Lab B Parallel Functional Programming

Part 1

Write a tutorial about parallel functional programming, concentrating on a particular language or library. You are free to choose your own title. Check with Mary if you are unsure about your choice. Some possible titles might be

The Par Monad for parallel Haskell programming: a tutorial

A tutorial on Parallel Strategies in Haskell

How to use **`par`** and **`pseq`** for parallel programming in Haskell

Parallel functional programming in Java 8: a tutorial

GPU programming in Haskell: a tutorial on Accelerate

Parallel sorting in Haskell: how well can we do?

Parallel programming in F#: a tutorial

Deterministic parallel programming in Erlang

The main point, remember, is to make a clear and simple tutorial (more like a technical blog post than a paper). Good tutorials often contain nice pictures! Give readers some information about sequential and parallel running times. Submit your document or web page with all associated images and code. One option would be to implement an interesting deterministic parallel program in Haskell or some other functional language, and to write a blog post documenting your work in developing and optimising the program. This task might be a good way to start exploring possible Masters thesis topics. It gives you the opportunity to explore one approach to parallel functional programming more deeply. It is important to avoid plagiarism!

Lab B Part 2
Exercises in Futhark for Parallel Functional Programming at Chalmers

Troels Henriksen
based on work by Martin Elsman
DIKU, University of Copenhagen

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Introduction

These exercises explore the use of Futhark for writing parallel programs in a functional setting. The exercises assume access to a computer with Futhark installed. For information about installing Futhark, please consult `https://futhark-lang.org`.

1 Getting started

This exercise aims at illustrating how simple parallel problems can be expressed in Futhark.

Exercise 1.1: Write a function

Create a Futhark function called `process` that takes as arguments two one-dimensional `i32` arrays (signals) of the same length and computes the maximum absolute difference (pointwise) between the signals (you should not use Futhark's `loop` construct). The function should return the value 0 if two empty signals are passed to the function.

Consider the following two signals:

```
let s1 = [23,45,-23,44,23,54,23,12,34,54,7,2, 4,67]
let s2 = [-2, 3,  4,57,34, 2, 5,56,56, 3,3,5,77,89]
```

What is the result of calling your function on `s1` and `s2`?

Hint: To run the function, define a `main` function that passes the two arrays to the function `process`, then compile this program with `futhark-c` or `futhark-opencl`.

Exercise 1.2: Run your function

Use the `futhark-dataset` tool to generate seven sets of test data of different length. Each set should contain a pair of one-dimensional `i32` arrays each containing integers in the range [-10000;10000]. The array lengths for the seven different sets should be 100, 1000, 10000, 100000, 1000000, 5000000, and 10000000.

Run the function `process` with the different data sets and with executables obtained both with using `futhark-c` and `futhark-opencl`. Map the timings (in microseconds) onto a chart and remember to specify the system on which you're running the executables. Follow the guidelines given in the Futhark book on benchmarking[1].

Exercise 1.3: Extend your function

Create a refined version of the `process` function, called `process_idx`, that also returns the index of the source signals for which the maximum absolute difference is found.

Report the result of calling `process_idx` on the signals `s1` and `s2`. Show evidence that your solution scales as the `process` function.

Hint: The lecture slides should give you a hint to solving this problem.

Exercise 1.4: Neutral elements

Assuming $\oplus$ is an associative operator with neutral element `0`, show that `(0,false)` is a left-neutral element of

$$(v_1, f_1) \oplus' (v_2, f_2) = (\texttt{if}\ f_2\ \texttt{then}\ v_2\ \texttt{else}\ v_1 \oplus v_2,\ f_1 \vee f_2)$$

Exercise 1.5: Segmented scans and reductions

The operator in the previous question can be used to implement a segmented scan. Specifically, a `scan` on an array of type `[]t` with operator $\oplus$ and neutral element $0_\oplus$ can be turned into a segmented scan on an array of type `[](t, bool)` with operator $\oplus'$ and neutral element `(`$0_\oplus$`, false)`. A

[1] `http://futhark-book.readthedocs.io/en/latest/benchmarking.html`

`true` indicates the beginning of a segment, and `false` the continuation of a segment.

Finish the following Futhark definition of segmented scan:

```
let segscan [n] 't (op: t -> t -> t) (ne: t)
                   (arr: [n](t, bool)): [n]t =
  ...
```

A segmented reduction is more complicated, but can be implemented by first performing a segmented scan, and then making use of the `scatter` operation.

Finish the following Futhark definition of segmented reduction:

```
let segreduce [n] 't (op: t -> t -> t) (ne: t)
                     (arr: [n](t, bool)): []t =
  ...
```

Note that we cannot provide the size of the returned array in the type, as we do not know the number of segments.

Benchmark the performance of segmented scan versus ordinary scan, and segmented reduce versus ordinary reduction and show the result.

2 Monte Carlo Simulation

In this exercise, we shall use the technique of Monte Carlo simulation for computing the value of π. We shall first use a simple technique based on an external generation of random numbers. We shall then use the concept of Sobol-numbers for approaching the real value of π with less work.

Exercise 2.1: Monte Carlo π

In this exercise we shall make use of the “dart-throwing” technique. Observe that if one randomly throws a dart on a square of size 2×2 then the chance of hitting within the enclosed circle of radius 1, provided one hits the square, is $\frac{\pi}{4}$. It is quite easy to determine, using Pythagoras's theorem, whether a throw (x, y) is successful, which it is if its distance to the center of the circle is less than or equal to 1, that is, if $(x-1)^2 + (y-1)^2 \leq 1$. Write a function `estimate_pi` that takes two arrays of `f32` values as arguments, corresponding to x and y coordinates for the throws, and, based on the above observations, gives an estimate on the value of π.

Exercise 2.2: External randomness

Using the `futhark-dataset` tool, generate three datasets of different sizes, each containing two arrays of type `[]f32` of the same size, containing `f32` values between 0.0 and 2.0. Use 100, 10000, and 1000000 as the sizes for the data sets. Both for executables generated with `futhark-c` and `futhark-opencl`, plot the time for computing π as a function of the different data set sizes. What do you see?

Hint: Use the following command to generate input for the 100-size case:

```
futhark-dataset --f32-bounds='0:2' -g [100]f32 -g [100]f32 > pi100.inp
```

Exercise 2.3: Monte Carlo integration

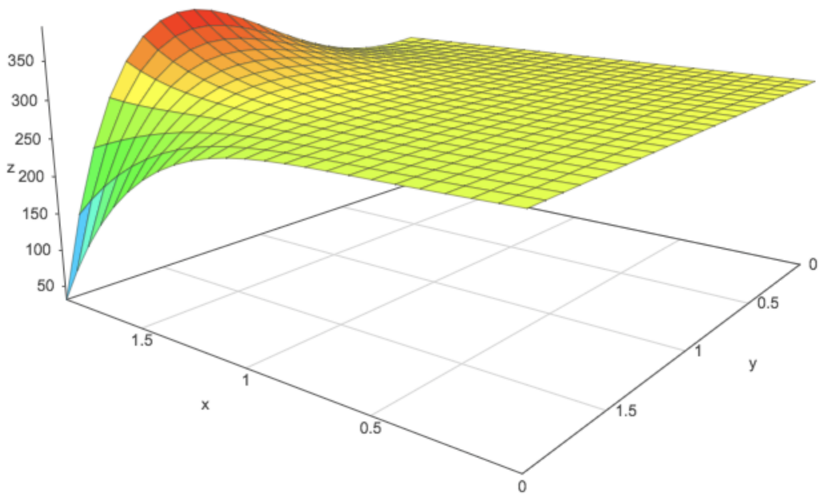
Use the same technique as above but for integrating (i.e., finding the volume under) the following mathematical function (a function of two variables) in the interval $x \in [0;2]$ and $y \in [0;2]$:

$$f(x,y) = 2x^6y^2 - x^6y + 3x^3y^3 - x^2y^3 + x^3y - 3xy^2 + xy - 5y + 2x^5y^4 - 2x^5y^5 + 250$$

A Futhark function resembling the mathematical function is given as follows:

```
let f (x:f32) (y:f32): f32 =
  2.0f32*x*x*x*x*x*x*y*y - x*x*x*x*x*x*y
  + 3.0f32*x*x*x*y*y*y - x*x*y*y*y +
  x*x*x*y - 3.0f32*x*y*y + x*y -
  5.0f32*y + 2.0f32*x*x*x*x*x*y*y*y*y -
  2.0f32*x*x*x*x*x*y*y*y*y*y + 250.0f32
```

Here is a graph showing the surface defined by the function in the interval:



Both for executables generated with `futhark-c` and `futhark-opencl`, plot the time for computing the integral as a function of the different data set sizes for the data sets generated in the previous exercise. What do you see now?

Hint: If n is the number of sample pairs S, the integral $\int_0^2 \int_0^2 f(x, y) dx dy$ can be approximated by $\frac{4}{n} \Sigma_{(x,y) \in S} f(x, y)$.

Exercise 2.4: Sobol numbers

Futhark features a standard library, which includes a module for generating so-called *Sobol* sequences, an example of quasi-random low-discrepancy sequences. Such sequences are particularly good for Monte-Carlo techniques in that results converge faster than if ordinary pseudo-random numbers are used.

The Futhark module `/futlib/sobol` includes a number of (higher-order) sub-modules, which can be composed to setup a module for implementing a Monte-Carlo simulation. An example use of the library, for establishing the value of π, is available in the file `https://github.com/diku-dk/futhark/blob/master/tests/futlib_tests/sobol.fut`. The documentation for the module is available online[2].

Use the sobol library for establishing a value for the integral in section 2.3 and investigate how fast (as a function of the number of sampling points)

[2] `https://futhark-lang.org/docs/futlib/sobol.html`

the integral value converges compared to using the technique presented in section 2.1.

Exercise 2.5: Conventional random numbers (optional)

The Futhark basis library also contains a more conventional random number generation facility, which is available as `/futlib/random`[3]. Rewrite the numerical integration function from before such that it uses this library instead, and compare performance and convergence rates.

3 2D Ising Simulation

The 2D Ising Simulation is a simulation of the behaviour of a simple idealised ferromagnet, in which we compute the *spin* of a grid of electrically charged atoms over a period of time. From a Futhark point of view, this grid is merely a two-dimensional array of integers that are either -1 or 1.

At any given discrete time step, the charge of a spin can be either positive or negative. A spin interacts only with its immediate neighbors, which makes Ising simulation a *stencil.* An atom prefers to have the same polarity as its neighbors, although it also has a small chance to flip polarity randomly, based on the temperature. This is the Monte Carlo aspect.

To update the grid, we compute for each spin c its corresponding Δ_e, as followed:

$$\Delta_e = 2c(u + d + l + r)$$

where u, d, l, r are the spins directly adjacent to c in the grid (we ignore diagonals). Further, for each spin we compute two random numbers, a and b, in the range $(0, 1)$. *It is very important that each spin receives its own* a, b. Then we compute the new values of c as

$$c' = \begin{cases} -c & \text{if } b < p \wedge (\Delta_e < -\Delta_e \vee b < e^{-\Delta_e \div t}) \\ c & \text{otherwise} \end{cases}$$

where $0 \leq p \leq 1$ is the *sample rate*, and $t \in \mathbb{R}$ is the *temperature.* Put in words, p is the fraction of spins that are candidates for flipping in a given time step. Of these, we flip those where it would locally reduce the energy of the system, or randomly, where the chance of random flips is proportional to the temperature. For more information on Ising models, I recommend the

[3]`https://futhark-lang.org/docs/futlib/random.html`

very readable 6-page article *The World in a Spin* by Brian Hayes[4]. However, we need not understand the physics to implement the model in Futhark.

For this exercise, you will be modifying a code handout, `ising.fut`, that contains the skeleton for an implementation of the 2D Ising model. Read the existing code carefully. The code handout also comes with a frontend, `ising-gui.py`, written in Python, that permits a visualisation of the computation.

Exercise 3.1: Generate initial state

The code handout defines the type of spins as follows:

```
type spin = i8
```

However, we also need to generate (potentially) distinct random numbers for every spin. For this exercise, we will use *one RNG state per spin.* Thus, the function for generating an initial grid state is:

```
entry random_grid (seed: i32) (w: i32) (h: i32)
  : ([w][h]rng_engine.rng, [w][h]spin) =
  ...
```

Where `rng_engine.rng` is the type of RNG states. See the code comments for more information.

Hint: Generate one-dimensional arrays of size $n \times m$, then use `reshape` *at the end.*

Exercise 3.2: Computing Δ_e

This is the stencil operation where we, for every spin, compute Δ_e as a value of type `i8`:

```
entry deltas [w][h] (spins: [w][h]spin): [w][h]i8 =
  ...
```

One question that must be answered for every stencil is how to handle the edges of the grid, where there are no neighbors. For the 2D Ising model, we pick the easy solution, and use *wraparound*, also known as a *torus world.* Simply put, when we go over one edge, we come out on the opposite edge. In Futhark, this is easily done with the `rotate` language construct. If `xss`

[4]`http://bit-player.org/wp-content/extras/bph-publications/AmSci-2000-09-Hayes-Ising.pdf`

is a two-dimensional array, then `rotate@0 1 xss` corresponds to rotating the array by one element horizontally (along the first dimension), while `rotate@1 (-1) xss` rotates it by negative one element vertically (along the second dimension).

Hint: Use the `map1`/`map2`/`map3`/`map4`/`map5` functions to easily map across multiple arrays simultaneously.

Exercise 3.3: The step function

Define the step function, which computes one time step of the simulation:

```
entry step [w][h] (abs_temp: f32) (samplerate: f32)
                  (rngs: [w][h]rng_engine.rng)
                  (spins: [w][h]spin)
  : ([w][h]rng_engine.rng, [w][h]spin) =
  ...
```

Note that it computes not just new spins, but also new RNG states.

Exercise 3.4: Benchmarking

If you have defined the above functions correctly, then you should now be able to use the predefined `main` function, which creates a grid and runs a few steps of the simulation; returning the final grid at the end:

```
let main (abs_temp: f32) (samplerate: f32)
         (w: i32) (h: i32) (n: i32): [w][h]spin =
  (loop (rngs, spins) = random_grid 1337 w h
   for _i < n do
     step abs_temp samplerate rngs spins).2
```

Show benchmarks that demonstrate how sequential versus parallel performance varies for different values of `w`, `h`, and `n`. Explain your results.

Hint: Use `futhark-bench` with the `--compiler` option to switch between `futhark-c` and `futhark-opencl`. Be aware that the Futhark compiler is not very good at generating sequential CPU code, and that stencils in particular are likely to have poor cache behaviour, so don't consider the `futhark-c` results indicative of the full power of your CPU.

Exercise 3.5: Now do it again (optional)

Rewrite the Ising simulation in another parallel functional language; reflect on how Futhark compares in convenience and performance.