Formal Methods for Software Development

Reasoning about Programs with Loops and Method Calls

Wolfgang Ahrendt

17 October 2017

Program Logic Calculus – Repetition

Calculus realises symbolic interpreter:

- works on first active statement

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- application of update computes weakest precondition of $\mathcal{U}'$ wrt. ϕ

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An Example

```
\javaSource "src/";

\programVariables{
 Person p;
 int j;
}

\problem {
  (\forall int i;
    (!p=null ->
      ({j := i}\<{p.setAge(j);}\>(p.age = i))))
}
```

Method Calls

Method Call with actual parameters $arg_0, \ldots, arg_n$

$$\langle \texttt{o.m}(arg_0, \ldots, arg_n)\texttt{;}\ \ \omega\rangle\phi$$

assume `m` declared as $\mathbf{void}\ \mathtt{m}(\tau_0\ \mathrm{p}_0, \ldots, \tau_\mathrm{n}\ \mathrm{p}_\mathrm{n})$

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Actions of rule methodCall

1. Declare new local variables $\mathtt{p\#i}$, initialize them with actual parameter: $\tau_i\ \mathtt{p\#i} = arg_i$;

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   split proof if implementation cannot be uniquely determined.

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3. Replace method call with implementation invocation
   `o.m(p#0,...,p#n)@`C

Method Calls Cont'd

After executing the initialisers: $\tau_{\mathtt{i}}$ `p#i` $= arg_i$; apply:

Method Body Expand

Call rule methodBodyExpand

$$\frac{\Gamma \Longrightarrow \langle \mathtt{method\text{-}frame(source=m(}\tau_0,\ldots,\tau_n\mathtt{)@C,\ this=o):\{}body\mathtt{\}}\ \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \mathtt{o.m(p\#0,\ldots,p\#n)@C;}\ \omega\rangle\phi, \Delta}$$

1. Replaces method invocation by method frame with method body
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Method frames:
Required in proof to represent call stack

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Demo

`methods/instanceMethodInlineSimple.key`
`methods/inlineDynamicDispatch.key`

Localisation of Fields and Method Implementations

JAVA has complex rules for localisation of fields and method implementations

- Polymorphism
- Late binding (dynamic dispatch)
- Scoping (class vs. instance)
- Visibility (private, protected, public)

Proof split into cases if implementation not statically determined

Object initialization

Java has complex rules for object initialization

- Chain of constructor calls until Object
- Implicit calls to **super**()
- Visibility issues
- Initialization sequence

Coding of initialization rules in methods `<createObject>()`, `<init>()`,... which are then symbolically executed

Limitations of Method Inlining: methodBodyExpand

- Source code might be unavailable
  - API method implementation vendor-specific
  - Source code often unavailable for commercial APIs
- Method is invoked multiple times in a program
  - Avoid multiple symbolic execution of identical code
- Cannot handle unbounded recursion
- Not modular:
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Use method contract instead of method implementation:

1. Show that requires clause is satisfied
2. Continue after method call:
   - assume ensures clause
   - forget prestate values of modifiable locations

Method Contract Rule: Normal Behavior Case

Warning: Simplified version

```
/*@ public normal_behavior
  @ requires preNormal;
  @ ensures postNormal;
  @ assignable mod;
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- π are openings of try blocks and method frames

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JML Method Contracts Revisited

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T m(T1 a1, ..., Tn an) { ... }
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Implicit Preconditions and Postconditions

- The object referenced by **this** is not **null**: **this != null** (precondition only; **this** cannot be changed by method)

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- The heap is wellformed: `wellFormed(heap)` (precondition only)
- Invariant for 'this': `\invariant_for(this)`

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- $\mathcal{V}_{\mathtt{mod}}$: anonymising update, *forgetting prevalues of modifiable locations*

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- Anonymising updates $\mathcal{V}$ erase information about modified locations

Anonymising Heap Locations

Define anonymising function `anon`: Heap $\times$ LocSet $\times$ Heap $\rightarrow$ Heap

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Definition:

$$\texttt{select}(\texttt{anon}(h1, locs, h2), o, f) = \begin{cases} \texttt{select}(h2, o, f) & \text{if } (o, f) \in locs \\ \texttt{select}(h1, o, f) & \text{otherwise} \end{cases}$$

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Effect: After $\mathcal{V}_{mod}$, modfied locations have unknown values

Anonymising Heap Locations: Example

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To erase all knowledge about the values of the locations of the assignable expression:

- Anonymise the current heap on the designated locations:

$$\mathtt{anon}(\mathtt{heap}, \{(\mathtt{o}, \mathtt{a})\} \cup \mathtt{allFields}(\mathtt{this}), \mathtt{h}_{an})$$

- Make that anonymised current heap the new current heap.

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Method Contract Rule: Exceptional Behavior Case

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(background only, no need to remember)

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Therefore translation of postcondition ϕ_{post} as follows (simplified):

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- $\mathcal{F}(\cdot)$: translation to Java DL
- $\mathcal{V}_{mod}$: anonymising update

Method Contract Rule: Example

```
class Person {
 private /*@ spec_public @*/ int age;
 /*@ public normal_behavior
   @ requires age < 29;
   @ ensures age == \old(age) + 1;
   @ assignable age;
   @ also
   @ public exceptional_behavior
   @ requires age >= 29;
   @ signals_only ForeverYoungException;
   @ assignable \nothing;
   @//allows object creation (not \strictly_nothing)
   @*/
 public void birthday() {
   if (age >= 29) throw new ForeverYoungException();
   age++;
 } }
```

Method Contract Rule: Example Cont'd

Demo

`methods/useContractForBirthday.key`

- Prove without contracts
  - Method treatment: Expand
- Prove with contracts (until method contract application)
  - Method treatment: Contract
- Prove used contracts
  - Method treatment: Expand
  - Select contracts for `birthday()` in `src/Person.java`
  - Prove both specification cases

Verification of Loops

Symbolic execution of loops: unwind

$$\text{unwindLoop}\ \frac{\Gamma \Longrightarrow \mathcal{U}[\pi\,\texttt{\textbf{if}(b)\{p; \textbf{while}(b) p\}}\,\omega]\phi, \Delta}{\Gamma \Longrightarrow \mathcal{U}[\pi\,\texttt{\textbf{while}(b) p}\,\omega]\phi, \Delta}$$

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How to handle a loop with...

- 0 iterations?

Verification of Loops

Symbolic execution of loops: unwind

$$\text{unwindLoop}\ \frac{\Gamma \Longrightarrow \mathcal{U}[\pi\ \texttt{\textbf{if}}\,\texttt{(b)\{p;}\ \texttt{\textbf{while}}\,\texttt{(b) p\}}\ \omega]\phi, \Delta}{\Gamma \Longrightarrow \mathcal{U}[\pi\ \texttt{\textbf{while}}\,\texttt{(b)}\ \texttt{p}\ \omega]\phi, \Delta}$$

How to handle a loop with...

- 0 iterations? Unwind $1\times$

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How to handle a loop with...

- 0 iterations? Unwind $1\times$
- 10 iterations? Unwind $11\times$

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How to handle a loop with...

- 0 iterations? Unwind $1\times$
- 10 iterations? Unwind $11\times$
- 10000 iterations? Unwind $10001\times$
- an unknown number of iterations?

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- an unknown number of iterations?

We need an invariant rule (or some form of induction)

Loop Invariants

Idea behind loop invariants

- A formula *Inv* whose validity is preserved by loop body whenever the loop guard is true

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How to Derive Loop Invariants Systematically?

Example (Active statement of symbolic execution is loop)

```
    n >= 0 & wellFormed(heap)
->  {i := 0}
       \[{ while (i < n) {
              i = i + 1;
          }
        }\] i = n
```

Look at desired postcondition `i = n`

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What, in addition to negated guard `i >= n`, is needed? `i <= n`

Is `i <= n` preserved by loop body?
Does it hold when entering loop?

Yes! We have found a suitable loop invariant!

Demo `loops/simple.key` (auto after inv)

Obtaining Invariants by Strengthening

Example (Slightly changed problem)

```
    n >= 0 & n = m & wellFormed(heap)
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`i <= n & n = m`

Is `i <= n & n = m` preserved by loop body?
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Yes! We have found a suitable loop invariant!

Generalization

Example (Addition: `x,y` program variables, `x0,y0` rigid constants)

```
x = x0 & y = y0 & y0 >= 0 & wellFormed(heap) ==>
\[{
  while (y > 0) {
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}\] (x = x0 + y0)
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Finding the invariant

First attempt: use postcondition `x = x0 + y0`

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Finding the invariant

First attempt: use postcondition `x = x0 + y0`

- Not true at start whenever `y0 > 0`
- Not preserved by loop, because `x` is increased

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What stays invariant?

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Finding the invariant

What stays invariant?

- The sum of `x` and `y`: `x + y = x0 + y0` "Generalization"
- Can help to think of "δ" between `x` and `x0 + y0`

Generalization

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Is `x + y = x0 + y0` a good invariant?

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- Holds in the beginning and is preserved by loop

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Checking the invariant

Is `x + y = x0 + y0` a good invariant?

- Holds in the beginning and is preserved by loop
- But postcondition not achieved by `x + y = x0 + y0` and exit condition `y <= 0`

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Strenghtening the invariant

Postcondition holds if `y = 0`

- Add `y >= 0` to invariant: `x + y = x0 + y0 & y >= 0`

Demo `loops/simple3.key`

Basic Loop Invariant: Context Loss

Problems with the Basic Invariant Rule

$$\text{loopInvariant}\ \frac{\begin{array}{ll}\Gamma \Longrightarrow \mathcal{U}\,\mathit{Inv}, \Delta & \text{(initially valid)} \\ \mathit{Inv},\ b = \mathtt{TRUE} \Longrightarrow [\mathrm{p}]\mathit{Inv} & \text{(preserved)} \\ \mathit{Inv},\ b = \mathtt{FALSE} \Longrightarrow [\pi\ \omega]\phi & \text{(use case)}\end{array}}{\Gamma \Longrightarrow \mathcal{U}[\pi\ \mathbf{while}(\mathtt{b})\ \mathtt{p}\ \omega]\phi, \Delta}$$

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- Context Γ, Δ, $\mathcal{U}$ must be omitted in 2nd and 3rd premise:

  $\Gamma, \neg\Delta$ cannot be assumed for arbitrary iterations or at loop exit

  2nd premise State after some loop iterations is not $\mathcal{U}$

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- Context contains preconditions and class invariants
- Only way to propagate context: add to loop invariant *Inv*

Example

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
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Example

Precondition: a $\neq$ **null**

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- How to erase all values of `assignable` locations?
- Anonymising updates $\mathcal{V}$ erase information about modified locations

Anonymising JAVA Locations

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```

To erase all knowledge about these assignable lactions:

- introduce a new (not yet used) constant of type `int`, e.g., c
- introduce a new (not yet used) constant of type Heap, e.g., $\mathtt{h}_{an}$
  - anonymise the current heap: $\mathtt{anon}(\mathtt{heap}, \mathtt{allFields}(\mathtt{a}), \mathtt{h}_{an})$
- compute anonymizing update for assignable locations

$$\mathcal{V} = \{\mathtt{i} := \mathtt{c} \parallel \mathtt{heap} := \mathtt{anon}(\mathtt{heap}, \mathtt{allFields}(\mathtt{a}), \mathtt{h}_{an})\}$$

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$$\mathcal{V} = \{\mathtt{i} := \mathtt{c} \mid\mid \mathtt{heap} := \mathtt{anon}(\mathtt{heap}, \mathtt{allFields}(\mathtt{a}), \mathtt{h}_{an})\}$$

For local program variables (e.g., `i`) KeY computes assignable clause automatically

Loop Invariants Cont'd

Improved Invariant Rule

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- Context is kept as far as possible:
  $\mathcal{V}$ erases only information in locations assignable in the loop
- Invariant *Inv* does not need to include unmodified locations
- For **assignable \everything** (the default):
  - $\mathtt{heap} := \mathtt{anon}(\mathtt{heap}, \mathtt{allLocs}, \mathtt{h}_{an})$ wipes out **all** heap information
  - Equivalent to basic invariant rule
  - Avoid this! Always give a specific **assignable** clause

Example with Improved Invariant Rule

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Precondition: $a \neq$ **null**

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Loop invariant: $0 \leq \texttt{i}\ \ \&\ \ \texttt{i} \leq \texttt{a.length}$
$\&\ \ \forall\, \texttt{int}\ x;\ (0 \leq x\ \&\ x < \texttt{i} \rightarrow \texttt{a}[x] = 1)$

Example with Improved Invariant Rule

Precondition: $\texttt{a} \neq \texttt{null}$ & *ClassInv*

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Postcondition: $\forall\, \texttt{int}\ x; (0 \leq x \ \&\ x < \texttt{a.length} \rightarrow \texttt{a}[x] = 1)$

Loop invariant: $0 \leq \texttt{i} \ \&\ \texttt{i} \leq \texttt{a.length}$
$\&\ \forall\, \texttt{int}\ x; (0 \leq x \ \&\ x < \texttt{i} \rightarrow \texttt{a}[x] = 1)$

Example in JML/Java – Loop.java Demo

```
public int[] a;
/*@ public normal_behavior
  @  ensures (\forall int x; 0<=x && x<a.length; a[x]==1);
  @  diverges true;
  @*/
public void m() {
  int i = 0;
  /*@ loop_invariant
    @  0 <= i && i <= a.length &&
    @  (\forall int x; 0<=x && x<i; a[x]==1);
    @ assignable a[*];
    @*/
  while(i < a.length) {
    a[i] = 1;
    i++;
  }
}
```

Example from an ealier Lecture

$\forall$ **int** x;
  $(x = \mathtt{n} \land x >= 0 \rightarrow$
    $[$ `i = 0; r = 0;`
      `while (i<n) { i = i + 1; r = r + i;}`
      `r=r+r-n;`
    $]\,(\mathtt{r} = x * x)$

How can we prove that the above formula is valid
(i.e., satisfied in all states)?

Example from an ealier Lecture

$\forall$ **int** x;
  $(x = \mathtt{n} \land x >= 0 \rightarrow$
    $[$
```
i = 0; r = 0;
while (i<n) { i = i + 1; r = r + i;}
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Needed Invariant:

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How can we prove that the above formula is valid (i.e., satisfied in all states)?

Needed Invariant:

```
@ loop_invariant
@     i>=0  && i <= n && 2*r == i*(i + 1);
@ assignable \nothing; // no heap locations changed
```

Example from an ealier Lecture

$\forall$ **int** x;
$(x = \mathtt{n} \land x >= 0 \rightarrow$
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```
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```

Demo Loop2.java

Hints

Proving `assignable`

- Invariant rule above assumes that **`assignable`** is correct
  E.g., possible to prove nonsense with incorrect
  `assignable \nothing;`
- Invariant rule of KeY generates proof obligation that ensures correctness of **`assignable`**
  This proof obligation is part of 'Body Preserves Invariant' branch

Hints

Proving `assignable`

- Invariant rule above assumes that **`assignable`** is correct
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  This proof obligation is part of 'Body Preserves Invariant' branch

Setting in the KeY Prover when proving loops w. given invariant

- Loop treatment: Invariant
- Quantifier treatment: No Splits with Progs
- If program contains `*`, `/`: Arithmetic treatment: DefOps
- Is search limit high enough (time out, rule apps.)?
- To prove only partial correctness, add `diverges true;`

Total Correctness

Is the sequent

$$\Longrightarrow [\texttt{i = -1; while (true)\{\}}]\texttt{i} = 4711$$

provable?

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Yes, e.g.,

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@ loop_invariant true;
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```

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Yes, e.g.,

```
@ loop_invariant true;
@ assignable \nothing;
```

With this, correctness of non-terminating loop is provable:

- Invariant trivially initially valid and preserved:
  Initial Case and Preserved Case close immediately
- Negated loop condition is `false`: Use case close immediately

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With this, correctness of non-terminating loop is provable:

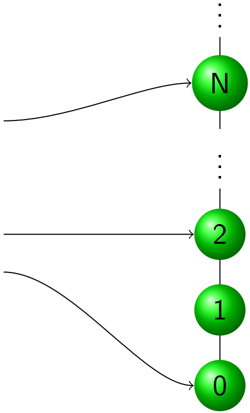
- Invariant trivially initially valid and preserved:
  Initial Case and Preserved Case close immediately
- Negated loop condition is `false`: Use case close immediately

But need a method to prove termination of loops

Mapping Loop Execution to Well-Founded Order

```
while (b) {
  body
}
```

```
if (b) { body }_1
⋮
if (b) { body }_17
if (b) { body }_18
```



Need to find expression getting smaller wrt $\mathbb{N}$ in each iteration

Such an expression is called a decreasing term or variant

Total Correctness: Decreasing Term (Variant)

Find a decreasing integer term v (called variant)

Add the following premisses to the invariant rule:

- $v \geq 0$ is initially valid
- $v \geq 0$ is preserved by the loop body
- v is *strictly* decreased by the loop body

Total Correctness: Decreasing Term (Variant)

Find a decreasing integer term v (called variant)

Add the following premisses to the invariant rule:

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Proving termination in JML/Java

- Remove `diverges true;` from contract
- Add `decreasing v;` to loop invariant
- KeY creates suitable invariant rule and PO (with $\langle \ldots \rangle \phi$)

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Example (The `array` loop)

```
@ decreasing
```

Total Correctness: Decreasing Term (Variant)

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Example (The `array` loop)

```
@ decreasing  a.length - i;
```

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Example (The `array` loop)

```
@ decreasing  a.length - i;
```

Files:

- `LoopT.java`
- `Loop2T.java`

Final Example: Computing the GCD(see 16.3.8 [KeYbook])

```
public class Gcd {
 /*@ public normal_behavior
   @ requires _small>=0 && _big>=_small;
   @ ensures _big!=0 ==>
   @  (_big % \result == 0 && _small % \result == 0 &&
   @    (\forall int x; x>0 && _big % x == 0
   @       && _small % x == 0; \result % x == 0));
   @ assignable \nothing;
   @*/
 private static int gcdHelp(int _big, int _small) {
   int big = _big; int small = _small;
   while (small != 0) {
     final int t = big % small;
     big = small;
     small = t;
   }
   return big;
 }
}
```

Computing the GCD: Method Specification

```
public class Gcd {
 /*@ public normal_behavior
   @ requires _small>=0 && _big>=_small;
   @ ensures _big!=0 ==>
   @ (_big % \result == 0 && _small % \result == 0 &&
   @   (\forall int x; x>0 && _big % x == 0
   @      && _small % x == 0; \result % x == 0));
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 private static int gcdHelp(int _big, int _small) {...}
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   @     && _small % x == 0; \result % x == 0));
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 private static int gcdHelp(int _big, int _small) {...}
```

requires normalization assumptions on method parameters (both non-negative and `_big` $\geq$ `_small`)

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ensures if `_big` positive, then

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   @     && _small % x == 0; \result % x == 0));
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 private static int gcdHelp(int _big, int _small) {...}
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requires normalization assumptions on method parameters (both non-negative and `_big` $\geq$ `_small`)

ensures if `_big` positive, then

- the return value `\result` is a divisor of both arguments

Computing the GCD: Method Specification

```
public class Gcd {
 /*@ public normal_behavior
   @ requires _small>=0 && _big>=_small;
   @ ensures _big!=0 ==>
   @ (_big % \result == 0 && _small % \result == 0 &&
   @   (\forall int x; x>0 && _big % x == 0
   @      && _small % x == 0; \result % x == 0));
   @ assignable \nothing;
   @*/
 private static int gcdHelp(int _big, int _small) {...}
```

requires normalization assumptions on method parameters (both non-negative and `_big` $\geq$ `_small`)

ensures if `_big` positive, then

- the return value `\result` is a divisor of both arguments
- all other divisors `x` of the arguments are also dividers of `\result` and thus smaller or equal to `\result`

Computing the GCD: Specify the Loop Body

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Which locations are changed (at most)?

Computing the GCD: Specify the Loop Body

```
int big = _big; int small = _small;
while (small != 0) {
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  big = small;
  small = t;
}
return big;
```

Which locations are changed (at most)?

```
@ assignable \nothing; // no heap locations changed
```

What is the variant?

Computing the GCD: Specify the Loop Body

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
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}
return big;
```

Which locations are changed (at most)?

```
@ assignable \nothing; // no heap locations changed
```

What is the variant?

```
@ decreases small;
```

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Possible for `big` to become 0 in a loop iteration?

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Possible for `big` to become 0 in a loop iteration? No.

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Adding `big>0` to loop invariant?

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Adding `big>0` to loop invariant? No. Not initially valid.

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Weaker condition necessary: `big==0 ==> _big==0`

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
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  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Weaker condition necessary: `big==0 ==> _big==0`
- What does the loop preserve?

Computing the GCD: Specify the Loop Body Cont'd

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int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
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```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Weaker condition necessary: `big==0 ==> _big==0`
- What does the loop preserve? The set of dividers!
  All common dividers of `_big, _small` are also dividers of `big, small`

Computing the GCD: Specify the Loop Body Cont'd

```
int big = _big; int small = _small;
while (small != 0) {
  final int t = big % small;
  big = small;
  small = t;
}
return big;
```

Loop Invariant

- Order between `small` and `big` preserved by loop: `big>=small`
- Weaker condition necessary: `big==0 ==> _big==0`
- What does the loop preserve? The set of dividers!

  All common dividers of `_big, _small` are also dividers of `big, small`

```
(\forall int x; x > 0;
                  (_big%x == 0 && _small%x == 0)
                  <==>
                  (big%x == 0 && small%x == 0));
```

Computing the GCD: Final Specification

```
int big = _big; int small = _small;
/*@ loop_invariant small >= 0 && big >= small &&
  @   (big == 0 ==> _big == 0) &&
  @   (\forall int x; x > 0; (_big % x == 0 && _small % x == 0)
  @                          <==>
  @                          (big % x == 0 && small % x == 0));
  @ decreases small;
  @ assignable \nothing;
  @*/
  while (small != 0) {
    final int t = big % small;
    big = small;
    small = t;
  }
  return big; // assigned to \result
```

Computing the GCD: Final Specification

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int big = _big; int small = _small;
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```

Why does big divides _small and _big follow from the loop invariant?

Computing the GCD: Final Specification

```
int big = _big; int small = _small;
/*@ loop_invariant small >= 0 && big >= small &&
  @   (big == 0 ==> _big == 0) &&
  @   (\forall int x; x > 0; (_big % x == 0 && _small % x == 0)
  @                          <==>
  @                          (big % x == 0 && small % x == 0));
  @ decreases small;
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  @*/
  while (small != 0) {
    final int t = big % small;
    big = small;
    small = t;
  }
  return big; // assigned to \result
```

Why does `big` divides `_small` and `_big` follow from the loop invariant?

If `big` is positive, one can instantiate `x` with it, and use `small == 0`

Computing the GCD: Demo

Demo `loops/Gcd.java`

1. Show `Gcd.java` and `gcd(a,b)`
2. Ensure that "DefOps" and "Contract" is selected, $\geq$ 10,000 steps
3. Proof contract of `gcd()`, using contract of `gcdHelp()`
4. Note KeY check sign in parentheses:
   4.1 Click "Proof Management"
   4.2 Choose tab "By Proof"
   4.3 Select proof of `gcd()`
   4.4 Select used method contract of `gcdHelp()`
   4.5 Click "Start Proof"
5. After finishing proof obligations of `gcdHelp()` parentheses are gone

Some Hints On Finding Invariants

General Advice

- Invariants must be developed, they don't come out of thin air!
- Be as systematic in deriving invariants as when debugging a program

Some Hints On Finding Invariants, Cont'd

Technical Hints

- Good starting point: desired postcondition (of the loop!)
  - What, in addition to negated loop guard, is needed for it to hold?

Some Hints On Finding Invariants, Cont'd

Technical Hints

- Good starting point: desired postcondition (of the loop!)
  - What, in addition to negated loop guard, is needed for it to hold?
- If the invariant candidate is not preserved by the loop body:
  - Can you add stuff from the precondition?
  - Does it need strengthening?
  - Try to express the relation between partial and final result

Some Hints On Finding Invariants, Cont'd

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- If the invariant is not initially valid:
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  - Did you forget an assumption in the requires clause?

Some Hints On Finding Invariants, Cont'd

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- Several "rounds" of weakening/strengthening might be required

Some Hints On Finding Invariants, Cont'd

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  - What, in addition to negated loop guard, is needed for it to hold?
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  - Try to express the relation between partial and final result
- Simulate a few loop body executions to discover invariant patterns
- If the invariant is not initially valid:
  - Can it be weakened such that the postcondition still follows?
  - Did you forget an assumption in the requires clause?
- Several "rounds" of weakening/strengthening might be required
- Use the KeY tool to iteratively try invariants:
  - Loop treatment: None
  - apply **Loop Invariant** $\rightarrow$ **Enter Loop Specification**
  - After each change of invariant make sure all cases are ok
  - If not, prue and retry

Understanding Unclosed Proofs (see also p.528ff [KeYbook])

Reasons why a proof may not close

- Buggy or incomplete specification
- Bug in program
- Maximal number of steps reached: restart or increase # of steps
- Automatic proof search fails: manual rule applications necessary

Understanding Unclosed Proofs (see also p.528ff [KeYbook])

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- Bug in program
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- Automatic proof search fails: manual rule applications necessary

Understanding open proof goals

- Follow the control flow from the proof root to the open goal
- Branch labels give useful hints
- Identify unprovable part of post condition or invariant
- Sequent remains always in "pre-state"
  Constraints on program variables refer to value at start of program
  (exception: formula is behind update or modality)
- NB: $\Gamma \Longrightarrow o = \texttt{null}, \Delta$ is equivalent to $\Gamma, o \neq \texttt{null} \Longrightarrow \Delta$

Literature for this Lecture

KeYbook *W. Ahrendt*, B. Beckert, R. Bubel, R. Hähnle, P. Schmitt, M. Ulbrich, editors.
Deductive Software Verification - The KeY Book
Vol 10001 of *LNCS*, Springer, 2016
(E-book at `link.springer.com`)

- *W. Ahrendt*, S. Grebing, Using the KeY Prover
  Chapter 15 in [KeYbook], p.528ff + Section 15.3 (also for Lab2)
- B. Beckert, R. Hähnle, M. Hentschel, P.H. Schmitt,
  Formal Verification with KeY: A Tutorial
  Chapter 16 in [KeYbook], except Section 16.6

further reading:

- B. Beckert, V. Klebanov, B. Weiß, Dynamic Logic for Java
  Chapter 3 in [KeYbook], Section 3.7

Master's Thesis Projects in Formal Methods

Examples:

- Extracting tests from runtime traces
- JM^2L: a library of specification models for Java
- Tests from failed proofs
- Dynamic invariants for runtime verification
- Runtime verification of reactive systems
- Compositional runtime enforcement of reactive systems
- Learning automata (properties) from example traces
- Application of runtime monitoring for automotive systems
- Malware on Chrome: Modifying Secure Preferences
- Browser Extensions Metadata Correlation
- Chromium Modification to Parallelise Browser Extensions
- Various master theses on Blockchain, Bitcoin, and Smart Contracts

Master's Thesis Projects in Formal Methods

see Formal Methods Master Theses on the web (cklick here).
(and come back, many edits on that page these days)