Overview Sequent Calculus Rules

Propositional logic

notLeft $\dfrac{\Gamma \Longrightarrow \phi, \Delta}{\Gamma, \neg\phi \Longrightarrow \Delta}$ notRight $\dfrac{\Gamma, \phi \Longrightarrow \Delta}{\Gamma \Longrightarrow \neg\phi, \Delta}$

andLeft $\dfrac{\Gamma, \phi, \psi \Longrightarrow \Delta}{\Gamma, \phi \wedge \psi \Longrightarrow \Delta}$ andRight $\dfrac{\Gamma \Longrightarrow \phi, \Delta \qquad \Gamma \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \phi \wedge \psi, \Delta}$

orLeft $\dfrac{\Gamma, \phi \Longrightarrow \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \phi \vee \psi \Longrightarrow \Delta}$ orRight $\dfrac{\Gamma \Longrightarrow \phi, \psi, \Delta}{\Gamma \Longrightarrow \phi \vee \psi, \Delta}$

impLeft $\dfrac{\Gamma \Longrightarrow \phi, \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \phi \to \psi \Longrightarrow \Delta}$ impRight $\dfrac{\Gamma, \phi \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \phi \to \psi, \Delta}$

close $\dfrac{}{\Gamma, \phi \Longrightarrow \phi, \Delta}$ true $\dfrac{}{\Gamma \Longrightarrow \mathrm{true}, \Delta}$ false $\dfrac{}{\Gamma, \mathrm{false} \Longrightarrow \Delta}$

First-order logic

allLeft $\dfrac{\Gamma, \forall \tau\, x;\, \phi,\, [x/t']\,\phi \Longrightarrow \Delta}{\Gamma, \forall \tau\, x;\, \phi \Longrightarrow \Delta}$ allRight $\dfrac{\Gamma \Longrightarrow [x/c]\,\phi, \Delta}{\Gamma \Longrightarrow \forall \tau\, x;\, \phi, \Delta}$

exLeft $\dfrac{\Gamma, [x/c]\,\phi \Longrightarrow \Delta}{\Gamma, \exists \tau\, x;\, \phi \Longrightarrow \Delta}$ exRight $\dfrac{\Gamma \Longrightarrow [x/t']\,\phi,\, \exists \tau\, x;\, \phi, \Delta}{\Gamma \Longrightarrow \exists \tau\, x;\, \phi, \Delta}$

applyEq $\dfrac{\Gamma, t \doteq t' \Longrightarrow [t/t']\,\phi, \Delta}{\Gamma, t \doteq t' \Longrightarrow \phi, \Delta}$ applyEq $\dfrac{\Gamma, t \doteq t', [t/t']\,\phi \Longrightarrow \Delta}{\Gamma, t \doteq t', \phi \Longrightarrow \Delta}$ introEq $\dfrac{}{\Gamma \Longrightarrow t \doteq t, \Delta}$

- $[t/t']\,\phi$ is result of replacing each occurrence of t in ϕ with t'
- In allLeft and exRight t' is any variable-free term of type τ
- c is a **fresh** constant of type τ (i.e., it does not occur on the current proof branch)
- Equations can be reversed by commutativity

Dynamic logic

$$\textsf{assign}\quad \frac{\Gamma \Longrightarrow \{\mathcal{U}\}\{\mathtt{x} := t\}\langle \ldots \rangle \phi, \Delta}{\Gamma \Longrightarrow \{\mathcal{U}\}\langle \mathtt{x\ =\ t;}\ \ \ldots \rangle \phi, \Delta} \qquad \textsf{ifElse}\quad \frac{\begin{array}{c}\Gamma, \{\mathcal{U}\}\mathtt{b} \doteq \mathbf{true} \Longrightarrow \{\mathcal{U}\}\langle \mathtt{p;}\ \ \ldots \rangle \phi, \Delta \\ \Gamma, \{\mathcal{U}\}\mathtt{b} \doteq \mathbf{false} \Longrightarrow \{\mathcal{U}\}\langle \mathtt{q;}\ \ \ldots \rangle \phi, \Delta\end{array}}{\Gamma \Longrightarrow \{\mathcal{U}\}\langle \mathbf{if}\ \mathtt{(b)\ \{\ p\ \}}\ \mathbf{else}\ \mathtt{\{\ q\ \}\ ;}\ \ \ldots \rangle \phi, \Delta}$$

$$\textsf{unwindLoop}\quad \frac{\Gamma \Longrightarrow \langle \mathbf{if}\ \mathtt{(b)\ \{\ p;}\ \ \mathbf{while}\ \mathtt{(b)\ p\ \};}\ \ \ldots \rangle \phi, \Delta}{\Gamma \Longrightarrow \langle \mathbf{while}\ \mathtt{(b)\ \{p\};}\ \ \ldots \rangle \phi, \Delta}$$

$$\textsf{loopInvariant}\quad \frac{\begin{array}{l}\Gamma \Longrightarrow \{\mathcal{U}\}\mathit{inv}, \Delta \\ \Gamma, \{\mathcal{U}\}(\mathtt{b} \doteq \mathbf{true} \wedge \mathit{inv}) \Longrightarrow \{\mathcal{U}\}\langle \mathtt{p} \rangle \mathit{inv}, \Delta \\ \Gamma, \{\mathcal{U}\}(\mathtt{b} \doteq \mathbf{false} \wedge \mathit{inv}) \Longrightarrow \{\mathcal{U}\}\langle \ldots \rangle \phi, \Delta\end{array}}{\Gamma \Longrightarrow \{\mathcal{U}\}\langle \mathbf{while}\ \mathtt{(b)\ \{p\};}\ \ \ldots \rangle \phi, \Delta}$$

Update rewriting

$$\begin{array}{rcll}
\{\mathcal{U}\}\{\mathtt{x}_1 := t_1 \parallel \ldots \parallel \mathtt{x}_n := t_n\} & \rightsquigarrow & \{\mathcal{U} \parallel \mathtt{x}_1 := \{\mathcal{U}\}t_1 \parallel \ldots \parallel \mathtt{x}_n := \{\mathcal{U}\}t_n\} & \\
\{\mathcal{U}\}f(t_1, \ldots, t_n) & \rightsquigarrow & f(\{\mathcal{U}\}t_1, \ldots, \{\mathcal{U}\}t_n) & \\
\{\mathtt{x}_1 := t_1 \parallel \ldots \parallel \mathtt{x}_n := t_n\}\mathtt{x} & \rightsquigarrow & \left\{\begin{array}{ll} \mathtt{x} & \text{if } \mathtt{x} \notin \{\mathtt{x}_1, \ldots, \mathtt{x}_n\} \\ t_k & \text{if } \mathtt{x} = \mathtt{x}_k \text{ and } \mathtt{x} \notin \{\mathtt{x}_{k+1}, \ldots, \mathtt{x}_n\} \end{array}\right. & \\
\{\mathcal{U}\}(\varphi \wedge \phi) & \rightsquigarrow & \{\mathcal{U}\}\varphi \wedge \{\mathcal{U}\}\phi & \text{similar for } \vee, \rightarrow, \doteq, ! \\
\{\mathcal{U}\}\mathcal{Q}y.\varphi & \rightsquigarrow & \mathcal{Q}y.\{\mathcal{U}\}\varphi & \text{where } \mathcal{Q} \in \{\forall, \exists\}, y \notin \text{free}(\mathcal{U})
\end{array}$$