

# Formal Methods for Software Development

## Reasoning about Programs with Dynamic Logic

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# Dynamic Logic

## (JAVA) Dynamic Logic

### Typed FOL

- ▶ + (JAVA) programs  $p$
- ▶ + modalities  $\langle p \rangle \phi$ ,  $[p] \phi$  ( $p$  program,  $\phi$  DL formula)
- ▶ + ... (later)

### Remark on Hoare Logic and DL

**In Hoare logic**  $\{Pre\} p \{Post\}$

(Pre, Post must be FOL)

**In DL**  $Pre \rightarrow [p]Post$

(Pre, Post any DL formula)

# Proving DL Formulas

## An Example

```
∀ int x;  
(x ≥ 0 ∧ n = x →  
  [ i = 0; r = 0;  
    while(i < n){i = i + 1; r = r + i;}  
    r = r + r - n;  
  ]r = x * x)
```

How can we prove that the above formula is valid  
(i.e. satisfied in all states)?

# Semantics of DL Sequents

$\Gamma = \{\phi_1, \dots, \phi_n\}$  and  $\Delta = \{\psi_1, \dots, \psi_m\}$  sets of DL formulas where all logical variables occur bound.

Recall:  $\mathcal{S} \models (\Gamma \Rightarrow \Delta)$  iff  $\mathcal{S} \models (\phi_1 \wedge \dots \wedge \phi_n) \rightarrow (\psi_1 \vee \dots \vee \psi_m)$

Define semantics of DL sequents identical to semantics of FOL sequents

## Definition (Validity of Sequents over DL Formulas)

A sequent  $\Gamma \Rightarrow \Delta$  over DL formulas is **valid** iff

$$\mathcal{S} \models (\Gamma \Rightarrow \Delta) \text{ in all states } \mathcal{S}$$

## Consequence for program variables

Initial value of program variables implicitly “**universally** quantified”

# Symbolic Execution of Programs

Sequent calculus decomposes top-level operator in formula.  
What is “top-level” in a sequential program  $p; q; r; ?$

## Symbolic Execution

- ▶ Follow the **natural control flow** when analysing a program
- ▶ Values of some variables unknown: **symbolic state representation**

## Example

Compute the final state after termination of

$x = x + y; y = x - y; x = x - y;$

# Symbolic Execution of Programs Cont'd

## Typical form of DL formulas in symbolic execution

$$\langle \text{stmt}; \text{rest} \rangle \phi \quad [\text{stmt}; \text{rest}] \phi$$

- ▶ Rules symbolically execute *first* statement (“**active statement**”)
- ▶ Repeated application of such rules corresponds to **symbolic program execution**

**Example** (`symbolicExecution/simpleIf.key`,  
**Demo**, active statement only)

```
\programVariables {  
  int x; int y; boolean b;  
}  
\problem {  
  \<{ if (b) { x = 1; } else { x = 2; } y = 3; }\> y > x  
}
```

# Symbolic Execution of Programs Cont'd

## Symbolic execution of conditional

$$\text{if} \frac{\Gamma, b = \text{TRUE} \Rightarrow \langle p; \text{rest} \rangle \phi, \Delta \quad \Gamma, b = \text{FALSE} \Rightarrow \langle q; \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{if } (b) \{ p \} \text{ else } \{ q \} ; \text{rest} \rangle \phi, \Delta}$$

Symbolic execution must consider all possible execution branches

## Symbolic execution of loops: unwind

$$\text{unwindLoop} \frac{\Gamma \Rightarrow \langle \text{if } (b) \{ p; \text{while } (b) p \}; \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{while } (b) \{ p \}; \text{rest} \rangle \phi, \Delta}$$

# Updates for KeY-Style Symbolic Execution

## Needed: a Notation for Symbolic State Changes

- ▶ Symbolic execution should “walk” through program in natural **forward** direction
- ▶ Need **succinct representation** of state changes, effected by each symbolic execution step
- ▶ Want to **simplify** effects of program execution **early**
- ▶ Want to **apply** state changes **late**  
(to branching conditions and post condition)

We use dedicated notation for state changes: **updates**



# Explicit State Updates

## Definition (Syntax of Updates, Updated Terms/Formulas)

If  $v$  is program variable,  $t$  FOL term type-conformant to  $v$ ,  $t'$  any FOL term, and  $\phi$  any DL formula, then

- ▶  $\{v := t\}$  is an update
- ▶  $\{v := t\}t'$  is DL term
- ▶  $\{v := t\}\phi$  is DL formula

## Definition (Semantics of Updates)

State  $\mathcal{S}$  interprets program variables  $v$  with  $\mathcal{I}_{\mathcal{S}}(v)$ ,  $\beta$  variable assignment for logical variables in  $t$ , define semantics  $\rho$  as:

$$\rho_{\beta}(\{v := t\})(\mathcal{S}) = \mathcal{S}' \text{ where } \mathcal{S}' \text{ identical to } \mathcal{S} \text{ except } \mathcal{I}_{\mathcal{S}'}(v) = \text{val}_{\mathcal{S},\beta}(t)$$

# Explicit State Updates Cont'd

## Facts about updates $\{v := t\}$

- ▶ Update semantics similar to that of assignment
- ▶ Value of update also depends on  $\mathcal{S}$  and **logical** variables in  $t$ , i.e.,  $\beta$
- ▶ Updates are **not assignments**: right-hand side is FOL term
  - $\{x := n\}\phi$  cannot be turned into assignment if  $n$  is a logical variable
  - $\{x=i++;\}\phi$  cannot (immediately) be turned into update
- ▶ Updates are **not equations**: they **change** value of  $v$

# Computing Effect of Updates (Automated)

Rewrite rules for update followed by ...

program variable  $\left\{ \begin{array}{l} \{x := t\}x \rightsquigarrow t \\ \{x := t\}y \rightsquigarrow y \end{array} \right.$

logical variable  $\{x := t\}w \rightsquigarrow w$

complex term  $\{x := t\}f(t_1, \dots, t_n) \rightsquigarrow f(\{x := t\}t_1, \dots, \{x := t\}t_n)$

atomic formula  $\{x := t\}p(t_1, \dots, t_n) \rightsquigarrow p(\{x := t\}t_1, \dots, \{x := t\}t_n)$

FOL formula  $\left\{ \begin{array}{l} \{x := t\}(\phi \ \& \ \psi) \rightsquigarrow \{x := t\}\phi \ \& \ \{x := t\}\psi \\ \dots \\ \{x := t\}(\forall \tau y; \phi) \rightsquigarrow \forall \tau y; (\{x := t\}\phi) \end{array} \right.$

program formula No rewrite rule for  $\{x := t\}\langle prog \rangle\phi$

Substitution delayed until *prog* symbolically executed

# Assignment Rule Using Updates

## Symbolic execution of assignment using updates

$$\text{assign} \frac{\Gamma \Rightarrow \{x := t\} \langle rest \rangle \phi, \Delta}{\Gamma \Rightarrow \langle x = t; rest \rangle \phi, \Delta}$$

- ▶ Works as long as  $t$  is 'simple' (has no side effects)
- ▶ For every built-in Java operation, we need a separate rule (for  $x = t_1 + t_2$  and  $x = t_1 - t_2$  etc.)

## Demo

updates/assignmentToUpdate.key

How to apply updates on updates?

## Example

Symbolic execution of

```
t=x; x=y; y=t;
```

yields:

```
{t := x}{x := y}{y := t}
```

Need to compose three sequential state changes into a single one:

**parallel updates**

# Parallel Updates Cont'd

## Definition (Parallel Update)

A **parallel update** has the form  $\{v_1 := r_1 \parallel \dots \parallel v_n := r_n\}$ , where each  $\{v_i := r_i\}$  is simple update

- ▶ All  $r_i$  computed in **old state** before update is applied
- ▶ Updates of all program variables  $v_i$  executed **simultaneously**
- ▶ Upon **conflict**  $v_i = v_j, r_i \neq r_j$  later update ( $\max\{i, j\}$ ) wins

## Definition (Parallelising Updates, Conflict Resolution)

$$\{v_1 := r_1\}\{v_2 := r_2\} = \{v_1 := r_1 \parallel v_2 := \{v_1 := r_1\}r_2\}$$

$$\{v_1 := r_1 \parallel \dots \parallel v_n := r_n\}x = \begin{cases} x & \text{if } x \notin \{v_1, \dots, v_n\} \\ r_k & \text{if } x = v_k, x \notin \{v_{k+1}, \dots, v_n\} \end{cases}$$

# Symbolic Execution with Updates (by Example)

$$\begin{aligned} & x < y \implies x < y \\ & \vdots \\ & x < y \implies \{x:=y \parallel y:=x\} \langle \rangle y < x \\ & \vdots \\ & x < y \implies \{t:=x \parallel x:=y \parallel y:=x\} \langle \rangle y < x \\ & \vdots \\ & x < y \implies \{t:=x \parallel x:=y\} \{y:=t\} \langle \rangle y < x \\ & \vdots \\ & x < y \implies \{t:=x\} \{x:=y\} \langle y=t; \rangle y < x \\ & \vdots \\ & x < y \implies \{t:=x\} \langle x=y; y=t; \rangle y < x \\ & \vdots \\ & \implies x < y \rightarrow \langle t=x; x=y; y=t; \rangle y < x \end{aligned}$$

# Parallel Updates Cont'd

Demo

updates/swap1.key



# Parallel Updates Cont'd

## Example

symbolic execution of  $x=x+y; y=x-y; x=x-y;$  gives

$$(\{x := x+y\}\{y := x-y\})\{x := x-y\}$$
$$\{x := x+y \parallel y := (x+y)-y\}\{x := x-y\}$$
$$\{x := x+y \parallel y := (x+y)-y \parallel x := (x+y)-((x+y)-y)\}$$
$$\{x := x+y \parallel y := x \parallel x := y\}$$
$$\{y := x \parallel x := y\}$$

In case of conflict, KeY only keeps winning update

Parallel updates store intermediate state of symbolic computation

# Another use of Updates

If you would like to quantify over a program variable ...

**Not allowed:**  $\forall \tau i; \langle \dots i \dots \rangle \phi$   
(program variables  $\cap$  logical variables =  $\emptyset$ )

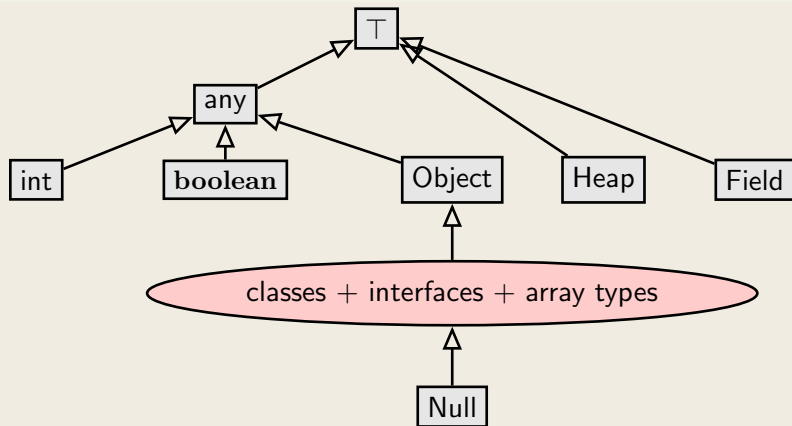
## Instead

Quantify over **value**, and **assign** it to program variable:

$\forall \tau x; \{i := x\} \langle \dots i \dots \rangle \phi$

# Modelling Java in FOL: Fixing a Type Hierarchy

Signature based on Java's type hierarchy



Each interface and class in API and in target program becomes type with appropriate subtype relation

# Modelling the Heap in FOL

## The Java Heap

Objects are stored on (i.e., in) the **heap**.

- ▶ Status of heap changes during execution
- ▶ Each heap associates values to object/field pairs

## The Heap Model of KeY-DL

Each element of data type Heap represents a certain heap status.

Two functions involving heaps:

- ▶ in  $F_{\Sigma}$ : Heap `store(Heap, Object, Field, any)` ;  
`store( $h, o, f, v$ )` returns heap like  $h$ , but with  $v$  associated to  $o.f$
- ▶ in  $F_{\Sigma}$ : any `select(Heap, Object, Field)` ;  
`select( $h, o, f$ )` returns value associated to  $o.f$  in  $h$

# Modelling the Heap in FOL

## Modelling instance fields

Person
<code>int age</code> <code>int id</code>
<code>int setAge(int newAge)</code> <code>int getId()</code>

- ▶ for each JAVA reference type  $C$  there is a type  $C \in T_\Sigma$ , for example, `Person`
- ▶ for each field  $f$  there is a **unique** constant  $f$  of type `Field`, for example, `id`
- ▶ domain of all `Person` objects:  $D^{\text{Person}}$
- ▶ a heap relates objects and fields to values

## Reading Field `id` of `Person p`

**FOL notation** `select(h, p, id)`

**Key notation** `p.id@h` ( abbreviating `select(h, p, id)` )  
`p.id` ( abbreviating `select(heap, p, id)` )<sup>a</sup>

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<sup>a</sup>`heap` is special program variable for “current” heap; mostly implicit in `o.f`

# Modelling the Heap in FOL

## Modelling instance fields

Person
int age int id
int setAge(int newAge) int getId()

- ▶ for each JAVA reference type  $C$  there is a type  $C \in T_{\Sigma}$ , for example, Person
- ▶ for each field  $f$  there is a **unique** constant  $f$  of type Field, for example, id
- ▶ domain of all Person objects:  $D^{\text{Person}}$
- ▶ a heap relates objects and fields to values

## Writing to Field id of Person p

**FOL notation**  $\text{store}(h, p, \text{id}, 6238)$

**KeY notation**  $h[p.\text{id} := 6238]$  ( notation for store, not update )

# The Algebra of Heaps

We do *not* formalise the *structure* (implementation) of heaps.  
We formalise the *behaviour*, with an algebra of heap operations:

$$\text{select}(\text{store}(h, o, f, v), o, f) = v$$

$$(o \neq o' \vee f \neq f') \rightarrow \text{select}(\text{store}(h, o, f, x), o', f') = \text{select}(h, o', f')$$

## Example

$$\text{select}(\text{store}(h, o, f, 15), o, f) \rightsquigarrow 15$$

$$\text{select}(\text{store}(h, o, f, 15), o, g) \rightsquigarrow \text{select}(h, o, g)$$

$$\text{select}(\text{store}(h, o, f, 15), u, f) \rightsquigarrow$$

$$\text{if } (o = u) \text{ then } (15) \text{ else } (\text{select}(h, u, f))$$

# Pretty Printing

## Shorthand Notations for Heap Operations

$o.f@h$  is `select(h, o, f)`

$h[o.f := v]$  is `store(h, o, f, v)`

*therefore:*

$u.f@h[o.f := v]$  is `select(store(h, o, f, v), u, f)`

$h[o.f := v][o'.f' := v']$  is `store(store(h, o, f, v), o', f', v')`

## Very-Shorthand Notations for **Current** Heap

Current heap always in special variable `heap`.

$o.f$  is `select(heap, o, f)`

$\{o.f := v\}$  is `update {heap := heap[o.f := v]}`



# Modelling the Heap in FOL—The Full Story

Is formula `select(h, p, id) >= 0` **type-safe?**

1. Return type is any—need to 'cast' to `int`
2. There can be many fields with name `id`

## Real Field Access

`int::select(h, p, Person::$id) >= 0` is type-safe

- ▶ `int::select` is a function name, not a cast
- ▶ can be understood *intuitively* as `(int)select`

## General

For each `T` typed field `f` of class `C`,  $F_\Sigma$  contains

- ▶ a constant declared as `Field C::$f`
- ▶ a function declared as `T T::select(Heap, C, Field)`

Everything **blue** is a function name

# Field Update Assignment Rule

## Changing the value of fields

How to (symbolically) execute assignment to field, e.g., `p.age=18;` ?

$$\text{assign} \frac{\Gamma \Rightarrow \{o.f := t\} \langle rest \rangle \phi, \Delta}{\Gamma \Rightarrow \langle o.f = t; rest \rangle \phi, \Delta}$$

Admit on left-hand side of update **JAVA location expressions**

But is this rule correct? See below.

# Field Update Assignment Rule

## Changing the value of fields

How to (symbolically) execute assignment to field, e.g., `p.age=18;` ?

$$\text{assign} \frac{\Gamma \Rightarrow \{p.age := 18\} \langle rest \rangle \phi, \Delta}{\Gamma \Rightarrow \langle p.age = 18; rest \rangle \phi, \Delta}$$

Admit on left-hand side of update **JAVA location expressions**

# Dynamic Logic: KeY input file

```
\javaSource "path to source code referenced in problem";  
  
\programVariables { Person p; }  
  
\problem {  
    \<{    p.age = 18;    }\> p.age = 18  
}
```

KeY reads in all source files and creates automatically the necessary signature (types, program variables, field constants)

## Demo

updates/firstAttributeExample.key

# Refined Semantics of Program Modalities

Does abrupt termination count as normal termination?

No! Need to distinguish **normal** and **exceptional** termination

- ▶  $\langle p \rangle \phi$ :  $p$  terminates **normally** and formula  $\phi$  holds in final state (total correctness)
- ▶  $[p] \phi$ : If  $p$  terminates **normally** then formula  $\phi$  holds in final state (partial correctness)

Abrupt termination on top-level counts as non-termination!

## Example Reconsidered: Exception Handling

```
\javaSource "path to source code";  
  
\programVariables {  
  ...  
}  
  
\problem {  
  p != null -> \<{   p.age = 18;   }\> p.age = 18  
}
```

Only provable when no top-level exception thrown

### Demo

updates/secondAttributeExample.key

# The Self Reference

## Modeling reference `this`

Special name for the object whose JAVA code is currently executed:

**in JML:** Object `this`;

**in Java:** Object `this`;

**in KeY:** Object `self`;

Default assumption in JML-KeY translation: `self != null`

# Which Objects do Exist?

How to model **object creation** with **new** ?

## Constant Domain Assumption

Assume that domain  $\mathcal{D}$  is the same in all states  $(\mathcal{D}, \delta, \mathcal{I}) \in States$

**Consequence:**

Quantifiers and modalities commute:

$$\models (\forall T x; [p]\phi) \leftrightarrow [p](\forall T x; \phi)$$



# Object Creation (background; no need to remember this)

## Realizing Constant Domain Assumption

- ▶ Implicitly declared field `boolean <created>` in class `Object`
- ▶ `<created>` has value `true` iff argument object has been created
- ▶ Object creation modeled as  $\{\text{heap} := \text{create}(\text{heap}, \text{ob})\}$  for not (yet) created `ob` (essentially sets `<created>` field of `ob` to `true`)

$$\frac{\Gamma, \text{ob}. \langle \text{created} \rangle = \text{FALSE} \Rightarrow \{\text{heap} := \text{create}(\text{heap}, \text{ob})\} \{o := \text{ob}\} \langle o. \langle \text{init} \rangle(\text{param}); \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle o = \text{new } T(\text{param}); \omega \rangle \phi, \Delta}$$

`ob` is a fresh program variable

Alternatives exist in the literature. E.g.:

[Ahrendt, de Boer, Grabe, *Abstract Object Creation in Dynamic Logic – To Be or Not To Be Created*, Springer, LNCS 5850]

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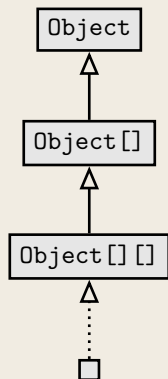
# Dynamic Logic to (almost) full Java

## KeY supports full **sequential** Java, with some limitations:

- ▶ Limited concurrency
- ▶ No generics
- ▶ No I/O
- ▶ No dynamic class loading or reflection
- ▶ Ongoing work to support floating point arithmetic
- ▶ API method calls: need either JML contract or implementation

# Java Features in Dynamic Logic: Arrays

## Arrays



- ▶ JAVA type hierarchy includes array types
- ▶ Types ordered according to JAVA subtyping rules
- ▶ Function  $\text{arr} : \text{int} \rightarrow \text{Field}$  turns integer index into type Field (required in store).
- ▶ Store array elements on heap
- ▶ Value of  $a[i]$  in heap  $\text{store}(\text{heap}, a, \text{arr}(i), 8)$  is 8
- ▶ Arrays  $a$  and  $b$  can refer to same object (aliasing)

# Java Features in Dynamic Logic: Complex Expressions

## Complex expressions with side effects

- ▶ JAVA expressions may have **side effects**, due to method calls, increment/decrement operators, nested assignments
- ▶ FOL terms have **no** side effect on the state

## Example (Complex expression with side effects in Java)

```
int i = 0; if ((i=2)>= 2) i++;   value of i ?
```

# Complex Expressions Cont'd

## Decomposition of complex terms by symbolic execution

Follow the rules laid down in JAVA Language Specification

### Local code transformations

$$\text{evalOrderIteratedAssgnmt} \frac{\Gamma \Rightarrow \langle y = t; x = y; \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle x = y = t; \omega \rangle \phi, \Delta} \quad t \text{ simple}$$

### Temporary variables store result of evaluating subexpression

$$\text{ifEval} \frac{\Gamma \Rightarrow \langle \text{boolean } v0; v0 = b; \text{if } (v0) \text{ p}; \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{if } (b) \text{ p}; \omega \rangle \phi, \Delta} \quad b \text{ complex}$$

# Java Features in Dynamic Logic: Abrupt Termination

## Abrupt Termination: Exceptions and Jumps

Redirection of control flow via return, break, continue, **exceptions**

$$\langle \text{try } \{p\} \text{ catch}(T \ e) \{q\} \text{ finally } \{r\} \ \omega \rangle \phi$$

Rule tryThrow matches **try-catch** in pre-/postfix and active throw

$$\Rightarrow \langle \text{if } (e \text{ instanceof } T) \{ \text{try} \{x=e; q\} \text{ finally } \{r\} \} \text{ else } \{r; \text{throw } e; \} \ \omega \rangle \phi$$
$$\Rightarrow \langle \text{try } \{ \text{throw } e; p \} \text{ catch}(T \ x) \{q\} \text{ finally } \{r\} \ \omega \rangle \phi$$

Demo

exceptions/try-catch.key

## Null pointer exceptions

There are no “exceptions” in FOL:  $\mathcal{I}$  total on FSym

Need to model possibility that  $o = \mathbf{null}$  in  $o.a$

- ▶ KeY branches over  $o = \mathbf{null}$  and  $o \neq \mathbf{null}$  upon each field access *within modalities*<sup>a</sup>
- ▶ Thereby,  $o.a$  appears *outside modalities* mostly under assumption  $o \neq \mathbf{null}$
- ▶  $\mathbf{null}.a$  *outside modalities* **has a value**, which is unknown

---

<sup>a</sup>Can be changed with Taclet Option runtimeExceptions



# Field Update Assignment Rule Revisited (A)

## Changing the value of fields

How to (symbolically) execute assignment to field?

$$\frac{\Gamma, o \neq \mathbf{null} \Rightarrow \{o.f := e\} \langle \pi \omega \rangle \phi, \Delta \quad \Gamma, o = \mathbf{null} \Rightarrow \langle \pi \mathbf{throw} \text{ new NullPointerException}(); \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \pi o.f = e; \omega \rangle \phi, \Delta}$$

$\pi$  is the “inactive prefix”, any number of opening try blocks:  $(\text{try}\{\})^*$

# Field Update Assignment Rule Revisited (B)

## Changing the value of fields

How to (symbolically) execute assignment to field?

$$\frac{\Gamma \Rightarrow \mathbf{o = null}, \{o.f := e\} \langle \pi \ \omega \rangle \phi, \Delta \quad \Gamma, \mathbf{o = null} \Rightarrow \langle \pi \ \mathbf{throw} \ \text{new} \ \text{NullPointerException}(); \ \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \pi \ \mathbf{o.f} = e; \ \omega \rangle \phi, \Delta}$$

$\pi$  is the “inactive prefix”, any number of opening try blocks:  $(\text{try}\{\})^*$

# Revisit: Field Assignment Demo

```
\javaSource "path to source code referenced in problem";  
  
\programVariables { Person p; }  
  
\problem {  
    \<{    p.age = 18;    }\> p.age = 18  
}
```

## Demo

updates/firstAttributeExample.key

# Java Features in Dynamic Logic: Aliasing

## Demo

aliasing/attributeAlias1.key

## Reference Aliasing

Alias resolution causes **proof split**

# Summary

- ▶ Most JAVA features covered in KeY
- ▶ Degree of automation for loop-free programs is very high
- ▶ Handling of loops: last lecture

# Literature for this Lecture

**KeYbook** *W. Ahrendt, B. Beckert, R. Bubel, R. Hähnle, P. Schmitt, M. Ulbrich, editors.*

**Deductive Software Verification - The KeY Book**

Vol 10001 of *LNCS*, Springer, 2016

(E-book at [link.springer.com](http://link.springer.com))

- ▶ B. Beckert, V. Klebanov, B. Weiß, **Dynamic Logic for Java**  
Chapter 3 in [KeYbook]  
on the surface only: Sections 3.1, 3.2, 3.4, 3.5.5, 3.5.6, 3.5.7, 3.6
- ▶ *W. Ahrendt, S. Grebing, Using the KeY Prover*  
Chapter 15 in [KeYbook]