Introduction to the Why tool

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Provers based on HOL are suitable tools to verify purely functional programs (see Tuesday's lecture)

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- Floyd-Hoare logic
- Dijkstra's weakest preconditions

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- could be applied by a tactic (shallow embedding)

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a realistic existing programming language such as C or Java?

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makes program verification

- prover-independent but prover-aware
- language-independent

so that we can use it to verify C, Java, etc. programs with HOL provers but also with FO decision procedures

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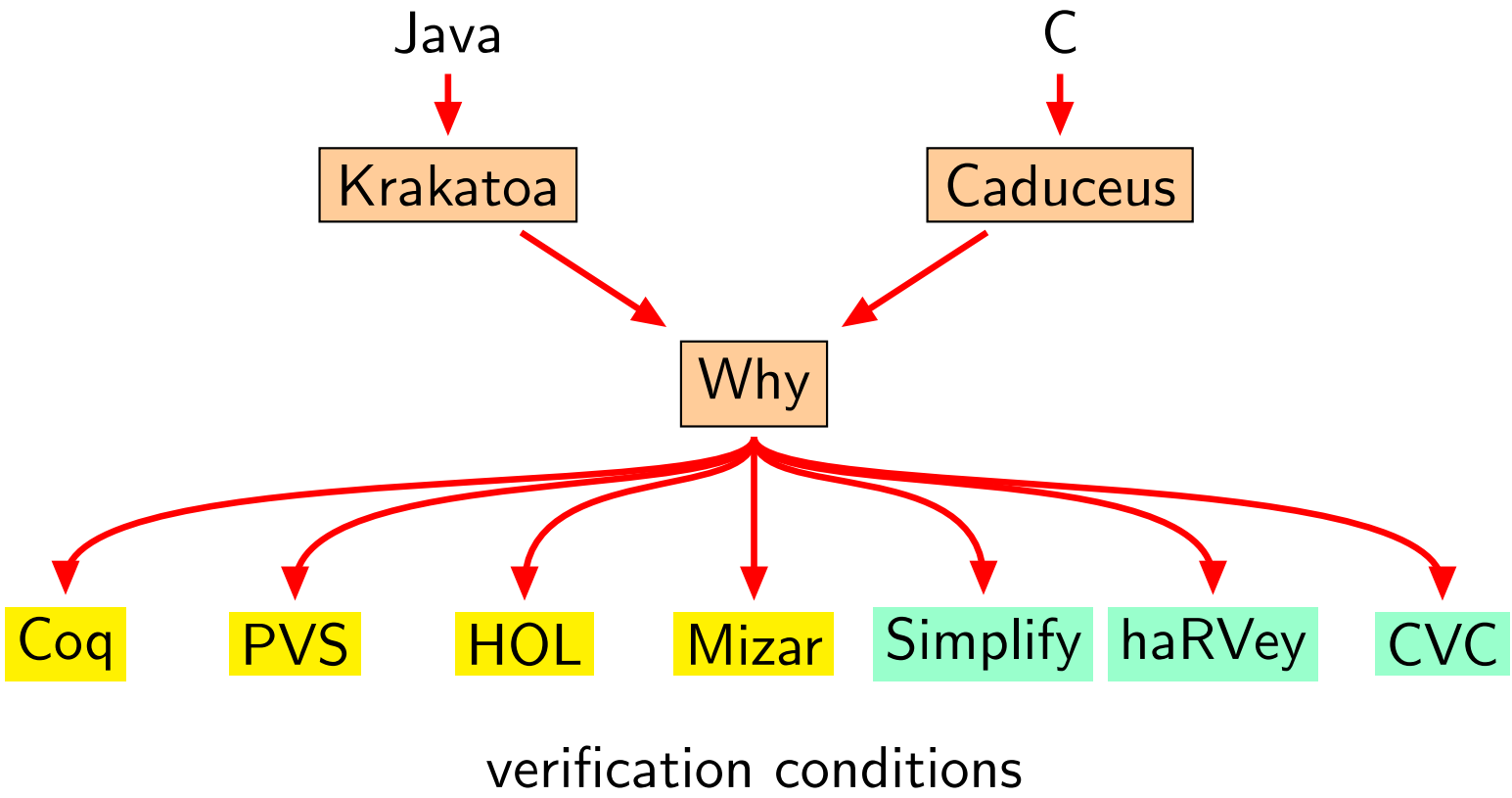
An intermediate language



Outline

1. A language for program verification
   - syntax
   - typing
   - semantics
2. Proof rules
3. The WHY tool
   - Dijkstra's Dutch flag
4. Verification of C programs

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Part I

A language for program verification

The essence of Hoare logic assignment rule

$$\{\ P[x \leftarrow E]\ \}\ x\ :=\ E\ \{\ P\ \}$$

1. absence of aliasing
2. side-effects free E shared between program and logic

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Data types

Any purely applicative data type from the logic can be used in programs

Example: a data type `int` for integers with constants 0, 1, etc. and operations +, *, etc.
The pure expression 1+2 belongs to both programs and logic

A single data structure: the reference (mutable variable) containing only pure values, with no possible alias between two different references

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ML syntax

No distinction between expressions and statements

$\Rightarrow$ less constructs

$\Rightarrow$ less rules

dereference	`!`x
assignment	x `:=` e
local variable	`let` x `=` e_1 `in` e_2
local reference	`let` x `= ref` e_1 `in` e_2
conditional	`if` e_1 `then` e_2 `else` e_3
loop	`while` e_1 `do` e_2 `done`

sequence e_1 `;` e_2 $\equiv$ `let _ =` e_1 `in` e_2

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Annotations

- $\texttt{assert}\ \{p\};\ e$
- $e\ \{p\}$

Examples:

- $\texttt{assert}\ \{x > 0\};\ 1/x$
- $x := 0\ \{!x = 0\}$
- $\texttt{if}\ !x > !y\ \texttt{then}\ !x\ \texttt{else}\ !y\ \{\mathit{result} \geq !x \wedge \mathit{result} \geq !y\}$
- $x := !x + 1\ \{!x > \texttt{old}(!x)\}$

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Annotations (cont'd)

Loop invariant and variant

- `while` e_1 `do` $\{$`invariant` p `variant` $t\}$ e_2 `done`

Example:

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while !x < N do
  { invariant !x ≤ N variant N − !x }
  x := !x + 1
done
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Annotations (cont'd)

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Functions

A function declaration introduces a precondition

- $\texttt{fun}\ (x : \tau) \rightarrow \{p\}\ e$
- $\texttt{rec}\ f\ (x_1 : \tau_1) \ldots (x_n : \tau_n) : \beta\ \{\texttt{variant}\ t\} = \{p\}\ e$

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Modularity

A function declaration extends the ML function type with a precondition, an effect and a postcondition

$$x : \tau_1 \to \{p\}\, \tau_2\ \texttt{reads}\, x_1, \ldots, x_n\ \texttt{writes}\, y_1, \ldots, y_m\, \{q\}$$

Example:

$$\mathit{swap} : \ x : \texttt{int ref} \to y : \texttt{int ref} \to \{\}\, \texttt{unit writes}\, x, y\, \{\texttt{!}x = \texttt{old(!}y\texttt{)} \wedge \texttt{!}y = \texttt{old(!}x\texttt{)}\}$$

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Auxiliary variables

Used to denote the intermediate values of variables

Example: $\ldots\ \{!x = X\}\ \ldots\ \{!x > X\}\ \ldots$

We will use labels instead

- new construct $L : e$
- new annotation $\mathtt{at}(t, L)$

Example:

$$\begin{array}{l} \vdots \\ L : \mathtt{while}\ \ldots\ \mathtt{do}\ \{\ \mathtt{invariant}\ !x \geq \mathtt{at}(!x, L)\ \ldots\ \} \\ \quad \ldots \\ \mathtt{done} \end{array}$$

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Finally, we introduce exceptions in our language

- a more realistic ML fragment
- to interpret abrupt statements like `return`, `break` or `continue`

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The notion of postcondition is extended

$$\texttt{if } x < 0 \texttt{ then raise Negative else } \mathit{sqrt}\ x$$
$$\{\ \mathit{result} \geq 0 \mid \text{Negative} \Rightarrow x < 0\ \}$$

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Loops and exceptions

We can replace the `while` loop by an infinite loop

- `loop` e `{invariant` p `variant` t`}`

and simulate the `while` loop using an exception

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while e1 do {invariant p variant t} e2 done ≡
  try
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Summary

Types

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\begin{array}{lcl}
\tau & ::= & \beta \mid \beta \text{ ref} \mid (x : \tau) \rightarrow \kappa \\
\kappa & ::= & \{p\}\, \tau\, \epsilon\, \{q\} \\
q & ::= & p; E \Rightarrow p; \ldots; E \Rightarrow p \\
\epsilon & ::= & \texttt{reads } x, \ldots, x \texttt{ writes } x, \ldots, x \texttt{ raises } E, \ldots, E
\end{array}
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Annotations

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\begin{array}{lcl}
t & ::= & c \mid x \mid \texttt{!}x \mid \phi(t, \ldots, t) \mid \texttt{old}(t) \mid \texttt{at}(t, L) \\
p & ::= & \text{True} \mid \text{False} \mid P(t, \ldots, t) \\
 & \mid & p \Rightarrow p \mid p \wedge p \mid p \vee p \mid \neg\, p \mid \forall x : \beta.p \mid \exists x : \beta.p
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Programs

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u & ::= & c \mid x \mid \,!x \mid \phi(u, \ldots, u) \\
e & ::= & u \\
& \mid & x := e \\
& \mid & \texttt{let } x = e \texttt{ in } e \\
& \mid & \texttt{let } x = \texttt{ref } e \texttt{ in } e \\
& \mid & \texttt{if } e \texttt{ then } e \texttt{ else } e \\
& \mid & \texttt{loop } e \,\{\texttt{invariant } p \texttt{ variant } t\} \\
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Typing

A typing judgment

$$\Gamma \vdash e : (\tau, \epsilon)$$

Rules given in the notes (page 24)

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Semantics

Call-by-value semantics, with left to right evalutation

Big-step operational semantics given in the notes (page 26)

Part II

Proof rules

Weakest preconditions

We define the predicate $wp(e, q)$, called the weakest precondition for program e and postcondition q

Property: If $wp(e, q)$ holds, then e terminates and q holds at the end of execution (and all inner annotations are verified)

The converse holds for the fragment without loops and functions

The correctness of an annotated program e is thus $wp(e, \mathrm{True})$

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Definition of $wp(e, q)$

We actually define $wp(e, q; r)$ where

- q is the "normal" postcondition
- $r \equiv E_1 \Rightarrow q_1; \ldots; E_n \Rightarrow q_n$ is the set of "exceptional" post.

Basic constructs

$$wp(u, q; r) = q[result \leftarrow u]$$

$$wp(x := e, q; r) = wp(e, q[result \leftarrow \texttt{void}; x \leftarrow result]; r)$$

$$wp(\texttt{let } x = e_1 \texttt{ in } e_2, q; r) = wp(e_1, wp(e_2, q; r)[x \leftarrow result]; r)$$

$$wp(\texttt{let } x = \texttt{ref } e_1 \texttt{ in } e_2, q; r) = wp(e_1, wp(e_2, q; r)[!x \leftarrow result]; r)$$

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Function call

Simplified using

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we define

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Part III

The WHY tool

Dijkstra's Dutch national flag

Goal: to sort an array where elements only have three different values (blue, white and red)

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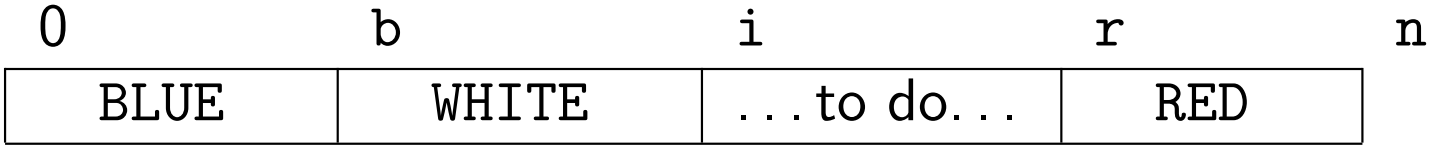

A few lines of C code

```
typedef enum { BLUE, WHITE, RED } color;

void swap(int t[], int i, int j) {
  color c = t[i]; t[i] = t[j]; t[j] = c;
}

void flag(int t[], int n) {
  int b = 0, i = 0, r = n;
  while (i < r) {
    switch (t[i]) {
    case BLUE: swap(t, b++, i++); break;
    case WHITE: i++; break;
    case RED: swap(t, --r, i); break;
    }
  }
}
```

Modelization

We are not verifying the C code, but rather the algorithm

We model

- colors with an abstract datetype
- arrays using references containing functional arrays

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An abstract type for colors

```
type color

logic blue : color
logic white : color
logic red : color

predicate is_color(c:color) = c=blue or c=white or c=red

parameter eq_color  :
  c1:color → c2:color →
    {} bool { if result then c1=c2 else c1≠c2 }
```

Functional arrays

```
type color_array

logic acc : color_array, int → color
logic upd : color_array, int, color → color_array

axiom acc_upd_eq :
  ∀a:color_array. ∀i:int. ∀c:color.
    acc(upd(a,i,c),i) = c

axiom acc_upd_neq :
  ∀a:color_array. ∀i,j:int. ∀c:color.
    i ≠ j → acc(upd(a,j,c),i) = acc(a,i)
```

Array bounds

```
logic length : color_array → int

axiom length_update :
  ∀a:color_array. ∀i:int. ∀c:color.
    length(upd(a,i,c)) = length(a)

parameter get :
  t:color_array ref → i:int →
    { 0<=i<length(t) } color reads t { result=acc(t,i) }

parameter set :
  t:color_array ref → i:int → c:color →
    { 0<=i<length(t) } unit writes t { t=upd(t@,i,c) }
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The swap function

```
let swap (t:color_array ref) (i:int) (j:int) =
  { 0 <= i < length(t) and 0 <= j < length(t) }
  let u = get t i in
  set t i (get t j);
  set t j u
  { t = upd(upd(t@,i,acc(t@,j)), j, acc(t@,i)) }
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5 proofs obligations

- 3 automatically discharged by WHY
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Function code

```
let dutch_flag (t:color_array ref) (n:int) =
  let b = ref 0 in
  let i = ref 0 in
  let r = ref n in
  while !i < !r do
     if eq_color (get t !i) blue then begin
       swap t !b !i;
       b := !b + 1;
       i := !i + 1
     end else if eq_color (get t !i) white then
       i := !i + 1
     else begin
       r := !r - 1;
       swap t !r !i
     end
  done
```

Function specification

```
predicate monochrome(t:color_array,i:int,j:int,c:color) =
  ∀k:int. i<=k<j → acc(t,k)=c

let dutch_flag (t:color_array ref) (n:int) =
  { 0 <= n and length(t) = n and
    ∀k:int. 0 <= k < n → is_color(acc(t,k)) }
  ⋮
  { ∃b:int. ∃r:int.
      monochrome(t,0,b,blue) and
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Loop invariant

```
  ⋮
while !i < !r do
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               monochrome(t,r,n,red) and
               length(t) = n and
               ∀k:int. 0 <= k < n → is_color(acc(t,k))
     variant r - i }
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done
```

Proof obligations

11 proof obligations

- loop invariant holds initially
- loop invariant is preserved and variant decreases (3 cases)
- `swap` precondition (twice)
- array access within bounds (twice)
- postcondition holds at the end of function execution

All automatically discharged by Simplify!

Note: to be exhaustive, one has to show that the set of elements was not changed

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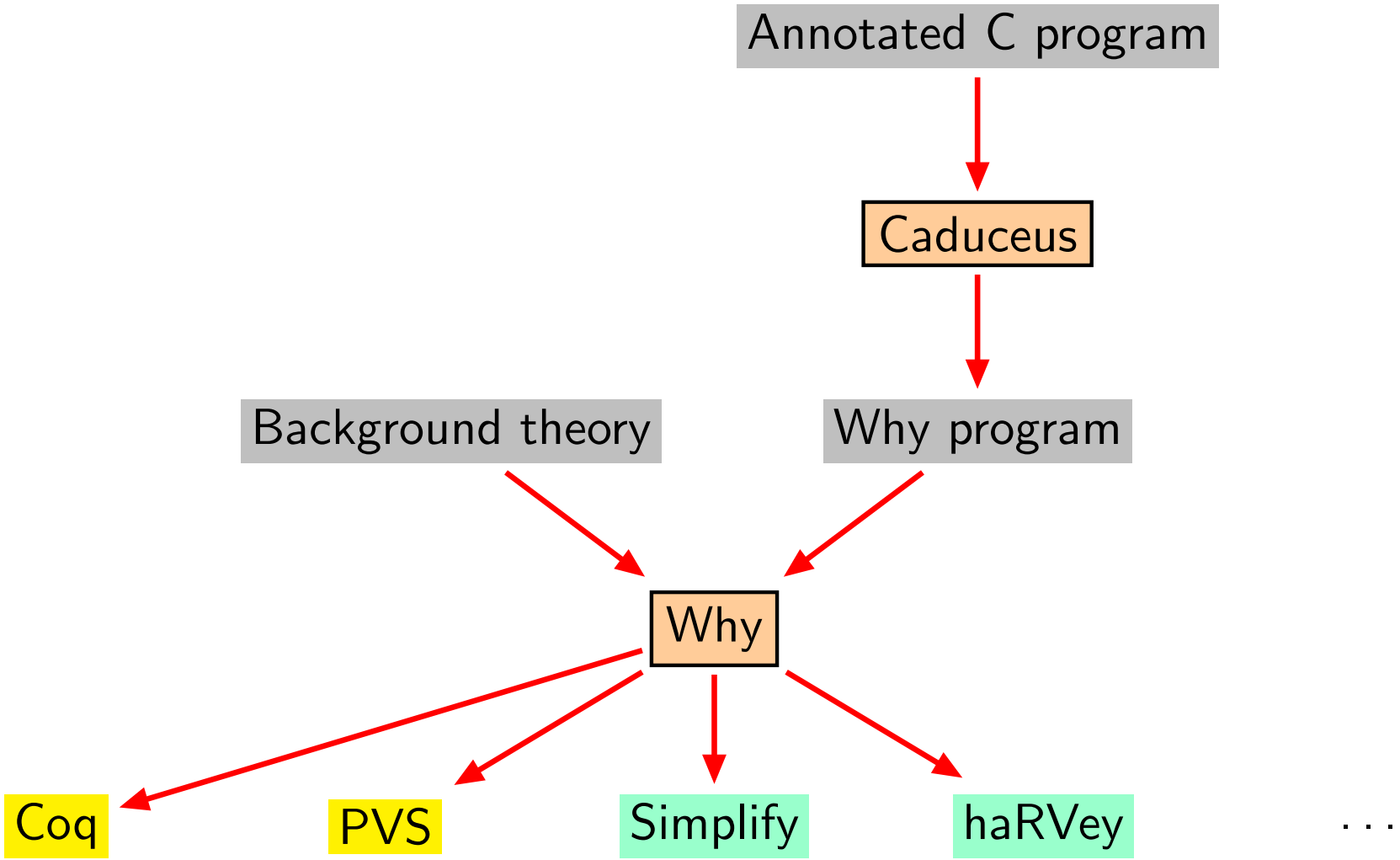
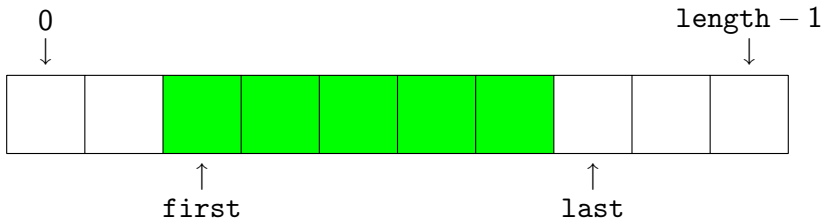
Part IV

Verification of C programs

Overview


Annotated C program
Caduceus
Background theory
Why program
Why
Coq
PVS
Simplify
haRVey
...

Example: character queue as circular array




```
struct queue {
  char contents[];
  int length;
  int first, last;
  unsigned int empty, full :1;
} q;
/*@ invariant q_invariant :
  @   \valid_range(q.contents, 0, q.length-1) &&
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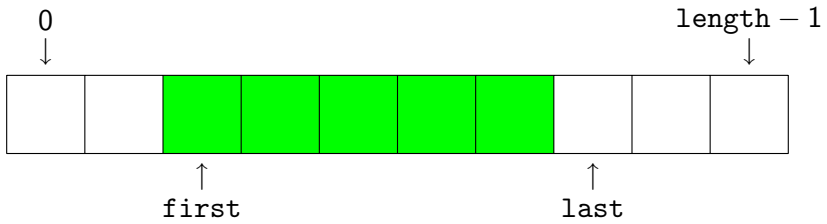


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  @*/
```

Example continued: specifying functions

```
/*@ requires !q.full
  @ assigns  q.empty, q.full, q.last, q.contents[q.last]
  @ ensures  !q.empty && q.contents[\old(q.last)] == c
  @*/
void push(char c);

/*@ requires !q.empty
  @ assigns q.empty, q.full, q.first
  @ ensures !q.full && \result == q.contents[\old(q.first)]
  @*/
char pop();
```

Example continued: body for push function

```
/*@ requires !q.full
  @ assigns  q.empty, q.full, q.last, q.contents[q.last]
  @ ensures  !q.empty && q.contents[\old(q.last)] == c
  @*/
void push(char c) {
  q.contents[q.last++] = c;      // insert 'c' in the queue
  if (q.last == q.length)
        q.last = 0;              // wrap if needed
  q.empty = 0;                   // queue is not empty
  q.full = (q.first == q.last); // queue is full if
                                 // 'last' reaches 'first'
}
```

Modeling C memory heap

- Burstall-Bornat model: memory partition according to structure fields
- We extend this idea to handle C arrays and pointer arithmetic: a memory block is

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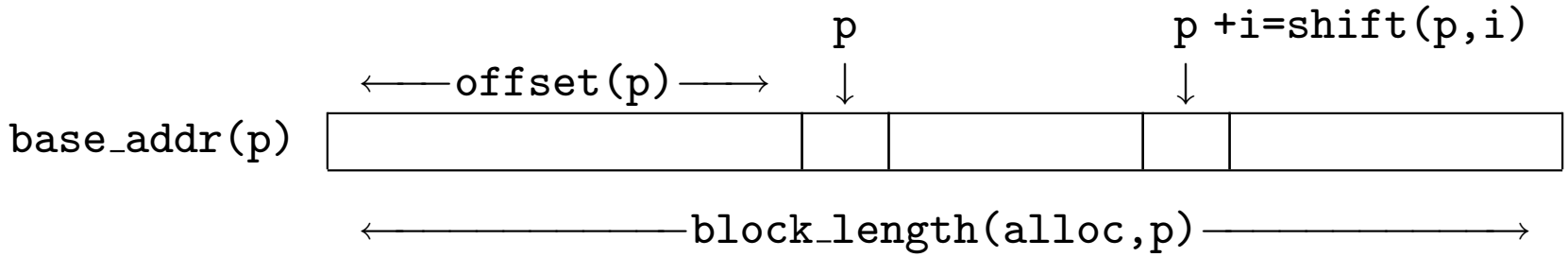

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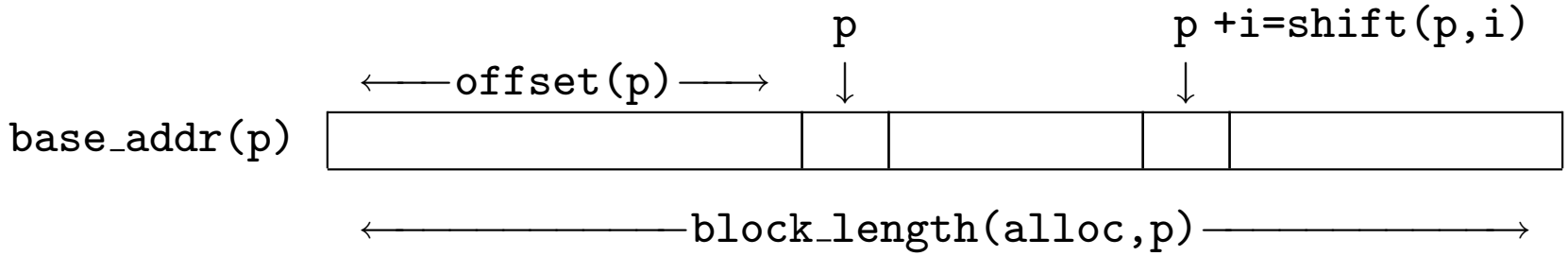

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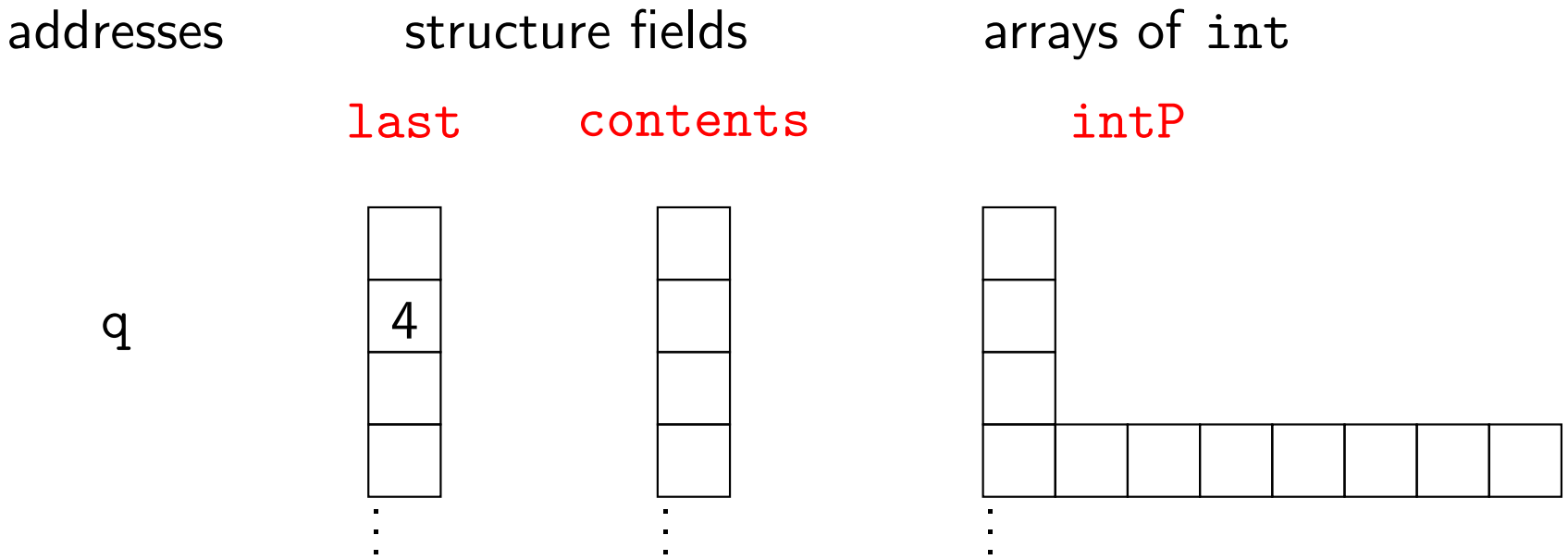
Burstall-Bornat model: example of character queue

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q.contents[q.last++] = c
    q.last <- 4 + 1
    *(q.contents+4) <- c
```

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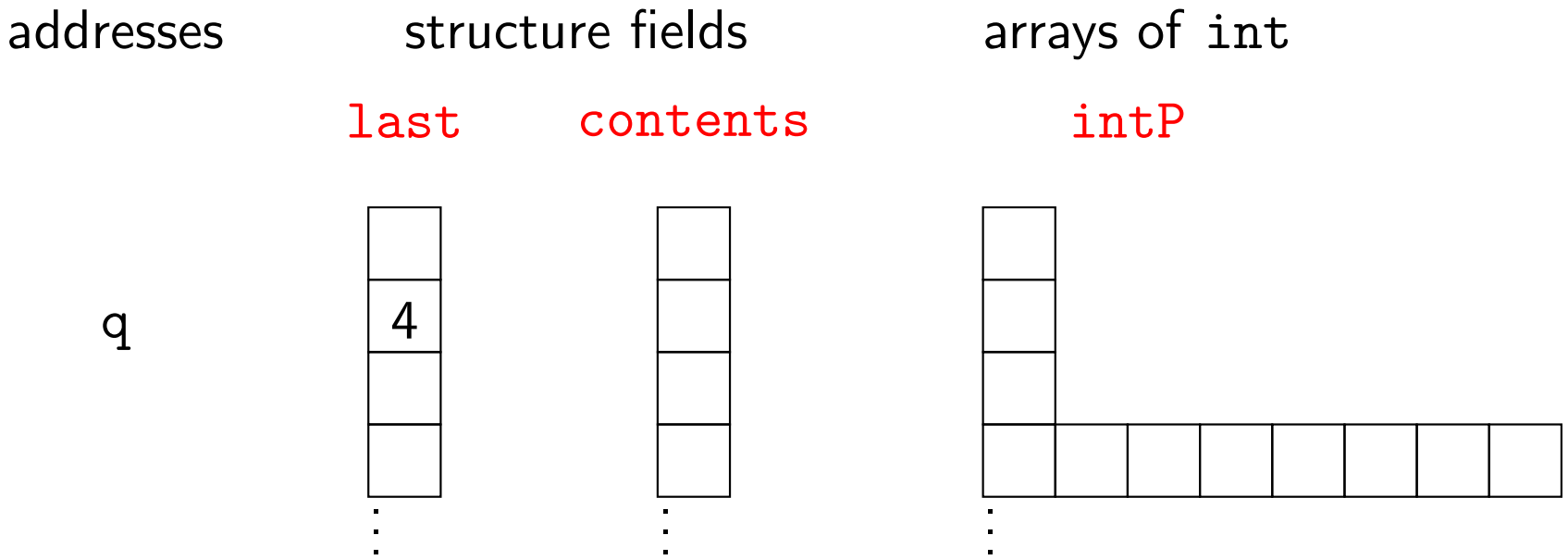


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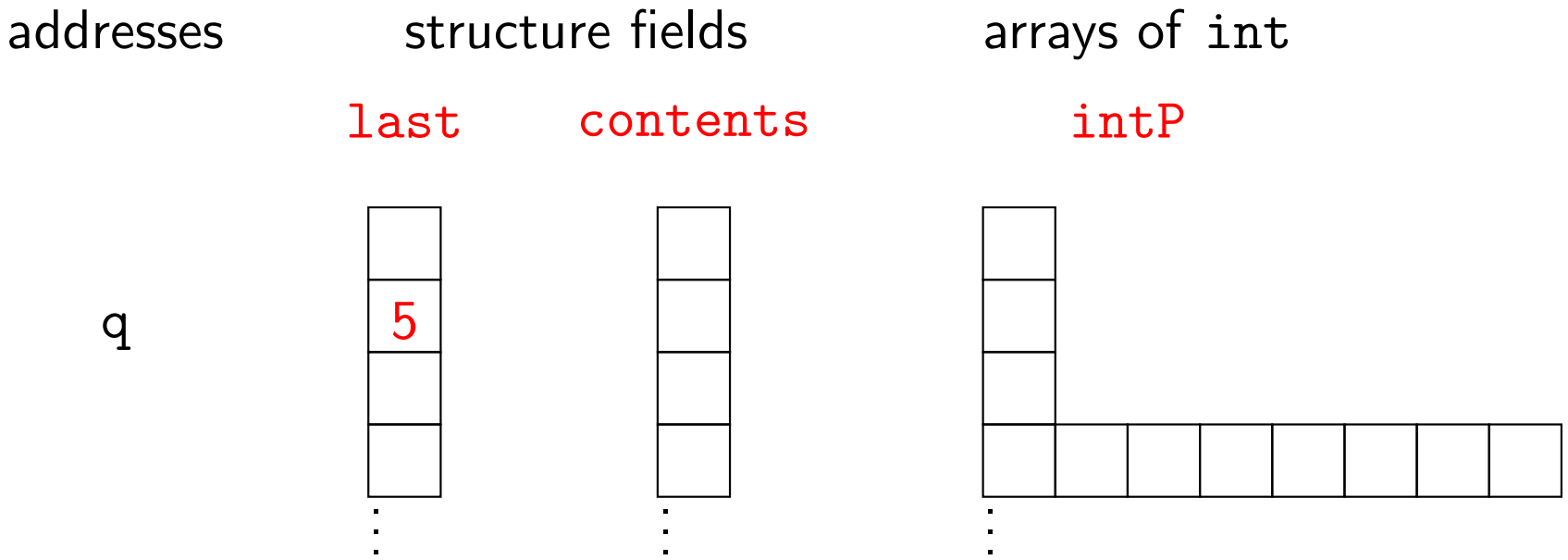


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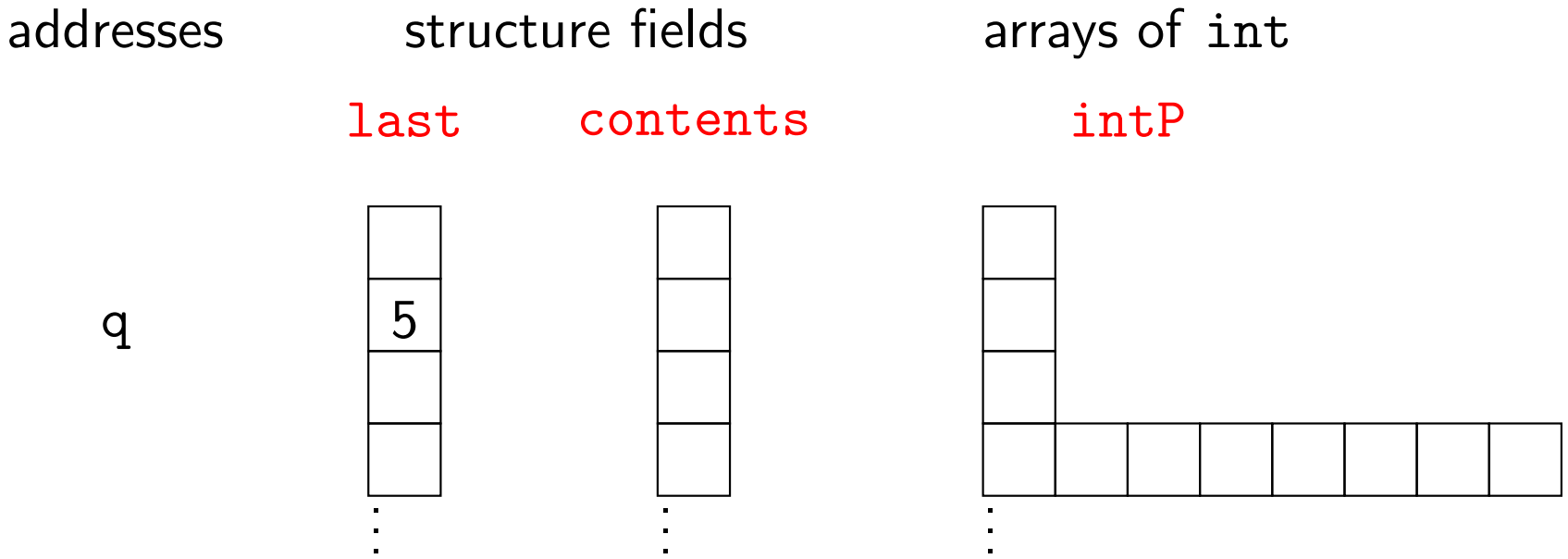


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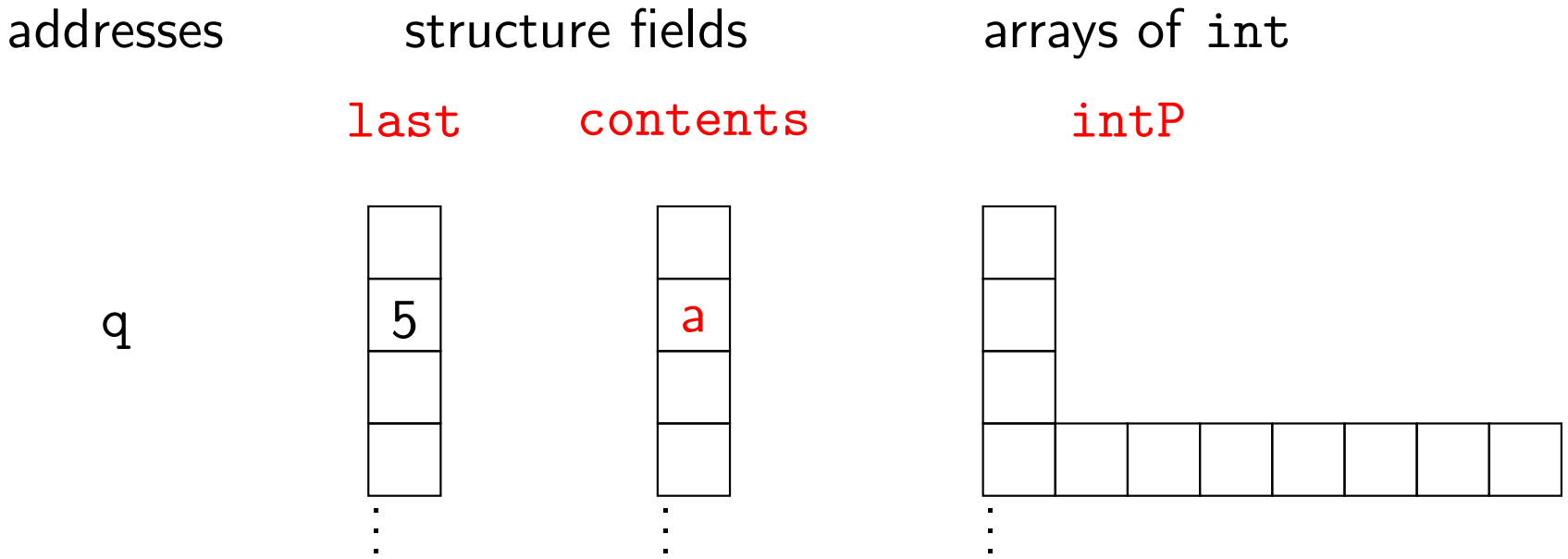


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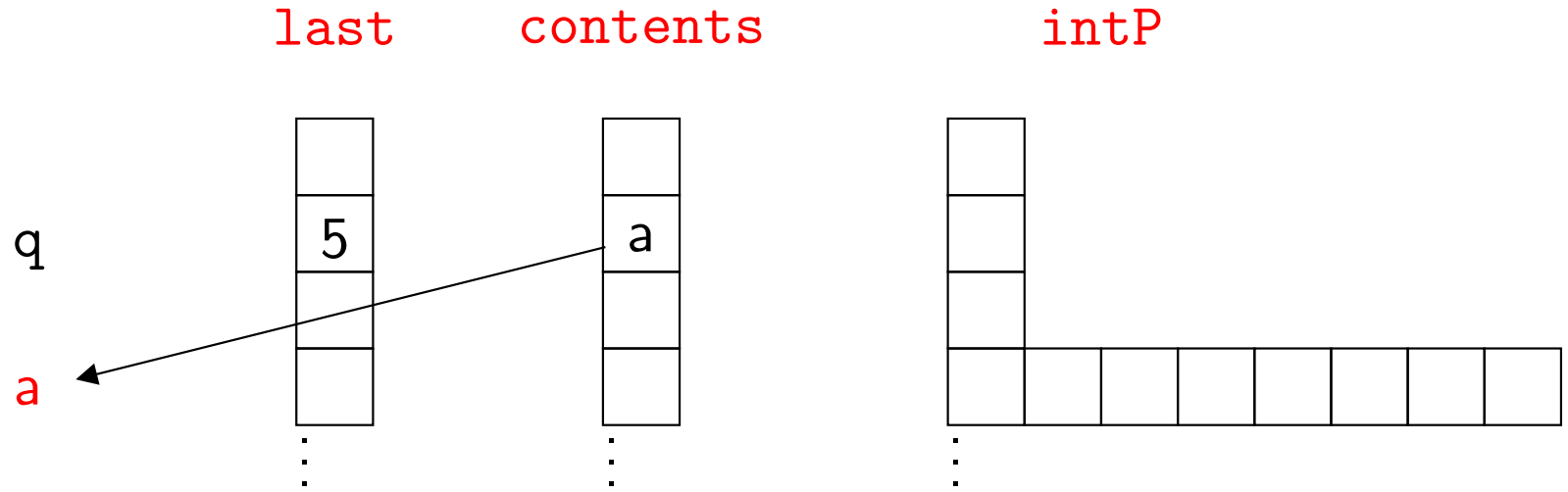
addresses structure fields arrays of `int`

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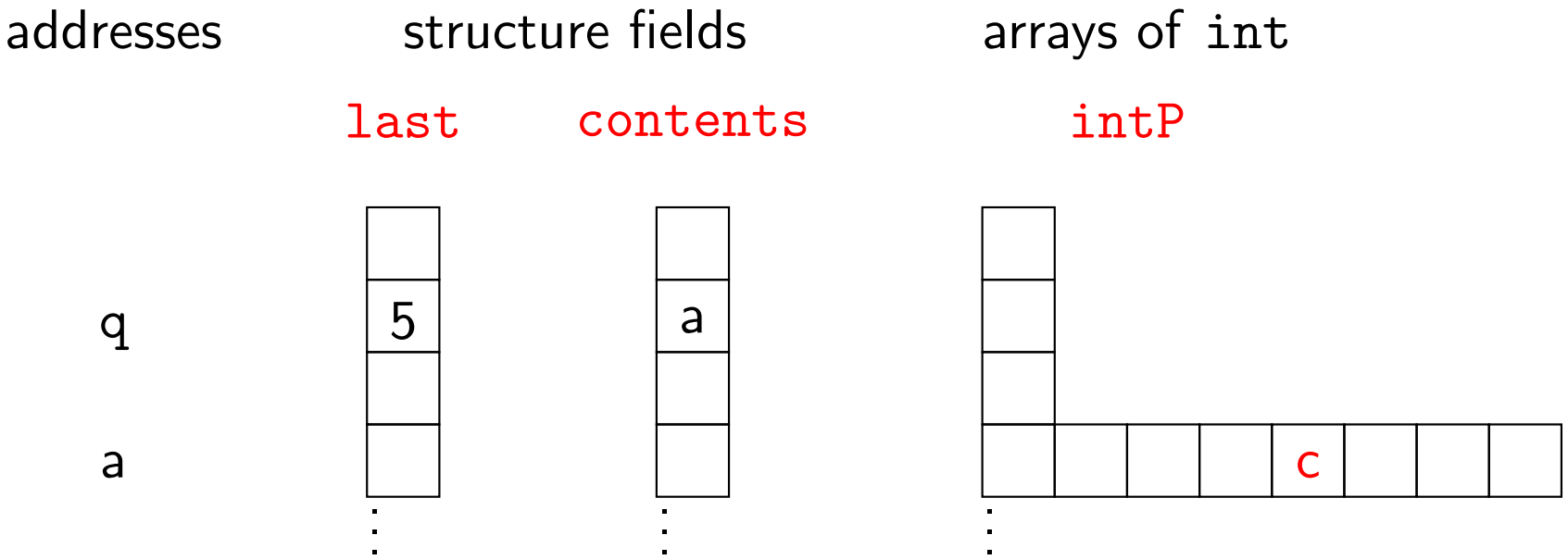
```

Burstall-Bornat model: example of character queue




```
q.contents[q.last++] = c
    q.last <- 4 + 1
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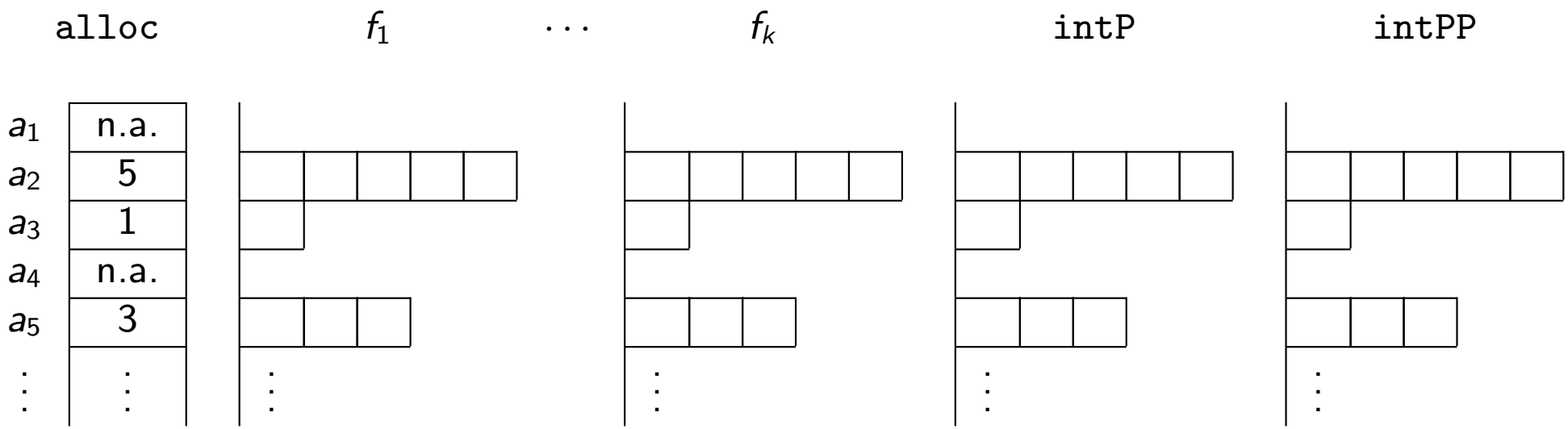
General structure of C memory heap



Translation of C statements into Why

The C statement

```
q.contents[q.last++] = c
```

becomes in Why:

```
assert { valid(!alloc,q) };    // proof obligation
let tmp1 = acc(!last,q) in     // tmp1 <- q.last
last := upd(!last,q,tmp1+1);   // q.last <- tmp1+1
let tmp2 = shift(acc(!contents,q),tmp1) in
                               // tmp2 <- q.contents + tmp1
assert { valid(!alloc,tmp2) };// proof obligation
intP := upd(!intP,tmp2,c)      // *tmp2 <- c
```

Axiomatization

- The abstract Why functions `acc`, `upd`, `shift`, etc. are specified by axioms: the background theory
- Excerpt from this theory:

```
acc(upd(t,i,v),i) = v
i <> j -> acc(upd(t,i,v),j) = acc(t,j)
shift(p,0) = p
shift(shift(p,i),j) = shift(p,i+j)
...
```

- An important part of this theory is dedicated to assigns clauses

Example continued: certification of `push` function

Caduceus produces 3 verification conditions expressing that

- the code of `push` contains no unallocated pointer dereference (e.g. assignment of `q.contents[q.last++]` is valid)
- the postcondition and the `assigns` clause of `push` are established
- the invariant `q_invariant` is preserved by `push`

Proofs of these obligations

- with Simplify (100%) and CVC Lite (67%)
- with Coq (100%), very easy (6 lines of tactics)

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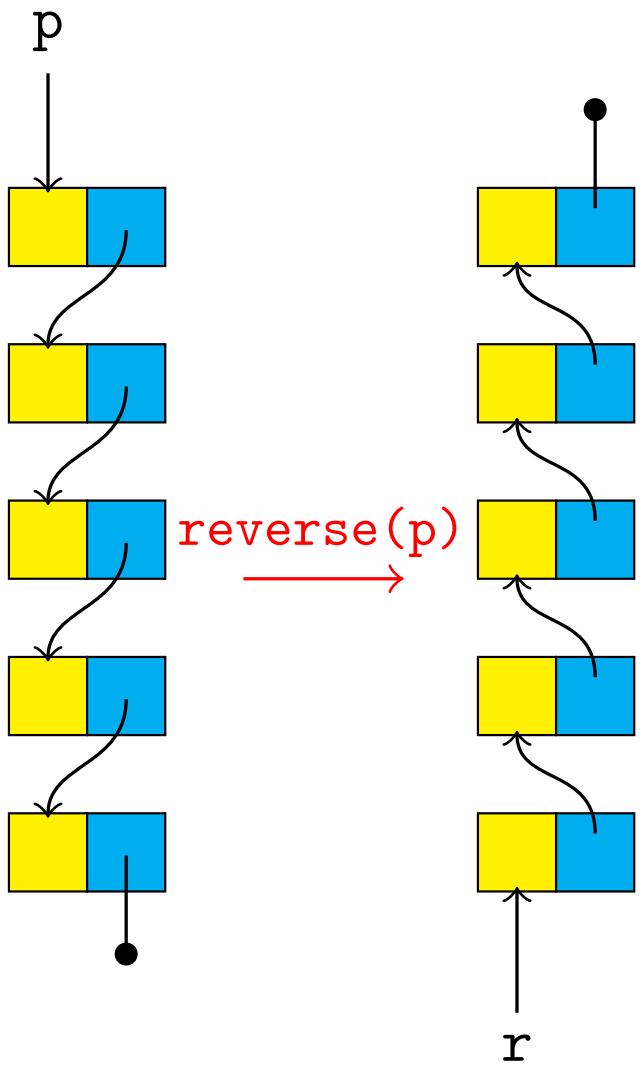
Example: in-place list reversal

```
typedef struct struct_list {
  int hd;
  struct struct_list *tl;
} *list;

list reverse(list p) {
  list r = NULL;
  while (p != NULL) {
    list q = p ;
    p = p->tl;
    q->tl = r;
    r = q;
  }
  return r;
}
```



Introduction of new logical types and functions

- New predicates and functions can be introduced

```
// logical finite list of pointers
//@ logic plist nil()
//@ logic plist cons(list p, plist l)

// concatenation and reversal
//@ logic plist app(plist l1, plist l2)
//@ logic plist rev(plist pl)
```

- Axioms may be given, e.g.

```
//@ axiom app_nil : \forall plist l; app(nil(),l) == l
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Specification of list reversal

```
/* llist(p,l) specifies that l is the list of pointers
   from p to NULL following tl fields */
//@ predicate llist(list p, plist l) reads p->tl

// is_list(p) specifies that p is finite
//@ predicate is_list(list p) { \exists plist l ; llist(p,l) }

/*@ requires is_list(p)
  @ ensures \forall plist l;
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list reverse(list p);
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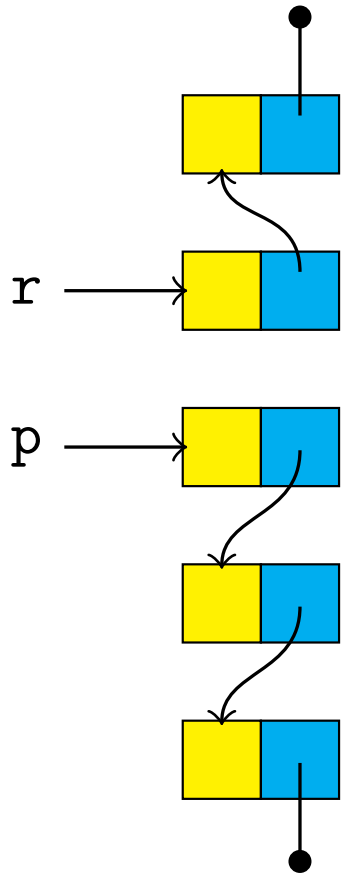
Annotating the code of list reversal

```
list reverse(list p) {
  list r = NULL;
/*@ invariant
    \exists plist lp; \exists plist lr;
      llist(p, lp) && llist(r, lr) &&
      disjoint(lp, lr) &&
      \forall plist l; \old(llist(p, l)) =>
        app(rev(lp), lr) == rev(l)
  @ variant length(p) for length_order */
  while (p != NULL) {
    list q = p;
    p = p->tl; q->tl = r; r = q;
  }
  return r;
}
```



Certification of list reversal

- 7 verification conditions
- With Simplify: 71%
- With Coq: 100%, with 661 lines of tactics

Example: Schorr-Waite algorithm

- Graph marking algorithm
- Considered as a benchmark for the verification of pointer programs (Bornat, 1999, Jape system) (Nipkow-Mehta, 2003, Isabelle/HOL)
- 12 verification conditions
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Part V

Conclusion

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- We support a large subset of ANSI C and Java/JML
- Tools freely available
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But scaling up issues show up on large programs:

- Generated proof obligations can get large
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- Need for more automation of proofs, cooperation of provers

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- arithmetic overflow
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