Finite Automata Theory and Formal Languages
TMV027/DIT321

DFA, NFA and ϵ-NFA

Exercise 2

In these exercises, book sections, exercise numbers and pages refer to those in the third edition of the course book.

DFA

Basic exercises

1. Do the exercise 2.2.4.
2. Show that the language with words over $\{0, 1\}^*$ ending with the string 0100 is a *regular* language.
3. Suppose $\Sigma = \{a, b\}$. Build a DFA that accepts the words containing bba as a subword. Build then a DFA that accepts the words *not* containing bba as a subword.
4. Let $\Sigma = \{a, b, c\}$. Build a DFA D_1 that accepts the words containing ac as a subword. Build a DFA D_2 that accepts the words containing ab as a subword.

   Using the product construction, build then a DFA that accepts the words containing both ab and ac as subwords, and another DFA that accepts the words containing ac but *not* ab as a subword.
5. Do exercise 2.2.10.

Additional exercises

1. In a factory, we have the possible events a, b, c. A constraint L_1 is that if the event b occurs after the event a, then the event c should occur in between. Represent this constraint L_1 as a DFA D_1.

   Suppose that there is another constraint L_2 stating that if the event b occurs after the event c, then the event a should occur in between.

   Explain intuitively why, if we have both constraints L_1 and L_2 then the event b cannot occur after the event a.

   Represent the constraint L_2 as a DFA D_2.

   Using the product construction of D_1 and D_2, produce an automaton representing the conjunction of the constraint L_1 and L_2. Verify in this automaton that b cannot occur after a or c.

2. Do exercises 2.2.1, 2.2.5, 2.2.7, 2.2.8, 2.2.9 and 2.2.11.

3. If $\mathcal{L} \subseteq \Sigma^*$ is a language, we define $\mathsf{Prefix}(\mathcal{L})$ to be the set of words that are prefix of a word in $\mathcal{L}$. Show that if $\mathcal{L}$ is regular then so is $\mathsf{Prefix}(\mathcal{L})$.

NFA and ϵ-NFA

Basic exercises

1. Do exercises 2.3.1 and 2.3.2.

2. Do exercise 2.3.4.

3. Do exercise 2.5.1 and 2.5.2.

4. Do exercise 2.5.3.

Additional exercises

1. Do exercise 2.3.3.

2. Do exercises 2.4.1 and 2.4.2.

3. Show that if $\mathcal{L} \subseteq \Sigma^*$ is regular then so is $\mathcal{L}^R = \{rev(x) \mid x \in \mathcal{L}\}$.
   (Hint: given an automaton for $\mathcal{L}$ build a NFA for $\mathcal{L}^R$)

4. A ship attempts to transmit data to shore at random intervals. The receiver must continually listen and recognise when an actual transmission starts so that it can record the data that follows. Let us assume that the start of the transmission is signaled by the string 010010 and the end of the transmission is signaled by the string 000111. Represent this behaviour with a DFA.

Programming Exercises

1. Write a program that runs/simulates a DFA.

2. Write a program that produces the product and the variation of the product of two DFA.

3. Write a program that produces the complement of a DFA.

4. Write a program that runs/simulates a NFA.

5. Write a program implementing the subset construction.

6. Write a program that runs/simulates an ϵ-NFA.

7. Write a program that computes the ϵ-closure of a set of states.

8. Write a program that transform an ϵ-NFA into its equivalent DFA.