Algorithms. Assignment 5

Problem 1: P-NP Thriller

Some definitions first: A Boolean *term* consists of literals connected by $\wedge$.
A *disjunctive normal form (DNF)* consists of terms connected by $\vee$.
Informally speaking, a DNF is a Boolean formula that looks like a CNF, except that the roles of $\wedge$ and $\vee$ are switched.

In the attic of an old house we found a manuscript saying:

"The SAT problem for DNF is easy to solve: It suffices to make one term true. Unless every term contains some pair of a variable and its negation, we can always pick some term and make it true, simply by setting all literals in this term true.
Furthermore, we can easily transform every CNF into an equivalent DNF, using the distributive laws, e.g.: $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$. Therefore, the SAT problem for CNF is easy to solve as well.
On the other hand, we know that SAT for CNF is NP-complete. This implies P=NP, which solves the famous P-NP-problem!!"

Unfortunately this manuscript is useless. Where exactly is the flaw in the reasoning? Explain your objection in detail.

Problem 2: Get Your Balance Back

In the Subset Sum problem we are given n integers w_i and another integer W, and the problem is to decide whether some subset has the sum W.
Now we define the Balanced Subset Sum problem, as the special case of Subset Sum where $W = \sum_{i=1}^{n} w_i/2$. (In words: Can we split the given set of integers half-half?)

Prove that Balanced Subset Sum is still NP-complete. In detail:
Give a reduction from the "unrestricted" Subset Sum problem and describe all necessary ingredients: How does your reduction work? Make sure that it runs in polynomial time. Show that the produced instance of Balanced Subset Sum is equivalent to the given Subset Sum instance.
And a hint: To come up with a reduction idea, do not think complicated.
It suffices to insert just one suitable extra number.