# TDA 231 Machine Learning: Homework 4

Goal: Support Vector Machines
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Due Date: Feb 27, 2017

**General guidelines:**

1. All solutions to theoretical problems, and discussion regarding practical problems, should be submitted in a single file named *report.pdf*
2. All matlab files have to be submitted as a single zip file named *code.zip*.
3. The report should clearly indicate your name, personal number and email address
4. All datasets can be downloaded from the course website.
5. All plots, tables and additional information should be included in *report.pdf*

## 1 Theoretical problems

**Problem 1.1** [SVM, 5 points]

Consider a (hard margin) SVM with the following training points from two classes:

$$\begin{aligned} +1: &\quad (2,2), (4,4), (4,0) \\ -1: &\quad (0,0), (2,0), (0,2) \end{aligned}$$

Plot these six training points, and construct by inspection the weight vector for the optimal hyperplane. In your solution, specify the hyperplane in terms of w and b such that $w_1 x_1 + w_2 x_2 + b = 0$. Calculate what the margin is (i.e., $2\gamma$ where $\gamma$ is the distance from the hyperplane to its closest data point), showing all of your work. (Hint: It may be useful to recall that the distance of a point $(a_1, a_2)$ from the line $w_1 x_1 + w_2 x_2 + b = 0$ is $|w_1 a_1 + w_2 a_2 + b| / \sqrt{w_1^2 + w_2^2}$.)

**Problem 1.2** [SVM cont'd, 5 points]

Consider the same problem from above.

(a) Write the primal formulation of the SVM *for this specific example* in terms of parameters $w_1, w_2, b$, including the constraints, inserting the given input. That is, you have to specialise the general formulation for the set of inputs given.

(b) Give the optimal primal solution *for this specific example*.

(c) Write the dual formulation *for this specific example.*

(d) Give the optimal dual solution, comment on support vectors.

## 2 Practical problems

**Useful matlab functions:**

- *General:* svmtrain, svmclassify, quadprog, feval
- *Kernel function example (linear kernel):* K= @(x, y) dot(x, y);
- All datasets have variables $X$ (data) and $Y$ (labels)

**Problem 2.1** [SVM, 5 points] Consider the dataset *d1b.mat* having two-dimensional points $X$ and corresponding labels $Y$.

(a) Train a soft-margin SVM (with linear kernel) on the above data. Report the box-constraint parameter $C$.

(b) Plot the data and the separating hyperplane of the trained classifier. Color the points of one class red and the other blue. Mark points that could not be separated correctly. Also, mark the support vectors.

(c) Does the classifier have bias, and if so, what is it?

(d) What is the (soft) margin?

Include all plots and results in the report.

**Problem 2.2** [Kernels, 5 points] Download the dataset *d2.mat*. Consider the following kernels.

1. Linear kernel
2. Quadratic kernel
3. Radial Basis Function (RBF) kernel

(a) Train a soft-margin SVM with the linear kernel on all of the data (d2.mat). Report the box-constraint parameter $C$. Plot the data. Color the points of one class red and the other blue. Mark points that could not be separated correctly by the trained model.

(b) Run *svmtrain* with optimization method 'SMO' as well as 'QP' for all of the kernels. The method is passed as a parameter. Type "help svmtrain" for details. Report the execution time and average classification accuracy (%) for the two optimization methods for 5-fold cross-validation, and all three kernels. Comment on the results.

(c) Plot the separating boundary of the RBF kernel

Include all plots and results in the report.