Advanced Algorithms. Home Exam 2018/19

The Problem

Suppose that kp objects of given sizes (which are positive numbers) shall be assigned to p persons such that: (1) each person gets exactly k objects, and (2) the total sizes of the objects assigned to the different persons are "as equal as possible". That is, no person should get a total amount being much larger than the amount received by another person, if that can be avoided. The objects cannot be split.
Condition (2) resembles the Load Balancing problem, but note that we also have the strict condition (1) here, which adds another twist to the problem.

Treat this problem, by using the concepts and methods from the course (at least: approximation, randomization) and write a report about your findings.

Submission

Mail your final report to `ptr@chalmers.se` as PDF attachment (no other formats please). Write your name, ID number, and study programme on the title page. The final submission deadline is annoucned on the course web page. Do not wait until the last minute, but submit when you are done. See also the further Instructions below.
Quality is more important than quantity. However, as a rule of thumb, your report should have at least 5 pages of text (with usual font size, spacing, and margins), plus the title page and possible references.

Some Detailed Suggestions

(This list is not exhaustive.)

- "as equal as possible" is vague. Define some suitable optimization goal(s) first.
- The resulting problems are most likely NP-complete. Perhaps you can prove this.
- Are there good approximation algorithms? Can they be based on some greedy rules, or other techniques? But how can we also guarantee equal numbers of objects for each person?
- Objects could be assigned by some randomized rule, which should intuitively yield a good balance. But again, how can we guarantee equal numbers of objects? What can be said about the expected amounts for each person? And how can we bound the probabilities of large deviations?
- Analyze running times (worst-case, expected, etc.) of your proposed algorithms.
- One can think of many natural special cases, e.g.: k is fixed; p is fixed; the objects have only a few different sizes; in particular, all sizes are integers up to a small constant; the sizes differ only by some additive or multiplicative constant, etc. All the above questions can be raised for such special cases. Do such restrictions enable more efficient algorithms?
- Are some special cases in XP or FPT, with suitable parameters?
- There may be additional constraints, e.g., certain pairs of objects cannot be given to the same person. How does this affect the problems? Can the algorithms be modified, or do the problems become harder by such constraints?

Criteria for a Good Report

- Breadth. You cover at least the main themes: approximation and randomization, ideally also some special cases or variants of the problem. (But the problem is apparently not related to network flows, unless you find an unexpected connection,)
- Correctness. There are no major factual mistakes.
- Depth. You provide some solid, substantial results that are fully worked out, not only some loose collection of trivial observations or vague guesses.
- Clarity. Algorithms, and proofs of their properties, are well described. See also the grading criteria on the course web page.
- Negative statements (that something is probably not possible to do) are motivated by good reasons.

Instructions

- In the beginning you may feel very uncertain about the expectations. Do not hesitate to discuss, and to send questions or drafts, in order to check whether you are on the right track, and to get early feedback. But take availability times into account; see the course web page.
- Do not misunderstand the suggestions as exercises that have to be solved. You can really choose some of your favourite directions, as long as you cover the main course topics.
- You can submit arbitrarily many drafts, at any time. Only the last version submitted before the deadline will count for the grade.
- Utmost academic honesty is expected. The words about cheating (see the course web page) apply also here. In particular, you must cite all literature you have used, and acknowledge all sources of help, and always describe the contents in your own way.