Algorithms. Assignment 6

Because there remains little time for grading, this last assignment comprises only a few test questions that should be rather simple. Submit answers as "Problem 6.1", ignore 6.2.

We consider only undirected, connected graphs with n nodes and m edges.

(a) We claim that an $O(n + m)$ time bound for any graph algorithm can be simplified to $O(m)$. Why?

(b) We claim that an $O(m \log m)$ time bound for any graph algorithm can also be written as $O(m \log n)$. Why?

(c) Suppose that we have executed DFS in an undirected graph and produced a DFS tree. Is the following claim true or false? (Explain.)
"Every possible cycle in the graph consists of tree edges and exactly one back edge."
Then, briefly describe an $O(m)$ time algorithm that outputs a cycle in the given graph, provided that a cycle exists. It should be based either on this claim (if it is true), or on a suitable modification of this claim (if it is false).

(d) We have seen how BFS can be used to efficiently decide whether a given graph is bipartite (2-colorable). Briefly describe an alternative algorithm for this problem, that uses DFS rather than BFS. Do not forget to argue why your proposed algorithm is correct.