

# Sample problem

The Plainview Oil Company operates three oil fields: one in California producing up to 3500 barrels a day, one in Texas producing up to 2500 barrels a day, and one in New Mexico producing up to 1000 barrels a day.

It also has two refineries, one in Los Angeles that can process 3000 barrels a day and one in Houston that can process 5000 barrels a day. The company earns 12 dollars for each barrel processed and sold in Houston, and 10 dollars for each barrel processed and sold in Los Angeles.

The cost of transporting one barrel of crude oil to a refinery is given by the following table:

	California	Texas	New Mexico
Houston	5	1	2
Los Angeles	1	5	3

The company wants to know how to organize the production and refinement of oil to make the most profit.

1. Write a LP to answer this question.

## Solution:

For each production site, we introduce two variables: one indicating the number of barrels that it should produce and send to the Houston refinery, and another indicating the number of barrels that it should produce and send to Los Angeles.

For each field, we introduce a constraint corresponding to the production capacity.

For each refinery we introduce a constraint corresponding to the processing capacity.

$$\begin{aligned} \max \quad & 9x_{CL} + 7x_{CH} + 5x_{TL} + 11x_{TH} + 7x_{NL} + 10x_{NH} \\ \text{s.t.} \quad & x_{CL} + x_{CH} \leq 3500 \\ & x_{TL} + x_{TH} \leq 2500 \\ & x_{NL} + x_{NH} \leq 1000 \\ & x_{CL} + x_{TL} + x_{NL} \leq 3000 \\ & x_{CH} + x_{TH} + x_{NH} \leq 5000 \\ & \vec{x} \geq \vec{0} \end{aligned}$$

2. Solve the problem using CVX.

**Solution:**

Optimal solution:

$$\{x_{CL} = 3000, x_{CH} = 500, x_{TL} = 0, x_{TH} = 2500, x_{NL} = 0, x_{NH} = 1000\}$$

Optimal value: 68000

3. The company is looking to invest to increase its production capacity on existing oil fields. Which of the fields should they invest in?

**Solution:** The constraints corresponding to production capacities are all binding, indicating that all fields are exploited at full capacity in the optimal solution.

The constraint corresponding to the Los Angeles refinery is also binding, but the constraint corresponding to the Houston refinery has slack (1000 units). Therefore this refinery could process more oil. Oil processed in Houston is most profitable if it comes from the field in Texas (c.f. coefficients in the objective function) therefore increasing the production capacity of the Texas field by up to a 1000 barrels is the most profitable investment.