"Termination and Productivity Checking with Continuous Types" Second Thoughts

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19 May 2010

The last display before Section 5 characterizes the legal return types of corecursive functions. The second example, $\mathsf{Nat} \times Y$ does not fit the grammar, contrary to the claim in the paper. Consequently, the definition of fib' from the introduction is not legal according to the rules given in the paper. Indeed, if $\mathsf{Nat} \times Y$ was legal, then the term

$$\mathsf{fix}^\nu g.\,(\mathsf{fst}\, g, \mathsf{fst}\, g :: \mathsf{snd}\, g) : \forall Y \approx \mathsf{Stream}.\, \mathsf{Nat} \times Y$$

would be accepted, but it is not productive.

The grammar for legal result types is correct, the fib' example is wrong. The old errata below no longer apply.

Old, erroneous errata (8 January 2005)

The reduction rule for fix^ν is strongly normalizing, but it does not have enough reductions in case the result type of corecursion is not directly a coinductive type. This problem arises in case of the unfolded stream of Fibonacci numbers $\mathsf{fib}' : \forall Y \approx \mathsf{Stream}.\, \mathsf{Nat} \times Y$ (the type is a product of some type and a coinductive type): The term

$$\mathsf{fst}\,\mathsf{fib}' = \mathsf{fst}\ (\mathsf{fix}^\nu f.\ (0,\ 1 :: \mathsf{sum}\,(\mathsf{fold}\, f)\,(\mathsf{snd}\, f)))$$

does not reduce, since the evaluation context $\mathsf{fst}\,\bullet$ is not of the form $\mathsf{unfold}\, E[\bullet]$. Trying to get the other Fibonacci numbers out of the stream, we obtain

$$\begin{array}{lll} \mathsf{fst}\,(\mathsf{unfold}\,(\mathsf{snd}\,\mathsf{fib}')) & \longrightarrow^+ & 1, \qquad \text{and} \\ \mathsf{fst}\,(\mathsf{unfold}\,(\mathsf{snd}\,(\mathsf{unfold}\,(\mathsf{snd}\,\mathsf{fib}')))) & \longrightarrow^+ & \mathsf{fst}\,\mathsf{fib}' + 1. \end{array}$$

This means that we cannot compute the values of the Fibonacci numbers.

The problem cannot be mended by relaxing the reduction rule for corecursion to $E[\mathsf{fix}^\nu g.M] \longrightarrow_\beta E[[\mathsf{fix}^\nu g.M/g]M]$, since then $\mathsf{snd}\,\mathsf{fib}'$ loops immediately. It would be sound, though, to let both

$$\begin{array}{ll} \mathsf{fst}\,\mathsf{fib}' & \text{and} \\ \mathsf{unfold}\,(\mathsf{snd}\,\mathsf{fib}') & \end{array}$$

reduce. This, of course, complicates the term calculus, since fix^ν would need to carry the annotation under which evaluation contexts it should unfold. The list of evaluation contexts would depend on the derivation of $Y\ \mathsf{legal}^\nu\ \tau(Y)$ in the typing rule of fix^ν.