Finite Automata Theory and Formal Languages
TMV027/DIT321

Turing Machines

Exercise 7

Basic exercises

1. Given the alphabet $\Sigma = \{I\}$, the Natural number n can be represented in a tape as n consecutive occurrences of I.

   Give both a high-level description and a transition diagram of a Turing machine computing the following operations on Natural numbers:

   (a) Successor and predecessor;
   (b) Addition and subtraction;

   Consider the input numbers on the tape separated by a blank symbol ($\square$).

2. For each of the languages below, give both a high-level description and a transition diagram of a Turing machine accepting the language.

   In each case, state whether your Turing machine is also a Turing decider or not.

   (a) $\{\sharp w \sharp w \mid w \in \{0,1\}^*\}$;
   (b) $\{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } w_1 \neq w_2\}$;
   (c) $\{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } \text{length}(w_1) < \text{length}(w_2)\}$;
   (d) $\{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } \text{length}(w_1) = \text{length}(w_2)\}$;

Additional exercises

1. Give a Turing machine for the multiplication of numbers given as in exercise 1) above.
2. Do as in exercise 2) above for the following languages:

   (a) $\{0^n 1^n 2^n \mid n \geqslant 0\}$;
   (b) $\{0^i 1^j 2^k \mid k = i * j\}$;
   (c) $\{0^i 1^j \mid j = i^2\}$.

Programming Exercises

1. Write a program that runs/simulates a Turing machine.