Lecture 3: Semaphores

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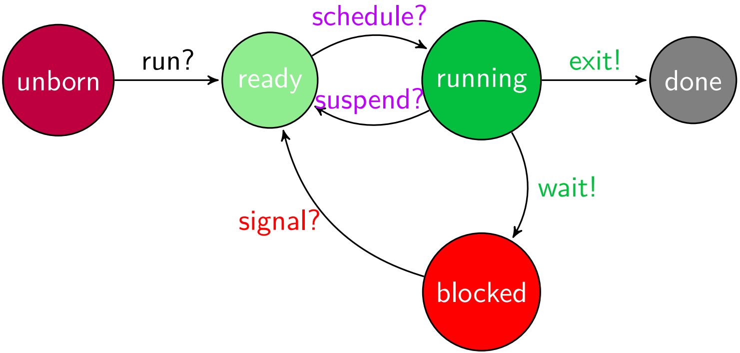
Process states, showing who causes the transitions

The picture as seen by process p.



Convention, borrowed from Hoare's CSP, is ! for speech and ? for hearing.

- The run? action is by the *parent* process (who creates p).
- exit! and wait! are the only actions taken p itself.
- The signal? action is taken by a process other than p.
- schedule? and suspend? are actions taken by the invisible scheduler. No process can tell whether p is ready or running.

Definition of general semaphore

semaphore is a type or class, with atomic methods *wait* and *signal*.

Data: A pair $\langle \text{int} \quad V, \text{set} \quad L \rangle$, where V is the number of *tokens* available, (each representing a *shared resource*) and L is the set of processes *blocked* on the semaphore.
Typically, V is initialised to the total number of tokens, and L to the empty set, $\emptyset$.

Method *wait*: if $V >$ 0 then V--
else $\{L := L \cup p$; //where p is the process doing the *wait*
block $p\}$ //when p is unblocked, it completes *wait*
//by simply exiting the method.

Method *signal*: if $L = \emptyset$ then V++
else $\{L := L$-q; //where q is an arbitrary process in L
make q ready$\}$

Writers often drop L, as though the semaphore is just V. But the blocking and unblocking of processes is associated with *wait* and *signal*.

Semaphore invariants

Let semaphore S be initialised to $\langle k, \emptyset \rangle$, where $k \geq 0$. Then the following are invariant:

1. $S.V \geq 0$
2. $S.V + \#wait(S) = k + \#signal(S)$

Proof by induction on number of semaphore instructions. (Other instructions do not affect $S.V$).

1. Base: True at initialisation.
   Step: *signal(S)* can only increase $S.V$;
   wait(S) decrements it by 1 only if $S.V > 0$.
2. Base: True at initialisation; no sem actions yet.
   Step: *wait* decrements $S.V$ only if it goes through. Otherwise neither $S.V$ nor $\#wait(S)$ change.
   signal always goes through. It increments either $S.V$ or $\#wait(S)$ by unblocking a process blocked on S.

Definition of binary semaphore

A *binary semaphore* is like a general semaphore, except that V can only be 0 or 1. Method *wait* is as for general semaphore, but *signal* changes.

Data: A pair $\langle$bool V, set $L\rangle$, where V=0 (resp. 1) means the *shared resource* is (un)available.
Typically, V is initialised to 1 (available), and L to $\emptyset$.

Method *wait*: if V = 1 then V:= 0
else {$L := L \cup p$; //where p is the process doing the *wait*
block p}

Method *signal*: if V = 1 then *undefined!*
else {if $L = \emptyset$ then V:= 1
else {$L := L$-q; //where q is an arbitrary process in L
make q ready}
}

The semaphore invariants hold for binary semaphores too.

CS problem for two processes, with semaphores

Reminder: we require that the program satisfy

- *mutex property*: if *p* is at p3 (abbr. "p3"), then $\neg$ q3
- *deadlock free*: p2 $\wedge$ q2 $\rightarrow$ p and q will not both be stuck waiting (i.e., p or q will progress to CS)
- *starvation free*: p2 $\rightarrow$ p will progress to CS

binary sem $S := \langle 0, \emptyset \rangle$	
process p	process q
while *true* {	while *true* {
p1: NCS;	q1: NCS;
p2: *wait(S)*; //entry protocol	q2: *wait(S)*; //entry protocol
p3: CS;	q3: CS;
p4: *signal(S)*; //exit protocol	q4: *signal(S)*; //exit protocol
};	}

Abbreviated CS program with binary semaphore

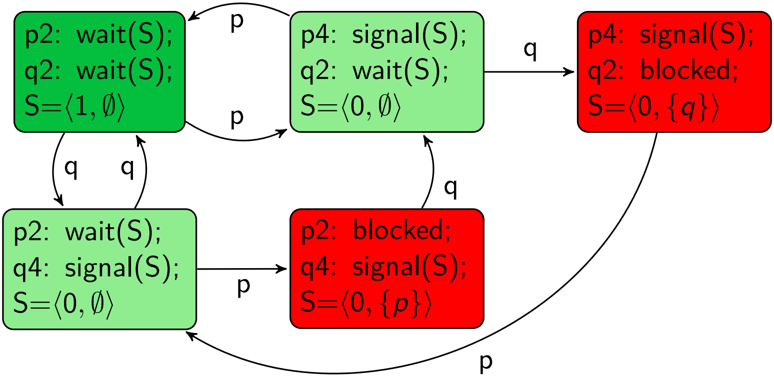
We remove the uninteresting commands to reduce the number of states we have to reason about, giving

binary sem $S := \langle 0, \emptyset \rangle$	
process p	process q
while *true* {	while *true* {
p2: *wait(S)*; //entry protocol	q2: *wait(S)*; //entry protocol
p4: *signal(S)*; //exit protocol	q4: *signal(S)*; //exit protocol
};	}

Reminder: At p4, *p* is yet to execute its exit protocol, so it is in its CS. Thus we require that the program satisfy

- *mutex property*: if *p* is at p4 (abbr. "p4"), then $\neg$ q4
- *deadlock free*: p2 $\wedge$ q2 $\rightarrow$ p and q will not both be stuck waiting (i.e., p or q will progress to CS)
- *starvation free*: p2 $\rightarrow$ p will progress to CS

State diagram, abbreviated CS program with binary sem



- The start state is at top left
- In the red states
  - one process is blocked, so only the other can move
  - only one move, by the process blocking itself, leads to a red state.
- In the green states, both can move
  - From the light green states, the system either moves back to the start state, or to a blocking state.

Correctness of semaphore CS program from state diagram

Mutex There is no state with p4 and q4. We can draw such a state, but it is not *reachable* from the start state of the program.

Deadlock There is no state where both processes are blocked. There is always a move from every reachable state.

Starvation If *p* is blocked, then *q* is poised to do a *signal*, i.e., *q* is in its CS. So it must in a finite time exit its CS (i.e., do the *signal*), and so in a finite time lead *p* into its CS.
If *p* is poised to do a *wait*, an unfair scheduler may let *q* loop around *wait* and *signal*. This is the only loop where *p* makes no progress to its CS. Since *p* is always ready to do its *wait*, a fair scheduler must let it act eventually and lead to ts CS.

Questions to ponder

Why is the semaphore defined this way? Why not let *signal* always increment *S.V* and let someone else (who?) get a waiting process to retry *wait*?

There are many other patterns to discover in the state diagram. Why 5 states? How odd that such a symmetric program produces an odd number of states!

Correctness of semaphore CS program by invariants

- Lemma: The initial value k of $S.V$ is 1 in this program, so the 2nd semaphore invariant becomes $S.V + \#wait(S) = 1 + \#signal(S)$.

  Let $\#\text{CS} = \#wait(S) - \#signal(S)$; it is the number of processes in their CSs. Then $\#\text{CS} = 1 - S.V$.

  Then for this program, $\#\text{CS} + S.V = 1$ is another form of the 2nd semaphore invariant.

- Mutex: The 1st semaphore invariant is $S.V \geq 0$, so $\#\text{CS} \leq 1$.
- Deadlock: If we are in deadlock, both processes are blocked, so it must be that $S.V = 0$. But also $\#\text{CS} = 0$. Contradicts the above, so deadlock is not possible.
- Starvation: Suppose p is starved, so $S.V = 0$ and $p \in S.L$. Then because $\#\text{CS} + S.V = 1$, it follows that $\#\text{CS} = 1$ and q is in its CS, and $S.L = \{p\}$. Then q has to do a $signal(S)$ and thus lead p to its CS.

Producer-consumer (PC) infinite buffer, with semaphores

An infinite buffer B holds items produced by *producer*, p, and consumed by *consumer*, c. While p can always act, c must wait if B is empty. Semaphore N is used to ensure this.

queue of int $B := \emptyset$ sem $N := \langle 0, \emptyset \rangle$	
process p	process c
int *d*;	int *d*;
while *true* {	while *true* {
p1: *append(d, B)*;	c1: *wait(N)*;//cons protocol
p2: *signal(N)*; //prod protocol	c2: *d:= take(B)*;
};	};

Note: p does the *signal(N)*, while c does the *wait(N)*. Note also that the CS and protocols occur in different orders in p and c.

Since the buffer can grow indefinitely, the state diagram can too. So we cannot use that for proofs.

Producer-consumer (PC) infinite buffer, invariants

We begin with a simplifying assumption: make the two actions of p into one atomic action, and similarly for q. For pedagogical reasons; the assumption can be removed!

Then $N.V = \#B$ is an invariant. True initially. Every atomic action by p increments both $N.V$ and $\#B$. Every atomic action by c decrements both $N.V$ and $\#B$.

So *PC safety*: c never removes an item from an empty buffer.

Deadlock: Only c can block, and it won't as long as p produces. (p is allowed to stop; that is not a deadlock).

Starvation: Only c can block, and with a fair scheduler, it can always act as long as B is non-empty.

The last two arguments are degenerate cases.

Producer-consumer (PC) finite buffer, with semaphores

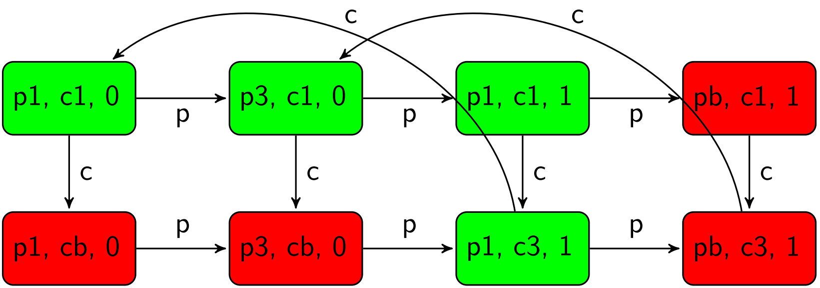
A finite buffer B holds up to N items produced by *producer*, p, and consumed by *consumer*, c. The conditions: c must wait if B is empty, and p must wait if B is full. Semaphores E and F are used to ensure this.

queue [capacity N] of int $B := \emptyset$ sem $E := \langle 0, \emptyset \rangle$, sem $F := \langle N, \emptyset \rangle$	
process p	process c
int *d*;	int *d*;
while *true* {	while *true* {
p1: *wait(F)*; //pre-protocol	c1: *wait(E)*; //pre-protocol
p2: *append(d, B)*;	c2: *d:= take(B)*;
p3: *signal(E)*; //post-protocol	c3: *signal(F)*; //post-protocol
};	};

NB: p does *wait(F)* and *signal(E)*, while q does *wait(E)* and *signal(F)*.

The *PC safety* requirement is that c never removes an item from an empty buffer, and that p never puts an item into a full buffer.

State diagram, abbreviated PC program, 1-place buffer



- *p1: wait(F)*, *p1: blocked* and *p1: signal(E)* are the three states of *p*, and similarly for *q*. The third parameter in each state notes whether there is an item in the buffer.
- The start state is at top left
- In the red states
  - one process is blocked, so only the other can move