Finite Automata Theory and Formal Languages
TMV027/DIT321 – LP4 2018

Context-free Grammars

Assignment 5 – Deadline: Sunday 13th of May 23:59

Assignments should be done and submitted individually!

For obtaining full points the answers should contain enough explanation/description so that they are easy to understand.

1. (3 pts) Consider the following grammar over the alphabet $\{0, 1\}$:

   $$S \to 010 \mid 0S \mid S10 \mid SS$$

   Prove that for any $w \in \mathcal{L}(S)$, $\#_1(w) \leqslant \#_0(w)$ where $\#_0$ and $\#_1$ are functions that count the number of 0's and of 1's, respectively, in w.

   Do not forget to clearly state the property you are proving, which kind of induction you are using, the base case and the inductive hypothesis!

2. Consider the language $\{a^i b^j c^k \mid 0 < i + j \leqslant k\}$.

   (a) (2 pts) Give a context-free grammar that generates the above language.

   (b) (0.75 pts) Explain your grammar and why it constructs the required language.

   (c) (0.5 pts) Is this grammar ambiguous? Justify your answer.

   (d) (0.75 pts) Give the recursive inference, the leftmost derivation and the parse tree for the word *abbcccc*.

3. Consider the following grammar with start variable S:

   $$S \to \epsilon \mid a \mid b \mid Ab \mid aB \qquad A \to aS \mid a \qquad B \to Sb \mid b$$

   (a) (1.25 pts) Describe as formal as you can the language generated by the grammar.

   (b) (0.5 pts) Show that this grammar is ambiguous.

   (c) (0.75 pts) Construct an unambiguous grammar which generates exactly this language.

   (d) (0.5 pts) Motivate why the new grammar is not ambiguous.