Z



Node Z chooses $X \sim \text{Binomial}(n,\ k/n)$ distinct nodes as in-neighbours, and *steal* one edge from each of them.

We need $k + \mathcal{O}(1/n)$ more calls in average to sample these vertices.

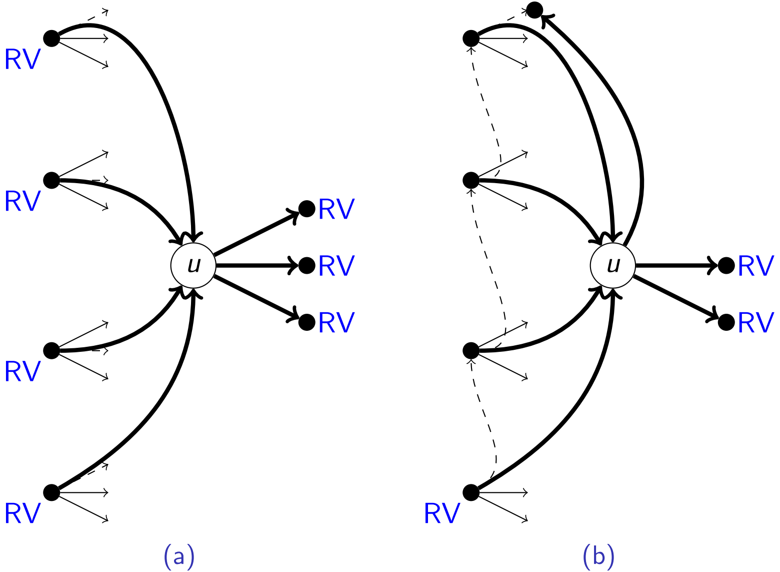


Figure : Typical instances of insertion of node u in the two algorithms.

Maintenance of k-out graphs

Theorem (Duchon, D., 14)

There exist local update algorithms in order to maintain uniform k-out graphs and:

- *modifying a minimal number of links,*
- *of average constant complexity, and using in average:*
  - $k + \mathcal{O}(1/n)$ *calls to* $\mathtt{RV}$ *for the insertion;*
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- **and these bounds are asymptotically optimal.**

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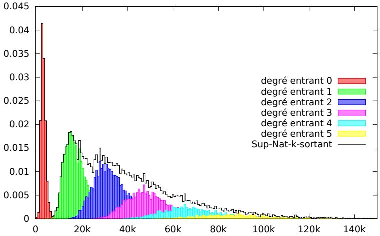
Simulation of Binomial(n, k/n) (D. 15)

This particular law can be simulated in our model with a non-trivial combinatorics-based algorithm using $< 5.2k + \mathcal{O}(1/n)$ expected calls to $\texttt{RV}$ without knowing n.

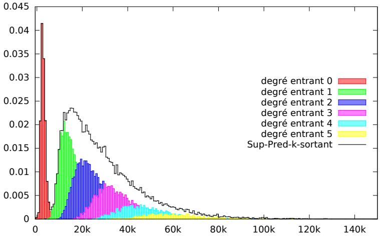
Algorithm for Binomial$(n, 1/n)$

```
m ← 1, g ← 0, j ← 1
S ← {Uniform(V)}
loop
    i ← Random(m + 1)
    if i = m + 1 then
        j ← j + 1
    else if i > g then
        j ← j − 1
        g ← m + 1
    else
        return j
    end if
    x ← Uniform(V)
    if x ∈ S then
        if g = m + 1 then
            return j + 1
        else
            return j − 1
        end if
    else
        S ← S + x
    end if
    m ← m + 1
end loop
```

Different algorithms have different consequences 1/2



(a) SUP-NAT-K-SORTANT 10^4 nodes



(b) SUP-PRED-K-SORTANT 10^4 nodes

Figure : Number of modified distances after one deletion, $k = 2$.

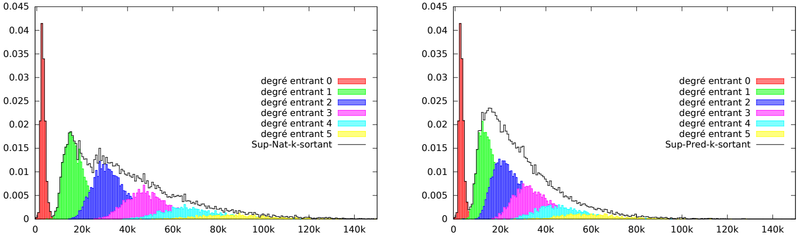


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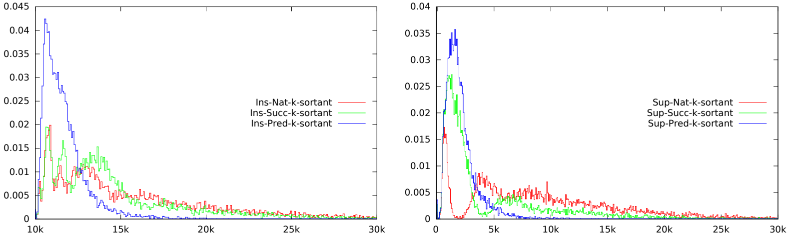
Figure : Number of modified distances after one deletion, $k = 2$.

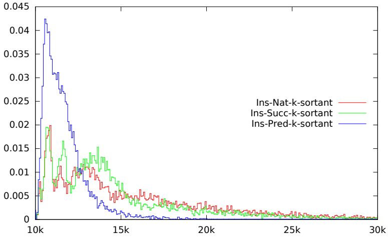
On these simulations, the second algorithm modifies 25% less distances in the graph.



(a) Insertion 10^4 nodes (b) Deletion 10^4 nodes

Figure : Number of modified distances by more than one unit, $k = 2$.

Different algorithms have different consequences 2/2



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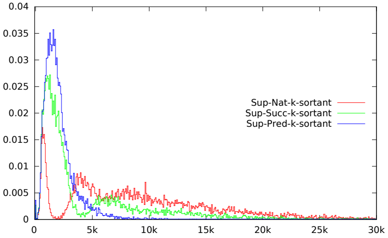

(b) Deletion 10^4 nodes

Figure : Number of modified distances by more than one unit, $k = 2$.

On these simulations, the second algorithms modify respectively 25% less distances during insertion and about 80% less distances during deletion.

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A general question as a starting point

Coul any **topology** of networks **arise**
and **last** in a **distributed** fashion ?

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random graphs

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Applications: model the evolution of P2P-like networks

Introduction: Networks modelled as Random Graphs

Some classical models of logical networks

- **Erdős–Rényi** $G(n, p)$:
  - p fixed: dense graphs, binomial degrees (*similar* degrees), ...
  - $p = p(n)$: phase transition around $\log(n)/n$, ...
- **Uniform** k**-regular graphs**:
  - constant degree, high connectivity (for $k \geq 3$), logarithmic diameter, ...
- **Barabási–Albert** (scale-free networks):
  - preferential attachment, power-law degree distribution, ...

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Could these classical models "appear" and somehow "perdure" in a decentralized evolution ?

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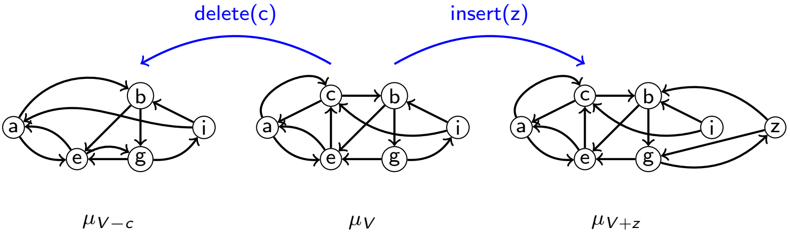

Summing up studied models

Distributions	Cost to insert		Cost to leave	
	with size	w/o size	with size	w/o size
Erdős–Rényi Graphs	$\Theta(n)$	impossible	0	
Pairing multigraph (degree m)	$m/2 + \mathcal{O}(1/n)$		0	
Preferential attachment (degree m)	$m + 1 + \mathcal{O}(1/n)$		not distributed	
Uniform k-out graphs	$k + \mathcal{O}(1/n)$	$k\frac{e^2+3}{2} - 1 + \mathcal{O}(1/n)$	$0 + \mathcal{O}(1/n)$	$k + \mathcal{O}(1/n)$
Uniform μ-out graphs	$\lvert\mu\rvert + \mathbb{E}(\mu)$	$\mathcal{O}(\lvert\mu\rvert)$	$\mu(0) + \mathcal{O}(1/n)$	$\mathbb{E}(\mu) + \mathcal{O}(1/n)$

Conclusion

Models analysed

- Erdős–Rényi: unmaintainable without knowledge of n when p is fixed
- Pairing models: efficient maintenance without size needed
- Uniform k-out Graphs: interesting model with efficient maintenance without size needed

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- Uniform k-out Graphs: interesting model with efficient maintenance without size needed

Many interesting open questions to investigate

- Maintability of Erdős–Rényi Random Graphs of varying density
- Full decentralized maintenance of Barabási–Albert model
- Other graph distributions (geometrical graphs, etc)
- What about approximate maintenance ?

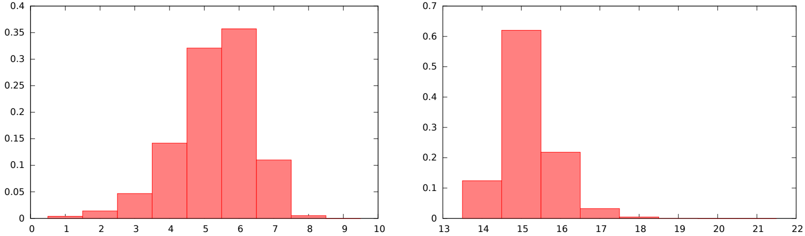
Thank you

Thank you for your attention.

Plan

1. **Uniform** k**-out Random Graphs**
2. Pairing and Barabási–Albert models

Why k-out random graphs ?



(a) Distances for 10^3 nodes (b) Max degree for 10^7 nodes

Figure : More statistics for k-out random graphs.

Plan

1. Uniform k-out Random Graphs
2. **Pairing and Barabási–Albert models**

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Need to remember the orientation of the edges. This information can be recomputed but may need linear time to do so.

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Correction and Cost

- Easy probability tricks :

$$\frac{1}{n}\frac{1}{2} + \sum_{u \in N^-(v)} \frac{1}{n}\frac{1}{2}\frac{\delta^{\to v}(u)}{k} = \frac{1}{2n} + \frac{\delta^-(v)}{2kn} = \frac{\delta(v)}{2kn}$$

- needs on average $k + \mathcal{O}(1/n)$ calls to RV

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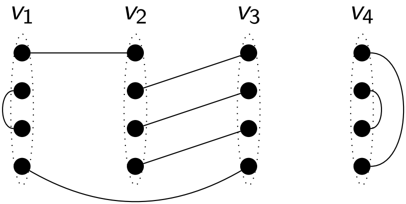
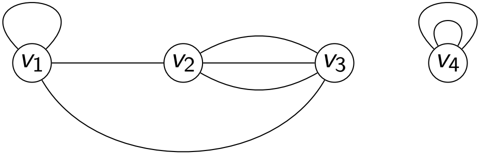
To account of edges not yet present (while inserting a new vertex), we use a simple rejection mechanism.

Maintenance of the pairing model

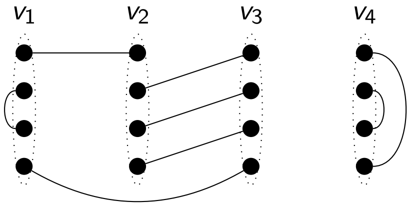


(a) Perfect matching M

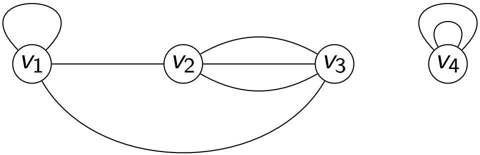


(b) Multigraph $P(M)$ of degree $k = 4$

Maintenance of the pairing model

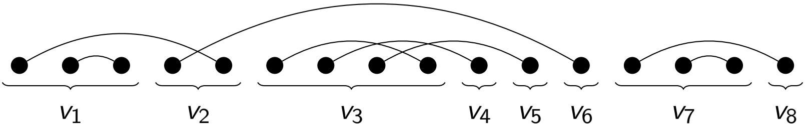


(c) Perfect matching M

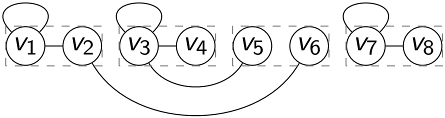


(d) Multigraph $P(M)$ of degree $k = 4$

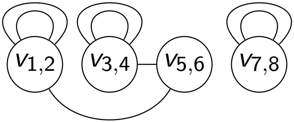
Only insertion needs $k/2 + \mathcal{O}(1/n)$ calls to RV, on average.



(e) Chord diagram with 16 points



(f) Constructed pseudograph



(g) after node merging

Figure : Example of Bollobás–Riordan construction.

Barabási–Albert model

Well known *scale-free* distribution used to model the Web, social networks, etc. Scale-free: proportion of node of degree k (for great k) is about $k^{-\gamma}$.

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Explicit model – Bollobás–Riordan Multigraph model

$$\mathbb{P}(v_i = v) \quad = \quad \frac{\delta_i(v)}{2(kn+i)-1} \qquad \text{pour } v \in V$$

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To forget the insertion order dependencies, we consider it to me uniform: exchange of neighbourhoods after each insertion.

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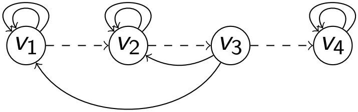
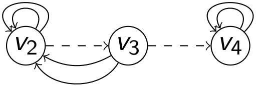
Deletion: needs to maintain a chain of the nodes and keep track of the "last inserted vertex".



(e) Augmented Multigraph



(f) After v_1's deletion

Barabási–Albert model conclusion

Maintenance of the model

- Efficient insertion is possible using extra structural information, but distributed deletion algorithms are not known
- Efficient maintenance is still open without extra information (edges orientation)
- Knowledge of n does not seem to help much

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Alternative model: Pairing model

- based on uniform pairing (as Barabási–Albert, but the construction of the final graph is different)
- is used to prove results on scale-free graphs