Home exam 1

1. Show that if T and T' are objects of a given category that are both *terminal* objects then T and T' are isomorphic.
2. If $f : A \to B$ and $g : B \to C$ are both monomorphisms, explain why $gf : A \to C$ is also a monomorphism.
3. Explain why the one-element group is an *initial* object in the category of groups
4. Any ordered set defines a category. Show that in this category any map is a monomorphism.
5. In a category with binary product and a terminal object T explain why the objects $A \times T$ and A are isomoprhic.
6. Show that a category that has both *products* and *equalizers* has also *pullbacks*
7. Explain why the pullback of a monomorphisms along *any* map is still a monomorphism. Does the same property also hold for epimorphisms?
8. Show that any mapping which has a section is an epimorphism.
9. Given $f : A \to B$ and $g : B \to A$ show that, if both $gf : A \to A$ and $fg : B \to B$ are isomorphisms then so are f and g.
10. Explain why any coequalizer is an epimorphism.
11. In a category with products, explain how to define the "diagonal" map $\Delta : A \to A \times A$ and explain why this map is always a monomorphism.
12. We assume that we have a category with an object which is both *initial* and *terminal*. Is it the case that all objects in this category are isomorphic?